

Homework Problem 1.1  
(10 points)Due Tuesday, January 31  
(at lecture)

## 1.1 Two-bit classical gates.

(a) Show that the most general two-bit classical gate can be written as

$$\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \oplus M \begin{pmatrix} x \\ y \end{pmatrix} \oplus \begin{pmatrix} p \\ q \end{pmatrix} xy ,$$

where  $a$ ,  $b$ ,  $p$ ,  $q$ , and the four entries in the  $2 \times 2$  matrix  $M$  can be either 0 or 1.(b) Show that the most general *reversible* two-bit classical gate can be written as

$$\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \oplus M \begin{pmatrix} x \\ y \end{pmatrix} ,$$

where  $a$  and  $b$  can be either 0 or 1, and where  $M$  can be any of the following matrices:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} , \quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \quad \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} , \quad \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} , \quad \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} , \quad \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} .$$