

Homework Problem 2.1
(10 points)

Due Tuesday, February 28
(at lecture)

2.1 Transformation of Pauli products by C-NOT and C-PHASE. Let C denote either the controlled-NOT gate or the controlled-PHASE gate, with the first qubit as control and the second qubit as target. In this problem we investigate how C-NOT and C-PHASE transform Pauli products.

(a) Use circuit identities to derive the following transformations for $C = \text{C-NOT}$:

$$C(X \otimes I)C^\dagger = X \otimes X$$

$$C(I \otimes X)C^\dagger = I \otimes X$$

$$C(Z \otimes I)C^\dagger = Z \otimes I$$

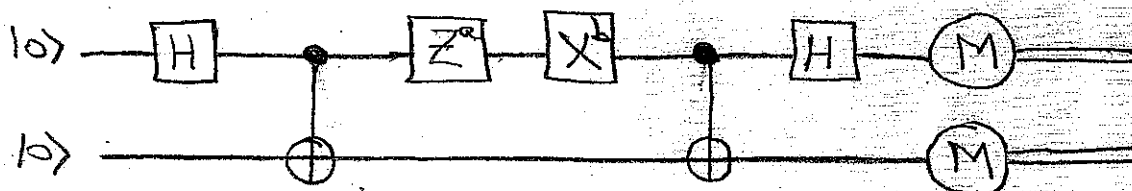
$$C(I \otimes Z)C^\dagger = Z \otimes Z.$$

(b) Use the transformations of part (a) to derive how the C-NOT transforms all Pauli products.

(c) Repeat parts (a) and (b) for the C-PHASE gate.

Notice that both C-NOT and C-PHASE take Pauli products to Pauli products.

(d) The circuit for superdense coding is shown below.



Use the transformation properties of the C-NOT to transform this circuit to an equivalent form for which it is obvious that the measurements yield the values of a and b .