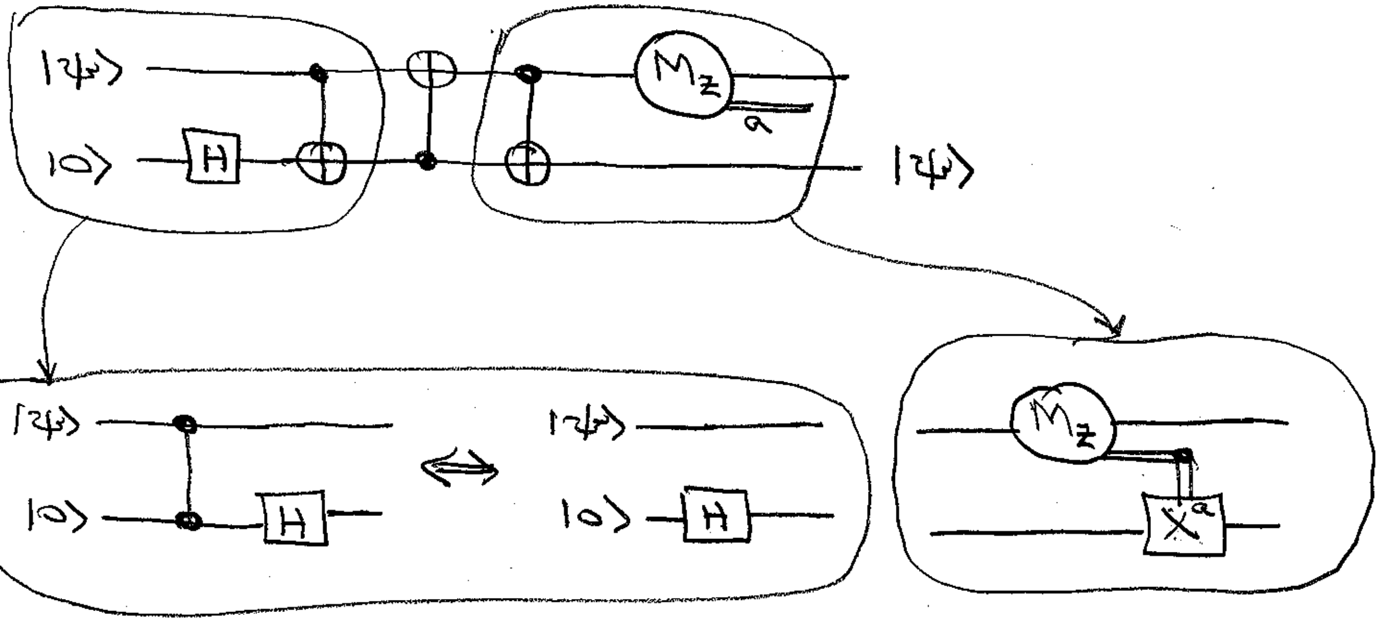
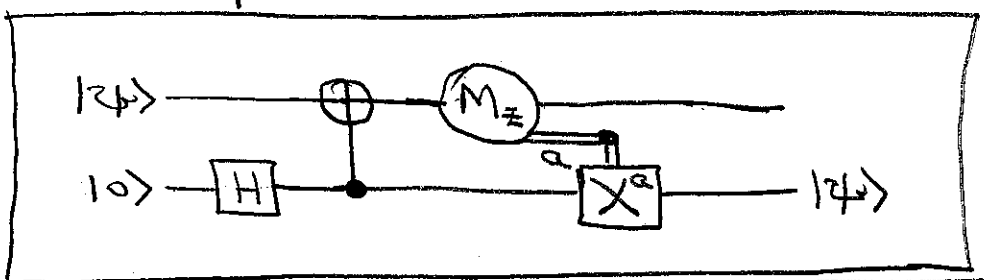
2.2 (a)



The top circuit is equivalent to



Check: that bottom circuit works:

$$|\psi\rangle\otimes|0\rangle = \sum_b c_b|b,0\rangle \xrightarrow[I\otimes H]{} \frac{1}{\sqrt{2}}\sum_{b,c} c_b|b,c\rangle \xrightarrow[V(X)]{} \frac{1}{\sqrt{2}}\sum_{b,c} c_b|b\oplus c, c\rangle$$

$$= \frac{1}{\sqrt{2}}\sum_{b,c} c_b|b,\bar{c}\rangle$$

$$= \frac{1}{\sqrt{2}}\sum_{b,c} c_{b\oplus c}|b,c\rangle$$

$$= \frac{1}{\sqrt{2}}|0\rangle\otimes(c_0|0\rangle + c_1|1\rangle) + \frac{1}{\sqrt{2}}|1\rangle\otimes(c_1|0\rangle + c_0|1\rangle)$$

$$\Big\downarrow M_z$$

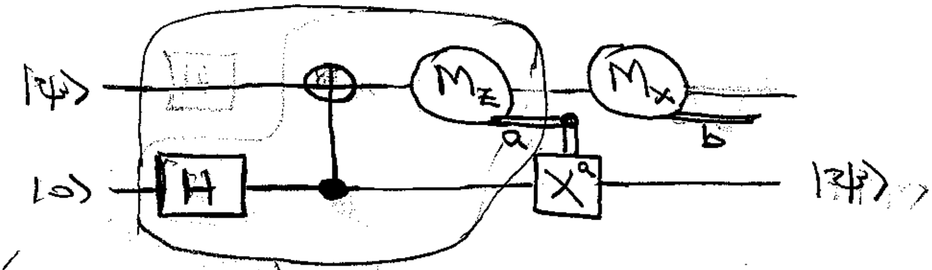
$$|a\rangle\otimes\sum_c c_{a\oplus c}|c\rangle$$

$$\Big\downarrow X^a$$

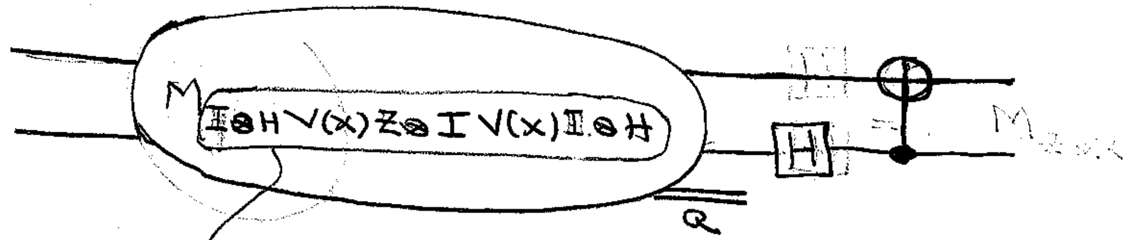
$$|a\rangle\otimes\sum_c c_{a\oplus c}|a\oplus c\rangle$$

$$= |a\rangle\otimes\sum_b c_b|b\rangle$$

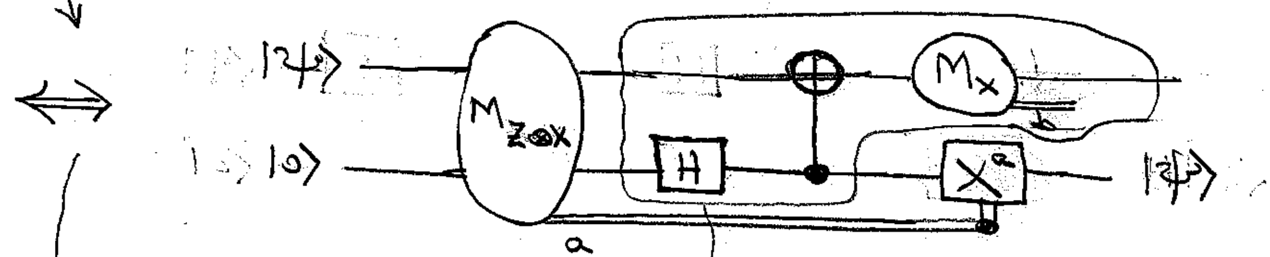
(b)

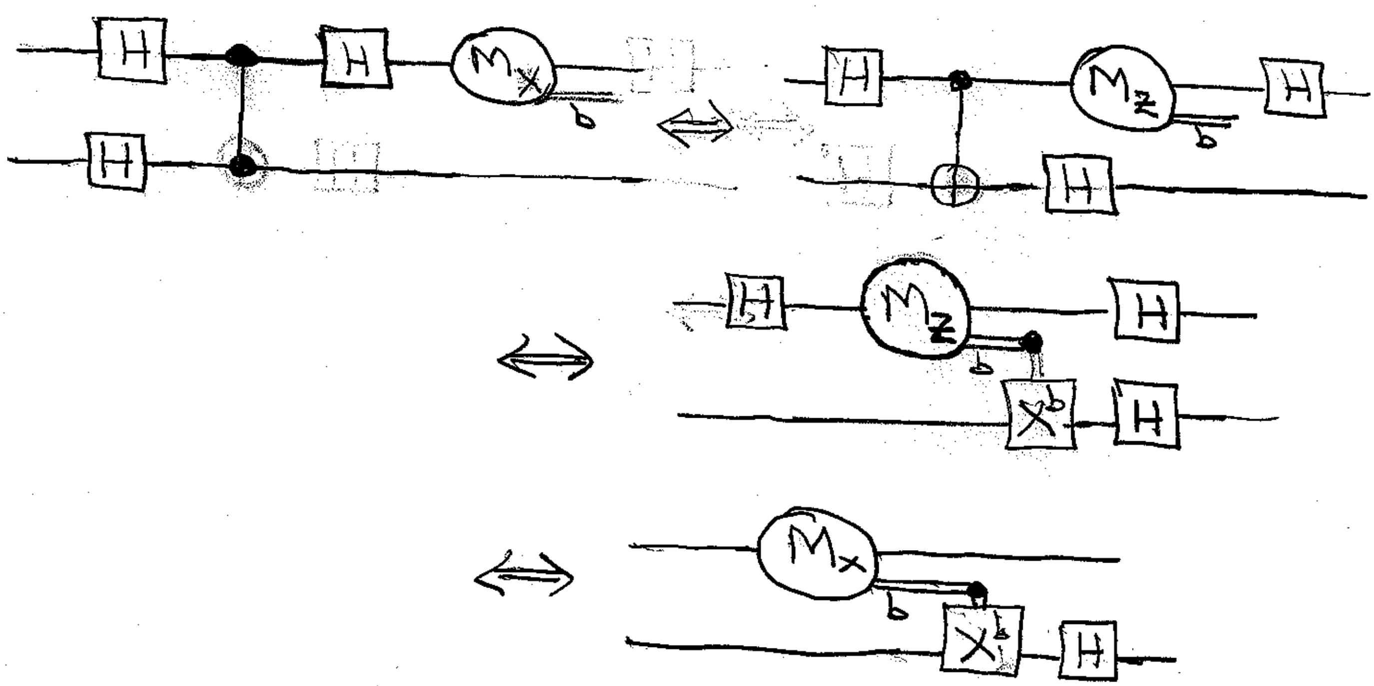


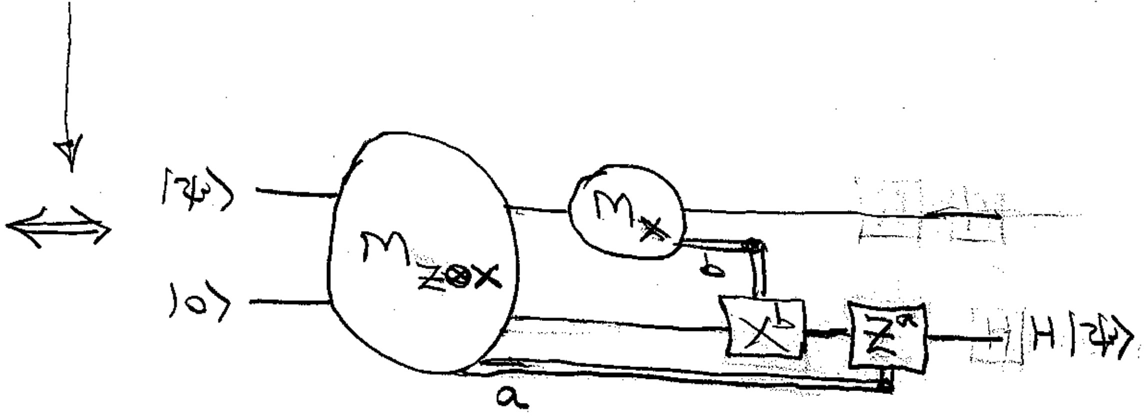
$V^\dagger = V(X)\, I \otimes H$



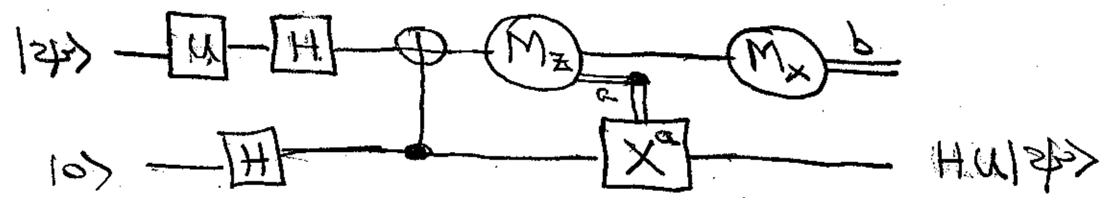
$I\otimes H \; Z\otimes Z \; I\otimes H = Z\otimes X$

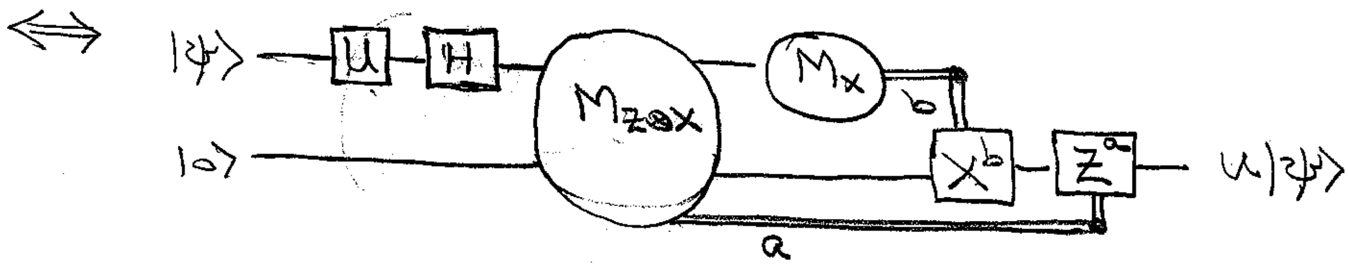


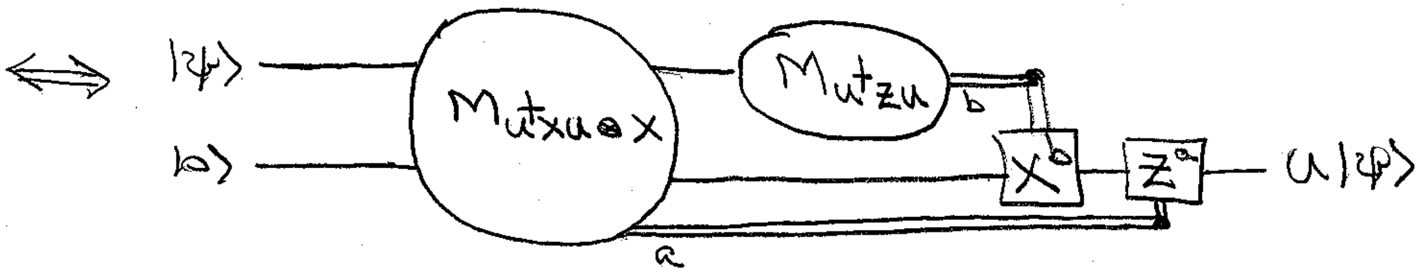




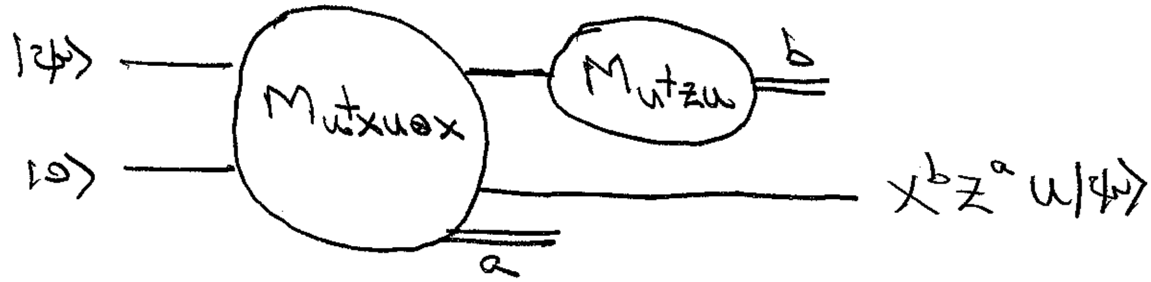
To do a unitary, we actually have to do HU at the input to get rid of the H at the output.







Leaving out the Pauli corrections we have our single-qubit-measurement circuit up to Pauli errors.



This is the same measurement circuit we got in the lectures by a different route.