1.2 **The Monty Hall problem.** On the ancient television show *Let's Make a Deal*, the host, Monty Hall, would show a contestant three doors. Behind one door was the grand prize; behind the other two was nothing. The contestant chose one of the doors. Monty then opened one of the remaining doors, revealing that it hid nothing. The contestant was then offered the opportunity to stick with his original choice or to switch to the one remaining door. Should he switch?

Let's do a generalization of this problem: there are N doors, the contestant guesses one, Monty opens n of the other doors, showing that none of them hides the prize, and the contestant is given the chance to switch to any of the $N - n - 1$ remaining doors.

(a) What is the probability $p(R)$ that the contestant's initial guess is right and the probability $p(W)$ that the initial guess is wrong.

Now consider the situation after Monty's revelation. Let g denote the door of the contestant's initial guess, and let r denote any one of the remaining doors.

(b) Given that the initial guess is right, what is the conditional probability $p(r|R)$ that the prize lies behind door r and the conditional probability $p(g|R)$ that the prize lies behind door g? What are the corresponding conditional probabilities, $p(r|W)$ and $p(g|W)$, given the that the initial guess is wrong?

(c) Since the contestant doesn't know whether his initial guess is right or wrong, what is relevant to his decision are the unconditioned probabilities $p(r)$ and $p(g)$. What are $p(r)$ and $p(g)$? For what values of n should the contestant change his initial guess to one of the remaining doors?