Phys 572 Quantum Information Theory **Fall 2014**

Homework Problem 2.2 **Discussion Friday, September 19**

2.2 Nielsen and Chuang Exercise 2.39 **(The Hilbert-Schmidt inner product on operators)**. The set L_V of linear operators on a Hilbert space V is obviously a vector space: the sum of two linear operators is a linear operator, zA is a linear operator if A is a linear operator and z is a complex number, and there is a zero element 0. An important additional result is that the vector space L_V can be given a natural inner product structure, turning it into a Hilbert space.

(1) *Show* that the function $(\cdot, \cdot)$ on $L_V \times L_V$ defined by

$$(A, B) \equiv \mathrm{tr}(A^\dagger B)$$

is an inner product function. This inner product is known as the *Hilbert-Schmidt* or *trace* inner product.

(2) If V has d dimensions, *show* that L_V has dimension d^2.

(3) *Find* an *orthonormal* basis of Hermitian matrices for the Hilbert space L_V.

plus

(4) *Find* an operator basis of d^2 pure states, i.e., one-dimensional projectors $P_\alpha = |\psi_\alpha\rangle\langle\psi_\alpha|$, $\alpha = 1, \ldots, d^2$, which are linearly independent and thus span the space of operators. Is it possible for such a basis to be orthonormal?