   5.2 **From SWAP to teleportation.** The SWAP circuit shown below exchanges Alice and Bob's quantum states.
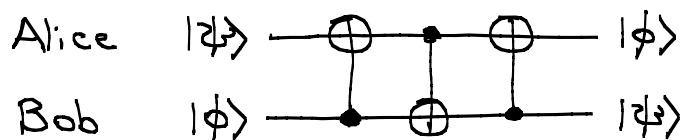

If Bob's qubit starts in the state $|0\rangle$, the first CNOT has no effect and can thus be deleted, leaving the circuit below, which has the same effect as teleportation in that the state $|\psi\rangle$ of Alice's qubit is transferred to Bob's qubit.
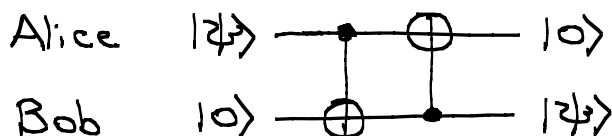
The reason this isn't teleportation is that Alice and Bob's qubits must directly interact during the process. In teleportation, quantum interaction between Alice and Bob is restricted to the interaction that creates the initial shared entanglement; during the teleportation process, Alice and Bob interact only when Alice sends Bob two bits of classical information. The goal of this problem is to show how, by introducing a third "middle" qubit, the circuit immediately above can be transformed through a series of steps to the teleportation circuit below.
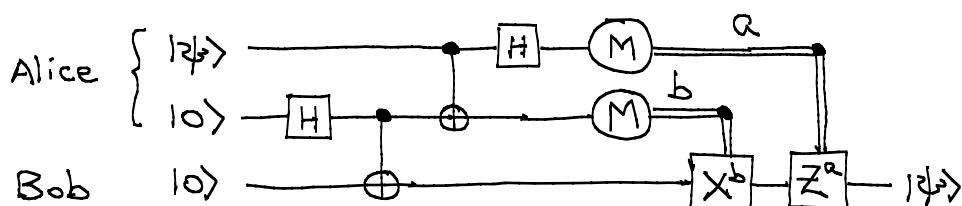
   (a) The first step in the transformation is to use the following circuit identity to replace the initial CNOT once you have introduced the middle qubit.
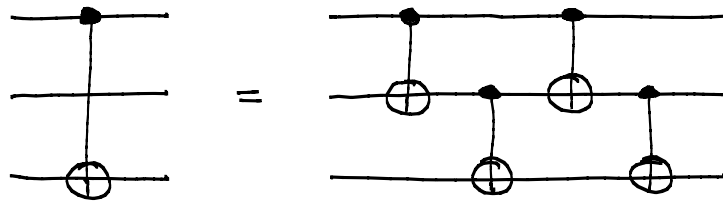
Show that this circuit identity is correct.

(b) *Carry out* the rest of the transformation to the teleportation circuit. This isn't easy, but if you succeed, you will have gained a better understanding of how to manipulate quantum circuit diagrams.