Homework Problem 7.2 Discussion Saturday, November 22

7.2 **Minimum error probability for two mixed states.** Consider two mixed states, ρ_1 and ρ_2, occurring with probabilities q_1 and q_2. Let E_1 and E_2 make up a two-outcome POVM such that on occurrence of outcome 1, you decide the state was ρ_1, and on outcome 2, you decide on ρ_2.

(a) *Show* that the error probability is

$$P_e = q_1 - \mathrm{tr}\big(E_1(q_1\rho_1 - q_2\rho_2)\big) \ .$$

(b) *Show* that

$$\max_{0\le E\le 1} \mathrm{tr}\big(E(q_1\rho_1 - q_2\rho_2)\big) = \frac{1}{2}\mathrm{tr}\big(|q_1\rho_1 - q_2\rho_2|\big) + \frac{1}{2}(q_1 - q_2) \ .$$

(c) Use the results of parts (a) and (b) to *show* that the minimum error probability is

$$(P_e)_{\min} = \frac{1}{2} - \frac{1}{2}\mathrm{tr}\big(|q_1\rho_1 - q_2\rho_2|\big) \ ,$$

and *find* a POVM that gives the minimum error probability.

(d) *Determine* the minimum error probability when the two states are pure, i.e., $\rho_1 = |\psi_1\rangle\langle\psi_1|$ and $\rho_2 = |\psi_2\rangle\langle\psi_2|$.