7.3 **Unambiguous state discrimination for two pure states with different probabilities.** If you know that a quantum system has one of two states, $|\psi_1\rangle$ or $|\psi_2\rangle$, there is a three-outcome measurement, two of whose outcomes conclusively identify one state or the other, but whose third outcome is inconclusive. If the states have equal probabilities of 1/2, the minimum probability for the inconclusive outcome is

$$(P_{\rm nd})_{\rm min} = |\langle\psi_1|\psi_2\rangle|\ ,$$

where "nd" stands for "no decision."

In this problem you generalize this result to the case of unequal prior probabilities, q_1 and q_2, for the two states. Without loss of generality, we assume that $q_1 \geq q_2$. *Find* the minimum no-decision probability $(P_{\rm nd})_{\rm min}$. [Hint: The answer here is more complicated than the minimum error probability for the same situation; you should find that $(P_{\rm nd})_{\rm min}$ is not an analytic function of $|\langle\psi_1|\psi_2\rangle|$, except when $q_1 = q_2$.]