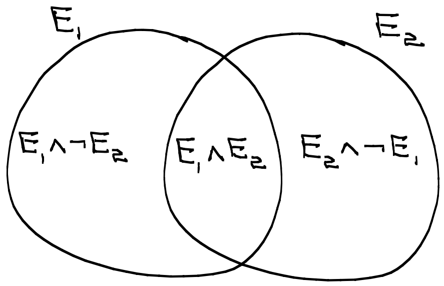
Phys 572 Quantum Information Theory Spring 2017

Homework Problem 1.1

Probabilities and Venn diagrams. The logical calculus is equivalent to the calculus of set-theoretic operations, so we can draw a Venn diagram (below) for two events (or alternatives), E_1 and E_2, which are not necessarily exclusive, i.e., $E_1 \wedge E_2 \neq \emptyset$.



Both circles: $E_1 \vee E_2$

Complement of both circles:

$\neg(E_1 \vee E_2) = \neg E_1 \wedge \neg E_2$

The four probability axioms are the following:

1. $0 \leq p(E) \leq 1$.
2. E is certain $\Longleftrightarrow P(E) = 1$.
3. $p(E_1 \vee E_2) = p(E_1) + p(E_2)$ if $E_1 \wedge E_2 = \emptyset$.
4. Bayes's theorem: $p(E_1 \wedge E_2) = p(E_1|E_2)p(E_2)$.

These axioms mean that probabilities are a measure on the space of events, so we should be able to regard the probability of an event as being represented by the area an event occupies in the Venn diagram. Thus it should be true that

$$p(E_1 \vee E_2) = p(E_1) + p(E_2) - p(E_1 \wedge E_2)\ .$$

Show this from from the probability axioms.