Phys 572 Quantum Information Theory Spring 2017

Homework Problem 3.5

Consider two systems, A of dimension d_A and B of dimension d_B. An arbitrary joint pure state $|\Psi\rangle$, when expanded in an arbitrary product basis $|e_j, f_k\rangle$, looks like

$$|\Psi\rangle = \sum_{j,k} c_{jk} |e_j, f_k\rangle \ .$$

(a) *Show* how $|\Psi\rangle$ can be brought into Schmidt form by using the singular-value decomposition of the matrix whose entries are c_{jk}, and *find* the Schmidt vectors for the two systems in terms of the unitary matrices involved in the singular-value decomposition.

(b) Now suppose the two systems have the same dimension d. A maximally entangled state of A and B is one such that the marginal density operators are maximally mixed, i.e., $\rho_A = I_A/d$ and $\rho_B = I_B/d$. *Find* the conditions on c_{jk} such that $|\Psi\rangle$ is maximally entangled, and discuss what this means for the singular values of c_{jk} and thus for the Schmidt coefficients.