Phys 572 Quantum Information Theory Homework Problem 3.6

A good example of a non-normal operator is the qubit raising operator

$$\sigma_{+} = \frac{1}{2}(\sigma_{x} + i\sigma_{y}) \leftrightarrow \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \ .$$

(a) Find the eigenvectors and eigenvalues of σ_+ .

(b) Find the polar decomposition of σ_+ . What is the relationship between the polar decomposition and the eigenvectors and eigenvalues?