What are the laws of physics? Resisting reification

Carlton M. Caves

C. M. Caves, C. A. Fuchs, and R. Schack, "Subjective probability and quantum certainty," Studies in History and Philosophy of Modern Physics **38**, 255-274 (2007).

C. G. Timpson, "Quantum Bayesianism: A Study," Studies in History and Philosophy of Modern Physics **39**, 579-609 (2008).

Department of Physics and Astronomy
University of New Mexico

caves@info.phys.unm.edu
http://info.phys.unm.edu/~caves

The laws are out there. Probabilities aren't.

Laws of physics?

Mechanics: $$\dot{q} = \frac{\partial H}{\partial p}\,, \qquad \dot{p} = -\frac{\partial H}{\partial q}$$

Maxwell equations: $$*F^{\alpha\beta}{}_{,\beta} = 0\,, \qquad F^{\alpha\beta}{}_{,\beta} = 4\pi J^{\alpha}$$

Liouville equation: $$\frac{\partial \rho}{\partial t} = \{H, \rho\} = \frac{\partial H}{\partial q}\frac{\partial \rho}{\partial p} - \frac{\partial H}{\partial p}\frac{\partial \rho}{\partial q}$$

von Neumann equation: $$i\frac{d\rho}{dt} = [H, \rho]$$

Open-system dynamics? Diffusion terms?

Some mathematical objects in a scientific theory are our tools; others correspond to reality. Which is which?

Subjective Bayesian probabilities

Oljeto Wash
Southern Utah

Objective probabilities

- Probabilities as frequencies: probability as verifiable fact
  - Probabilities are used routinely for individual systems.
  - Frequencies are observed facts, not probabilities.
  - Bigger sample space: exchangeability.

~~QM: Derivation of quantum probability rule from infinite frequencies?~~

C. M. Caves, R. Schack, ``Properties of the frequency operator do not imply the quantum probability postulate,'' Annals of Physics **315**, 123-146 (2005) [Corrigendum: **321**, 504--505 (2006)].

- Objective chance (propensity): probability as specified fact
  - Some probabilities are ignorance probabilities, but others are specified by the facts of a "chance situation."
  - Specification of "chance situation": same, but different.

objective *chance*

QM: Probabilities from physical law.
Salvation of objective chance?

- Logical probabilities (objective Bayesian): physical symmetry implies probability
  - Symmetries are applied to judgments, not to facts.

Subjective Bayesian probabilities

Category distinction

Facts

Outcomes of *events*
Truth values of *propositions*

Objective

Probabilities

Agent's *degree of belief*
in outcome of an event or
truth of a proposition

Subjective

Facts never imply (nontrivial) probabilities.

Two agents in possession of the same facts can assign different probabilities.

Subjective Bayesian probabilities

Probabilities

Agent's *degree of belief* in outcome of an event or truth of a proposition.

Consequence of ignorance

Agent's *betting odds*

Subjective

$\boxed{\text{Worth \$1 if } E \text{ is True}}$ Ticket price \$q

Agent *A* regards \$q as fair price for the ticket. $\Leftrightarrow$ ***A* assigns *p(E)=q*.**

Dutch-book consistency

***A*'s probability assignments, i.e., ticket prices, are *inconsistent* if they can lead to a sure loss.**

The standard rules for manipulating probabilities are *objective* consequences of requiring consistent betting behavior.

The usual argument: If *A* does not obey the probability rules, she will lose *in the long run.*

Dutch-book argument: If *A* does not obey the probability rules, she will lose *in one shot.*

Dutch-book argument: Rules (i) and (ii)

Rule (i): $0 \le p(E) \le 1$

$\boxed{\text{Worth \$1 if } E \text{ is True}}$ Ticket price $q < 0$

A **is willing to sell ticket for a negative amount.**
Sure loss.

Rule (ii): $p(E) = 1$ if A believes that E is certain to occur.

$\boxed{\text{Worth \$1 if } E \text{ is True}}$ Ticket price $q < 1$

A **is willing to sell ticket, which is definitely worth $1 to her, for less than $1.**
Sure loss.

Dutch-book argument: Rule (iii)

Rule (iii): $p(E \vee F) = p(E) + p(F)$, if A believes that E and F are mutually exclusive.

$\boxed{\text{Worth \$1 if } E \vee F \text{ is True}}$	Ticket price \$q
$\boxed{\text{Worth \$1 if } E \text{ is True}}$	Ticket price \$r
$\boxed{\text{Worth \$1 if } F \text{ is True}}$	Ticket price \$s

***A* would buy the purple ticket for \$q and sell the green tickets for \$r + \$s.**
If *q* > *r* + *s*, sure loss.

Dutch-book argument: Rule (iv)

Rule (iv): $p(E \wedge F) = p(E|F)p(F)$ Bayes's rule

$\boxed{\begin{array}{l}\text{Worth \$1 if } E \wedge F \\ \text{Worth \$}p(E|F) \text{ if } \neg F\end{array}}$	Ticket price $\$p(E|F)$
$\boxed{\text{Worth \$1 if } E \wedge F}$	Ticket price $\$p(E \wedge F)$
$\boxed{\text{Worth \$}p(E|F) \text{ if } \neg F}$	Ticket price $\$p(E|F)p(\neg F)$

Consistency implies

$$p(E|F) = p(E \wedge F) + p(E|F)p(\neg F) .$$

Rule (iv) follows from $p(F) + p(\neg F) = p(F \vee \neg F) = 1$.

Subjective Bayesian probabilities

The standard rules of probability theory are *objective* consequences of requiring consistent betting behavior.

Subjective Bayesian probabilities

Facts in the form of observed data *d* are used to update probabilities via Bayes's rule:

$$\underbrace{\boxed{p(h|d)}}_{\textbf{posterior}} = \frac{\overbrace{\boxed{p(d|h)}}^{\textbf{conditional (model, likelihood)}}\,\boxed{p(h)} \longleftarrow \boxed{\textbf{prior}}}{p(d)}$$

The posterior always depends on the prior, *except* when *d* logically implies h_0:

$$\mathrm{Pr}(d|h) = 0 \text{ for } h \neq h_0 \quad \Longrightarrow \quad \mathrm{Pr}(h_0|d) = 1\ .$$

Facts never determine (nontrivial) probabilities.

Are quantum probabilities subjective?

Bungle Bungle Range
Western Australia

	Classical (realistic, deterministic) world		Quantum world	
State space	Simplex of probabilities for microstates		Convex set of density operators	
State	Extreme point Microstate	Ensemble	Extreme point Pure state State vector	Ensemble Mixed state Density operator
	x_j	$p(x_j)$	$\|\psi\rangle$	$\rho = \sum_j p_j \|\psi_j\rangle\langle\psi_j\|$

Scorecard:

1. Predictions for fine-grained measurements
2. Verification (state determination)
3. State change on measurement
4. Uniqueness of ensembles
5. Nonlocal state change (steering)
6. Specification (state preparation)

Objective	**Subjective**	**Objective**	**Subjective**

	Classical (realistic, deterministic) world		Quantum world	
State space	Simplex of probabilities for microstates		Convex set of density operators	
State	Extreme point Microstate	Ensemble	Extreme point Pure state State vector	Ensemble Mixed state Density operator
Fine-grained measurement	Certainty	Probabilities	Certainty or Probabilities	Probabilities
	x_j	$p(x_j)$	$\|\psi\rangle$	$\rho = \sum_j p_j \|\psi_j\rangle\langle\psi_j\|$

Certainty:
Orthonormal measurement basis that contains $|\psi\rangle$.

Objective	**Subjective**	Objective	**Subjective**

	Classical (realistic, deterministic) world		Quantum world	
State space	Simplex of probabilities for microstates		Convex set of density operators	
State	Extreme point Microstate	Ensemble	Extreme point Pure state State vector	Ensemble Mixed state Density operator
Verification: state determination	Yes	No	No	No

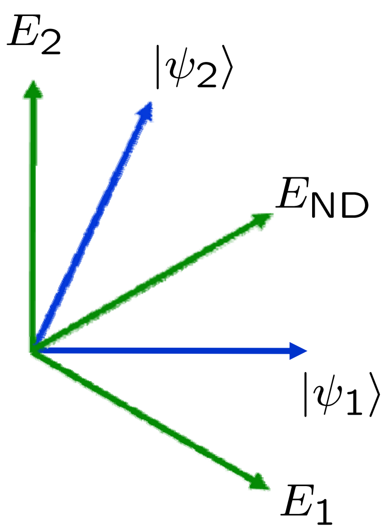
Whom do you ask for the system state? The system or an agent?

	Classical (realistic, deterministic) world		Quantum world	
State space	Simplex of probabilities for microstates		Convex set of density operators	
State	Extreme point Microstate	Ensemble	Extreme point Pure state State vector	Ensemble Mixed state Density operator

Can you reliably distinguish *two nonidentical* states?

iff orthogonal Always	iff orthogonal	iff orthogonal	iff orthogonal

	Classical (realistic, deterministic) world		Quantum world	
State space	**Simplex of probabilities for microstates**		**Convex set of density operators**	
State	Extreme point Microstate	Ensemble	Extreme point Pure state State vector	Ensemble Mixed state Density operator



Can you unambiguously distinguish *two nonidentical* states?

Always $p_{\mathrm{ND}} = 0$	Sometimes (iff supports not identical) $p_{\mathrm{ND}} = \left(\begin{array}{c}\text{average}\\ \text{probability}\\ \text{in support}\\ \text{overlap}\end{array}\right)$	Always (supports are not identical) $p_{\mathrm{ND}} = \lvert\langle\psi_1\vert\psi_2\rangle\rvert$	Sometimes (iff supports not identical) $p_{\mathrm{ND}} = ?$

	Classical (realistic, deterministic) world		Quantum world	
State space	**Simplex of probabilities for microstates**		**Convex set of density operators**	
State	Extreme point Microstate	Ensemble	Extreme point Pure state State vector	Ensemble Mixed state Density operator
Verification: state determination	Yes	No	No	No

Whom do you ask for the system state? The system or an agent?

	Objective	**Subjective**	Objective	**Subjective**

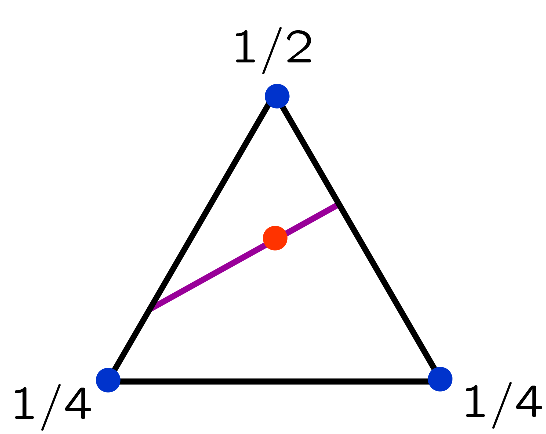
	Classical (realistic, deterministic) world		Quantum world	
State space	Simplex of probabilities for microstates		Convex set of density operators	
State	Extreme point Microstate	Ensemble	Extreme point Pure state State vector	Ensemble Mixed state Density operator
State change on measurement	No	Yes	Yes	Yes

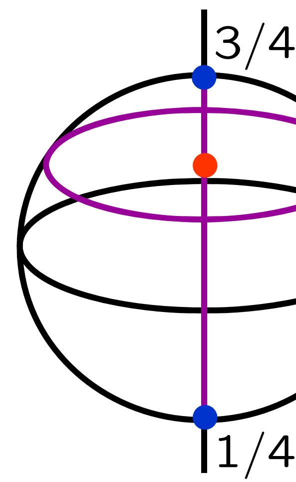
State-vector reduction or wave-function collapse

Real physical disturbance?

Objective	**Subjective**	Subjective	**Subjective**

	Classical (realistic, deterministic) world		Quantum world	
State space	Simplex of probabilities for microstates		Convex set of density operators	
State	Extreme point Microstate	Ensemble	Extreme point Pure state State vector	Ensemble Mixed state Density operator
Uniqueness of ensembles	Yes	No	No	No





	Objective	**Subjective**	Subjective	**Subjective**

	Classical (realistic, deterministic) world		Quantum world	
State space	Simplex of probabilities for microstates		Convex set of density operators	
State	Extreme point Microstate	Ensemble	Extreme point Pure state State vector	Ensemble Mixed state Density operator
Nonlocal state change (steering)	No	Yes	Yes	Yes

$$p_0 = 1/2 \qquad p_1 = 1/2$$

$$p_{0|0} = 3/4 \qquad p_{0|1} = 1/4$$
$$p_{1|0} = 1/4 \qquad p_{1|1} = 3/4$$

$$\begin{aligned} |\Psi\rangle &= \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \\ &= \frac{1}{\sqrt{2}}(|\mathbf{n}, -\mathbf{n}\rangle - |-\mathbf{n}, \mathbf{n}\rangle) \end{aligned}$$

Real nonlocal physical disturbance?

Objective	**Subjective**	**Subjective**	**Subjective**

	Classical (realistic, deterministic) world		Quantum world	
State space	**Simplex of probabilities for microstates**		**Convex set of density operators**	
State	Extreme point Microstate	Ensemble	Extreme point Pure state State vector	Ensemble Mixed state Density operator
Fine-grained measurement	Certainty	Probabilities	Certainty or Probabilities	Probabilities
Verification: state determination	Yes	No	No	No
State change on measurement	No	Yes	Yes	Yes
Uniqueness of ensembles	Yes	No	No	No
Nonlocal state change (steering)	No	Yes	Yes	Yes
Specification: state preparation	Yes	No	?	?
	Objective	**Subjective**	**Subjective**	**Subjective**

Copenhagen vs. Bayes

Truchas from East Pecos Baldy
Sangre de Cristo Range
Northern New Mexico

	Classical (realistic, deterministic) world		Quantum world	
State space	Simplex of probabilities for microstates		Convex set of density operators	
State	Extreme point Microstate	Ensemble	Extreme point Pure state State vector	Ensemble Mixed state Density operator
Specification: state preparation	Yes	No	Copenhagen: Yes	Copenhagen: Yes

Copenhagen interpretation: Classical facts specifying the properties of the preparation device determine a pure state.

Copenhagen (objective preparations view) becomes the home of objective chance, with nonlocal physical disturbances.

Objective	**Subjective**	**Objective**	**Objective**

Copenhagen	Classical (realistic, deterministic) world		Quantum world	
State space	Simplex of probabilities for microstates		Convex set of density operators	
State	Extreme point Microstate	Ensemble	Extreme point Pure state State vector	Ensemble Mixed state Density operator
Fine-grained measurement	Certainty	Probabilities	Certainty or Probabilities	Probabilities
Verification: state determination	Yes	No	No	No
State change on measurement	No	Yes	Yes	Yes
Uniqueness of ensembles	Yes	No	No	No
Nonlocal state change (steering)	No	Yes	Yes	Yes
Specification: state preparation	Yes	No	Yes	Yes
	Objective	**Subjective**	**Objective**	**Objective**

Classical and quantum updating

Facts in the form of observed data *d* are used to update probabilities via Bayes's rule:

conditional (model, likelihood)

prior

$$\boxed{p(h|d)} = \frac{\boxed{p(d|h)}\,\boxed{p(h)}}{p(d)}$$

posterior

The posterior always depends on the prior, *except* when *d* logically implies *h*$_0$:

$$\Pr(d|h) = 0 \text{ for } h \neq h_0$$
$$\Longrightarrow \quad \Pr(h_0|d) = 1 \; .$$

Facts in the form of observed data *d* are used to update quantum states:

quantum operation (model)

prior

$$\boxed{\rho_d} = \frac{\boxed{\mathcal{A}_d}(\boxed{\rho})}{p(d)}$$

posterior

Quantum state preparation:

ρ_d does not depend on ρ.

The posterior state *always* depends on prior beliefs, *even* for quantum state preparation, because there is a judgment involved in choosing the quantum operation.

Facts never determine probabilities or quantum states.

Where does Copenhagen go wrong?

The Copenhagen interpretation forgets that the preparation device is quantum mechanical. A detailed description of the operation of a preparation device (provably) involves prior judgments in the form of quantum state assignments.

It is possible to show that neither deterministic nor stochastic preparation devices can prepare the same system state independent of system and device initial states.

Subjective Bayesian	Classical (realistic, deterministic) world		Quantum world	
State space	Simplex of probabilities for microstates		Convex set of density operators	
State	Extreme point Microstate	Ensemble	Extreme point Pure state State vector	Ensemble Mixed state Density operator
Fine-grained measurement	Certainty	Probabilities	Certainty or Probabilities	Probabilities
Verification: state determination	Yes	No	No	No
State change on measurement	No	Yes	Yes	Yes
Uniqueness of ensembles	Yes	No	No	No
Nonlocal state change (steering)	No	Yes	Yes	Yes
Specification: state preparation	Yes	No	No	No
	Objective	**Subjective**	**Subjective**	**Subjective**

Bayesian quantum probabilities

Echidna Gorge
Bungle Bungle Range
Western Australia

Quantum states vs. probabilities

Are quantum states the same as probabilities? No, though both are subjective, there are differences, but these differences should be stated in Bayesian terms.

A quantum state is a catalogue of probabilities, but the rules for manipulating quantum states are different than for manipulating probabilities.

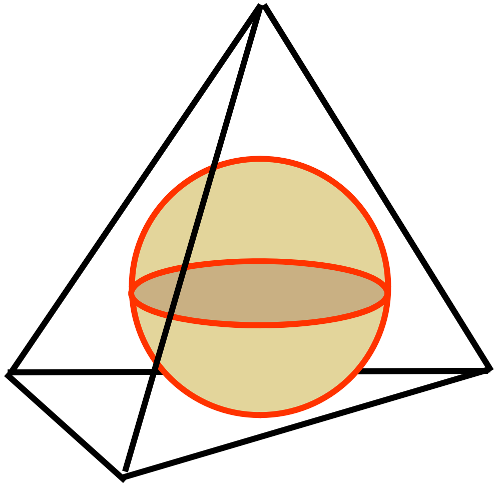
The rules for manipulating quantum states are *objective* consequences of restrictions on how agents interface with the real world.

Catalogue of probabilities: Fuchs's gold standard

Symmetric Informationally Complete (SIC)-POVM

$$E_\alpha = \frac{1}{D}\Pi_\alpha\,, \qquad \alpha = 1,\ldots,D^2 \qquad\qquad \mathrm{tr}(\Pi_\alpha\Pi_\beta) = \frac{1}{D+1}$$

$$p_\alpha = \frac{1}{D}\mathrm{tr}(\rho\Pi_\alpha) \qquad \Longleftrightarrow \qquad \rho = \sum_\alpha \left((D+1)p_\alpha - \frac{1}{D} \right) \Pi_\alpha$$



qubit: $D = 2$

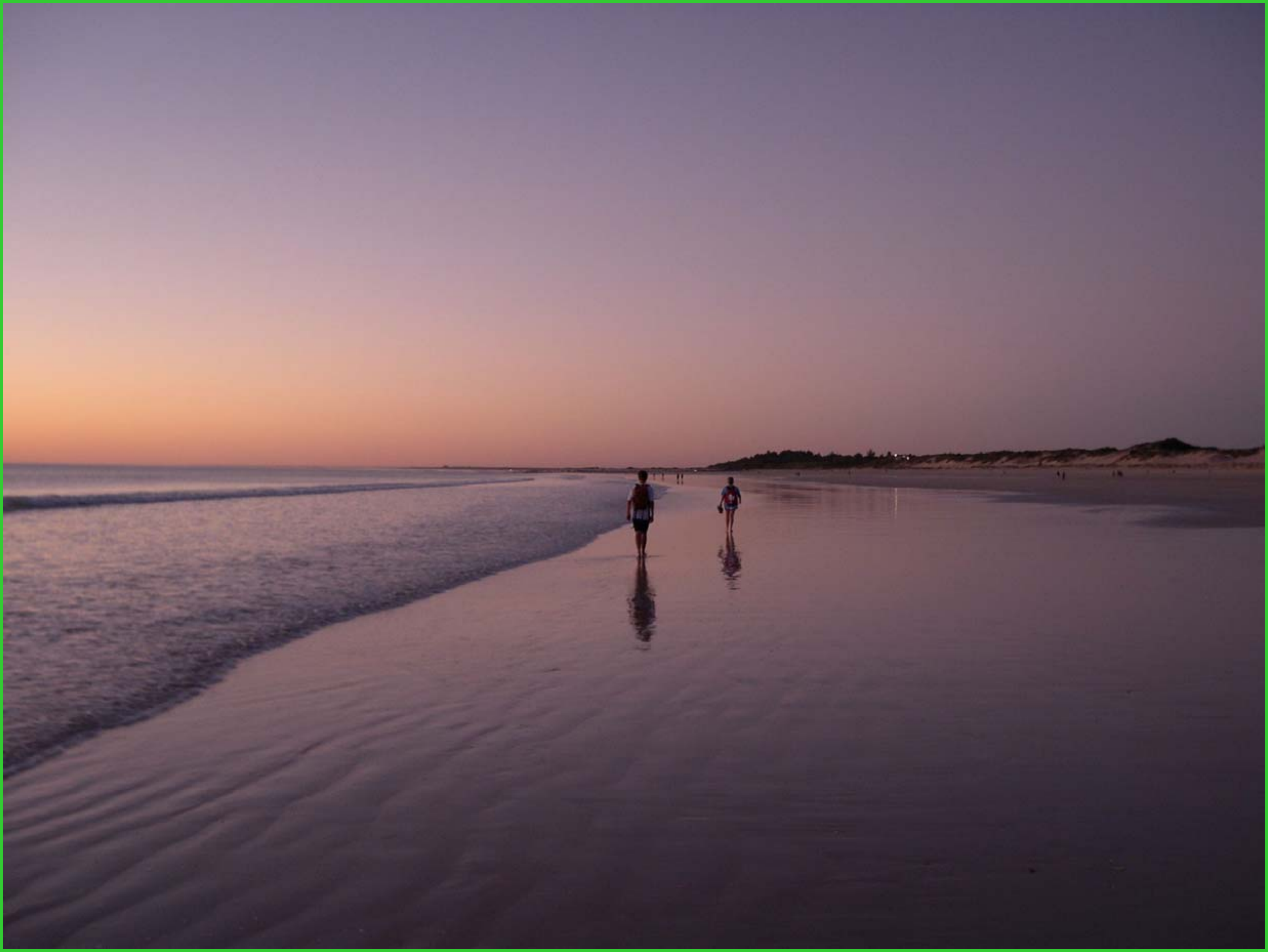
Unitary dynamics

$$\rho' = U\rho U^\dagger \qquad \Longleftrightarrow$$

$$p'_\alpha = -\frac{1}{D} + (D+1)\sum_\beta p_{\alpha|\beta}\, p_\beta$$

$$p_{\alpha|\beta} = \frac{1}{D}\mathrm{tr}(\Pi_\beta U \Pi_\alpha U^\dagger)$$

Quantum coin tossing



Cable Beach
Western Australia

Is a quantum coin toss more random than a classical one? Why trust a quantum random generator over a classical one?

$$|\psi\rangle = |\uparrow\rangle = (|\rightarrow\rangle + |\leftarrow\rangle)/\sqrt{2}$$

Measure spin along *z* axis: $p_\uparrow = 1$ $p_\downarrow = 0$

$\boxed{\text{Measure spin along } x \text{ axis: } \quad p_\rightarrow = 1/2 \quad p_\leftarrow = 1/2}$

C. M. Caves, R. Schack, "Quantum randomness," in preparation.

$\boxed{\text{quantum coin toss}}$

	Classical (realistic, deterministic) world		Quantum world	
State space	**Simplex of probabilities for microstates**		**Convex set of density operators**	
State	Extreme point Microstate	Ensemble	Extreme point Pure state State vector	Ensemble Mixed state Density operator
Fine-grained measurement	Certainty	Probabilities	**Certainty or Probabilities**	Probabilities

Is a quantum coin toss more random than a classical one? Why trust a quantum random generator over a classical one?

$$|\psi\rangle = |{\uparrow}\rangle = (|{\rightarrow}\rangle + |{\rightarrow}\rangle)/\sqrt{2}$$

Measure spin along *z* axis: $p_{\uparrow} = 1$ $p_{\downarrow} = 0$

Measure spin along *x* axis: $p_{\rightarrow} = 1/2$ $p_{\leftarrow} = 1/2$

quantum coin toss

Standard answer: The quantum coin toss is objective, with probabilities guaranteed by physical law.

Subjective Bayesian answer? No inside information.

Pure states and inside information

Party *B* has *inside information* about event *E,* relative to party *A*, if *A* is willing to agree to a bet on *E* that *B* believes to be a sure win. *B* has *one-way inside information* if *B* has inside information relative to *A*, but *A* does not have any inside information relative to *A*.

The unique situation in which *no other party can have one-way inside information* relative to a party *Z* is when *Z* assigns a pure state. *Z* is said to have a *maximal belief structure.*

Subjective Bayesian answer

We trust quantum over classical coin tossing because an agent who believes the coin is fair cannot rule out an insider attack, whereas the beliefs that lead to a pure-state assignment are inconsistent with any other party's being able to launch an insider attack.

A stab at ontology

Cape Hauy
Tasman Peninsula

A stab at ontology

Quantum systems are defined by *attributes*, such as position, momentum, angular momentum, and energy or Hamiltonian. These attributes—and thus the numerical particulars of their eigenvalues and eigenfunctions—are *objective* properties of the system.

The *value* assumed by an attribute is not an objective property, and the *quantum state* that we use to describe the system is purely subjective.

A stab at ontology

1. The attributes orient and give structure to a system's Hilbert space. Without them we are clueless as to how to manipulate and interact with a system.
2. The attributes are unchanging properties of a system, which can be determined from observable facts. The attributes determine the structure of the world.
3. The system Hamiltonian is one of the attributes, playing the special role of orienting a system's Hilbert space now with the same space later.
4. Convex combinations of Hamiltonian evolutions are essentially unique (up to degeneracies).

Why should you (I) care?
If you do care, how can this be made convincing?
Status of quantum operations?
Effective attributes and effective Hamiltonians? "Effective reality"?

Kookaburras in New Mexico