GHZ correlations are just a bit nonlocal

Carlton M. Caves
University of New Mexico
http://info.phys.unm.edu/~caves

Seminar date

Please join the
APS Topical Group on Quantum Information,
Concepts, and Computation
We consider the consequences of the observed violations of Bell’s inequalities. Two common responses are (i) the rejection of realism and the retention of locality and (ii) the rejection of locality and the retention of realism. Here we critique response (i). We argue that locality contains an implicit form of realism, since in a worldview that embraces locality, spacetime, with its usual, fixed topology, has properties independent of measurement. Hence we argue that response (i) is incomplete, in that its rejection of realism is only partial.

R. Y. Chiao and J. C. Garrison
“Realism or Locality: Which Should We Abandon?”
Locality, realism, or nihilism

Locality

No influences between spatially separated parts.
Violation of Bell inequalities.

Realism

Nonlocal HV models for entangled states.
Violation of Bell inequalities.

Nihilism

Local HV models for product states.
Bell inequalities satisfied.
Reductionism or realism

Reductionism
- Things made of parts.
- No influences between noninteracting parts.
- Violation of Bell inequalities.

Realism
- Holistic HV models for entangled states.
- Violation of Bell inequalities.

Reductionist HV models for product states.
- Bell inequalities satisfied.
Reductionism

Things made of parts.
Parts identified by the attributes we can manipulate and measure.
No influences between noninteracting parts.
Attributes do not have realistic values.
Subjective quantum states.

Realism

Holistic realistic account of states, dynamics, and measurements.
Holistic HV models.
Objective quantum states.

Quantum mechanics
or
Stories about a reality beneath quantum mechanics
Why not a different story, one that comes from quantum information science?
The old story

Local realistic description

Product states

Entangled states

Nonlocal realistic description
A new story from quantum information?

Local realistic description
Efficient realistic description
Product states

Globally entangled states

Inefficient realistic description
A new story from quantum information

Local realistic description
Product states

Efficient realistic description

How nonlocal is the realistic description of these states?

Globally entangled states

Realistic description
Modeling GHZ (cat) correlations

Measure XYY, YXY, and YYX: All yield result -1. *Local* realism implies XXX = -1, *but* quantum mechanics says XXX = +1.

Efficient (nonlocal) realistic description of states, dynamics, and measurements

Stabilizer formalism
Modeling GHZ (cat) correlations

\[ ZZI = ZIZ = IZZ = XXX = +1; \ YYY = YXY = YYX = -1. \]

To get correlations right requires 1 bit of classical communication: party 2 tells party 1 whether \( Y \) is measured on qubit 2; party 1 flips her result if \( Y \) is measured on either 1 or 2.

When party 1 flips her result, this can be thought of as a nonlocal disturbance that passes from qubit 2 to qubit 1. The communication protocol quantifies the required amount of nonlocality.
Modeling GHZ (cat) correlations

\[ ZZI = ZIZ = IZZ = XXX = +1; \ YXY = YXY = YYX = -1. \]

To get correlations right requires 1 bit of classical communication: party 2 tells party 1 whether \( Y \) is measured on qubit 2; party 1 flips her result if \( Y \) is measured on either 1 or 2.

GHZ (cat) entangled state

\[ \frac{1}{\sqrt{2}} (|000\rangle + |111\rangle) \]


For \( N \)-qubit GHZ states, this same procedure gives a local realistic description, aided by \( N-2 \) bits of classical communication (provably minimal), of states, dynamics, and measurements (of Pauli products).
Modeling GHZ (cat) correlations

Assume 1 bit of communication between qubits 1 and 2. Let $S=XXII$ and $T=XYII$ be Pauli products for qubits 1 and 2; then we have $SYY=TXY=TYX=-1$. Local realism implies $SXX=-1$, but quantum mechanics says $SXX=+1$.

For $N$-qubit GHZ states, a simple extension of this argument shows that $N-2$ bits of classical communication is the minimum required to mimic the predictions of quantum mechanics for measurements of Pauli products.
Clifford circuits: Gottesman-Knill theorem

- $N$ qubits in an initial product state in $Z$ basis
- Allowed gates: Pauli operators $X$, $Y$, and $Z$, plus $H$, $S$, and C-NOT
- Allowed measurements: Products of Pauli operators

**Global entanglement**

but

Efficient (nonlocal) realistic description of states, dynamics, and measurements (in terms of stabilizer generators)

This kind of global entanglement, when measurements are restricted to the Pauli group, can be simulated efficiently and thus does not provide an exponential speedup for quantum computation.
Graph states

All stabilizer (Clifford) states are related to graph states by \( Z\), Hadamard, and \( S \) gates.

\[
\begin{align*}
|\psi\rangle &= \frac{1}{2\sqrt{2}}( |00\bar{0}\bar{0}\bar{0}\rangle + |01\bar{0}\bar{1}\bar{0}\rangle + |00\bar{1}\bar{1}\bar{1}\rangle + |01\bar{1}\bar{0}\bar{1}\rangle \\
&\quad + |10\bar{1}\bar{1}\bar{0}\rangle - |11\bar{0}\bar{0}\bar{0}\rangle - |10\bar{0}\bar{0}\bar{1}\rangle + |11\bar{0}\bar{1}\bar{0}\rangle )
\end{align*}
\]

\( |\bar{a}\rangle \equiv H|a\rangle = (|0\rangle + (-1)^a|1\rangle)/\sqrt{2} \)
Graph states

4-qubit GHZ graph state

\[ |\psi\rangle = \frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle) \]
Graph states

2 x 2 cluster state

\[ |\psi\rangle = \frac{1}{2}(|0000\rangle + |1100\rangle + |0111\rangle + |1010\rangle) \]
Graph states: LHV model


\[ g_j = X_j \bigotimes_{k \in N(j)} Z_k \]

\[ x_j = \prod_{k \in N(j)} z_k , \]

\[ x_j y_j z_j = \pm 1 \]
Graph states: Nearest-neighbor (single-round) communication protocol

For qubit \( j \), let \( n_j \) be the number of neighbors that measure \( X \) or \( Y \). Certainty (stabilizer element) requires

\[
n_j = \begin{cases} 
0 \mod 2, & \text{if qubit } j \text{ measures } I \text{ or } X, \\
1 \mod 2, & \text{if qubit } j \text{ measures } Z \text{ or } Y.
\end{cases}
\]

\[
g_1 = XZZZZZ \\
g_2 = ZXIZI \\
g_3 = ZIXIZ \\
g_4 = ZZIXZ \\
g_5 = ZIZXX
\]

\[
g_1 g_2 g_5 = -XYIZY
\]

\[
\begin{pmatrix} \text{(overall)} \\ \text{(sign)} \end{pmatrix} = (-1)^{(# \text{ of } X \text{ qubits with } n_j = 2 \mod 4)}
\times (-1)^{(# \text{ of } Y \text{ qubits with } n_j = 3 \mod 4)}.
\]

All that matters is that a qubit measuring \( X(Y) \) doesn’t flip when \( n_j = 0(1) \mod 4 \) and does flip when \( n_j = 2(3) \mod 4 \).
Graph states: Nearest-neighbor communication protocol

\[ g_1 = XZZZZZ \]
\[ g_2 = ZXIZI \]
\[ g_3 = ZIXIZ \]
\[ g_4 = ZZIXZ \]
\[ g_5 = ZIZZX \]

\[ g_1g_2g_5 = -XYIZY \]

Each qubit tells its neighbors if it measures X or Y. A qubit flips its table entry if it measures X or Y and the number of neighbors measuring X or Y is 2, 3 mod 4.

OR

Each qubit tells its neighboring qubits if it measures X or Y. A qubit flips its table entry if it measures X (Y) and the number of neighbors measuring X or Y is 2, 3 mod 4 (0, 3 mod 4)

Site-invariant nearest-neighbor communication protocols
Graph states: Subcorrelations

Each qubit tells its neighbors if it measures $X$ or $Y$. A qubit flips its table entry if it measures $X$ or $Y$ and the number of neighbors measuring $X$ or $Y$ is $2, 3 \mod 4$.

Each qubit tells its neighboring qubits if it measures $X$ or $Y$. A qubit flips its table entry if it measures $X$ ($Y$) and the number of neighbors measuring $X$ or $Y$ is $2, 3 \mod 4$ ($0, 3 \mod 4$).
Graph states: Subcorrelations

Each qubit tells its neighboring qubits if it measures $X$ or $Y$. A qubit flips its table entry if it measures $X$ ($Y$) and the number of neighbors measuring $X$ or $Y$ is 2, 3 mod 4 (0, 3 mod 4)

Certain result: -1

Overall random result
Protocol gets submeasurement wrong.
Graph states: Site invariance and communication distance

A site-invariant protocol cannot introduce an overall sign flip when this measurement is viewed as a submeasurement of the one on the left.

Site-invariant protocols can get all correlations right, but even with unlimited-distance communication, such protocols fail on some subcorrelations for some graphs.
Nonetheless, any protocol with limited-distance communication, site-invariant or not, fails for some graphs; thus for a protocol to be successful for all graphs, it (i) must not be site invariant and (ii) must have unlimited-distance communication.
Graph states: Getting it all right

1. Select a special qubit that knows the adjacency matrix of the graph.

2. Each qubit tells the special qubit if it measures $X$ or $Y$.

3. From the adjacency matrix, the special qubit calculates a generating set of certain submeasurements (stabilizer elements), each of which has a representative qubit that participates in none of the other submeasurements. Since these submeasurements commute term by term, the overall sign for any certain submeasurement is a product of the signs for the participating submeasurements.

4. The special qubit tells each of the representative qubits whether to flip the sign of its table entry.

Non-site-invariant, unlimited-distance communication protocol

A non-site-invariant, unlimited-distance protocol like that for graph states, based on the adjacency matrix of the qubits connected by solid lines, gets everything right.
Stabilizer states

Simulation of Clifford circuits leads to local-complementation rules for generalized graphs subjected to Clifford gates, which can be expressed as powerful circuit identities.

\[ g_1 = XZIZII \]
\[ g_2 = -ZXZIZIX \]
\[ g_3 = IZYZXI \]
\[ g_4 = -ZZZYIX \]
\[ g_5 = IIIZIZI \]
\[ g_6 = -IZIZIZI \]
Stabilizer states

Simulation of Clifford circuits leads to local-complementation rules for generalized graphs subjected to Clifford gates, which can be expressed as powerful circuit identities.

\[
\begin{align*}
g_1 &= XZIZIZI \\
g_2 &= -ZXZZIX \\
g_3 &= IZYZXI \\
g_4 &= -ZZZYIX \\
g_5 &= IIIZIZI \\
g_6 &= -IZIZIZ
\end{align*}
\]
Clifford circuits: Gottesman-Knill theorem

- $N$ qubits in an initial product state in $Z$ basis
- Allowed gates: Pauli operators $X$, $Y$, and $Z$, plus $H$, $S$, and C-NOT
- Allowed measurements: Products of Pauli operators

Global entanglement

but

Efficient (nonlocal) realistic description of states, dynamics, and measurements (in terms of stabilizer generators)

This kind of global entanglement, when measurements are restricted to the Pauli group, can be simulated efficiently because it can be described efficiently by local hidden variables assisted by classical communication.
The problem is that it’s not just dogs, so ...

Quantum information science is the discipline that explores information processing within the quantum context where the mundane constraints of realism and determinism no longer apply. What better way could there be to explore the foundations of quantum mechanics?