Entanglement

Alice and Bob

Fidelity

Teleportation

Nonlocal influences

(a) Paranormal phenomena

(b) Men are from Mars. Women are from Venus

(c) Physics

Classical phase-space descriptions of continuous-variable teleportation

Carlton M. Caves and Krzysztof Wodkiewicz

PRL **69**, 040506 (2004), quant-ph/0401149)

Fidelity of Gaussian channels

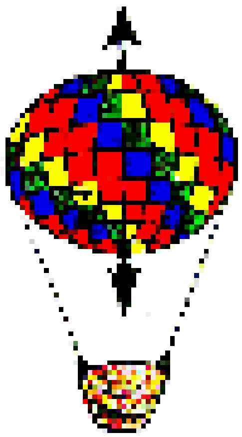
Carlton M. Caves and Krzysztof Wodkiewicz

“Fidelity of Gaussian channels,” Open Syst & Inf Dynamics **11**, 309 (2004)

Teleportation fidelity and sub-Planck phase-space structure

Carlton M. Caves and Andrew J. Scott

in preparation



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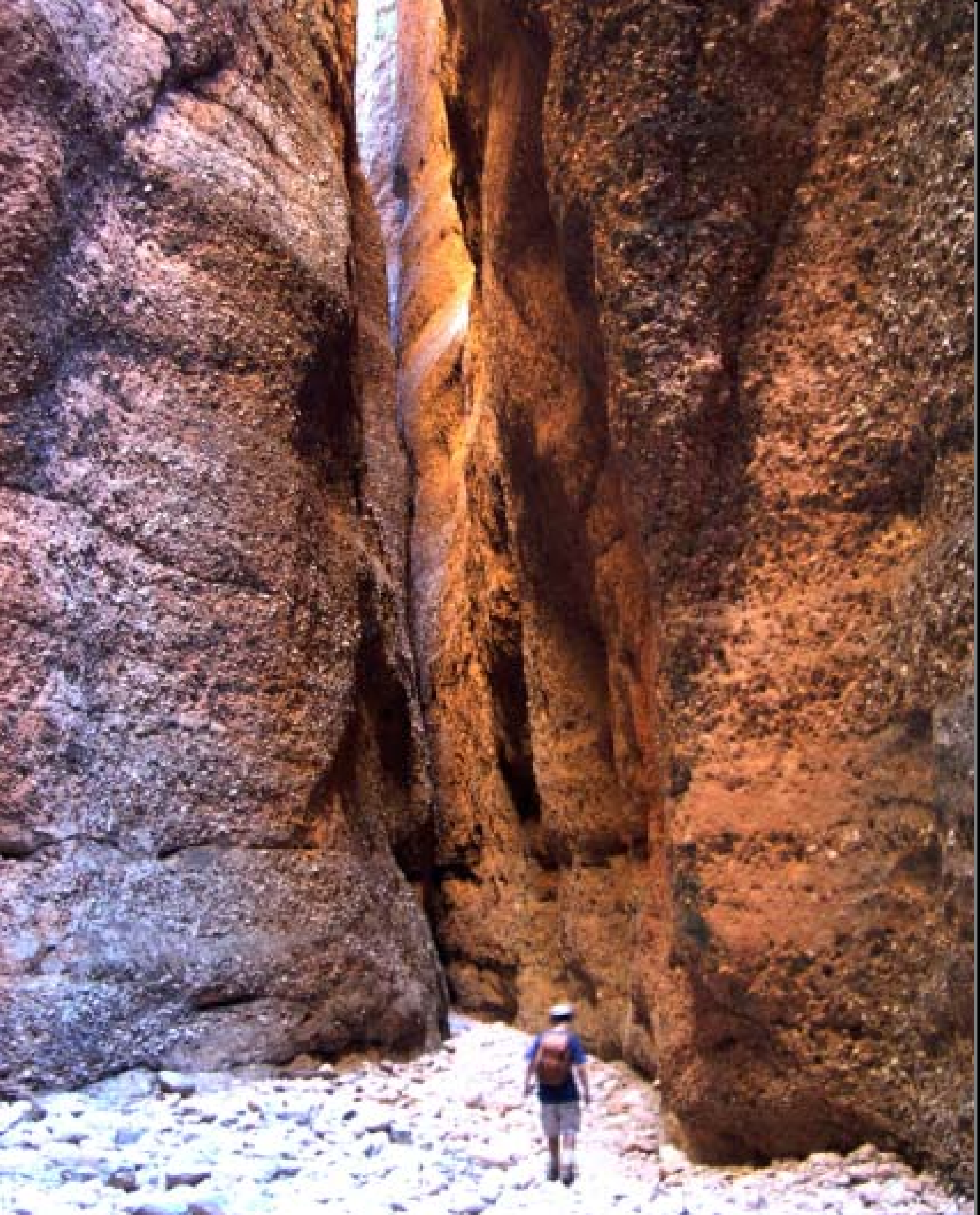


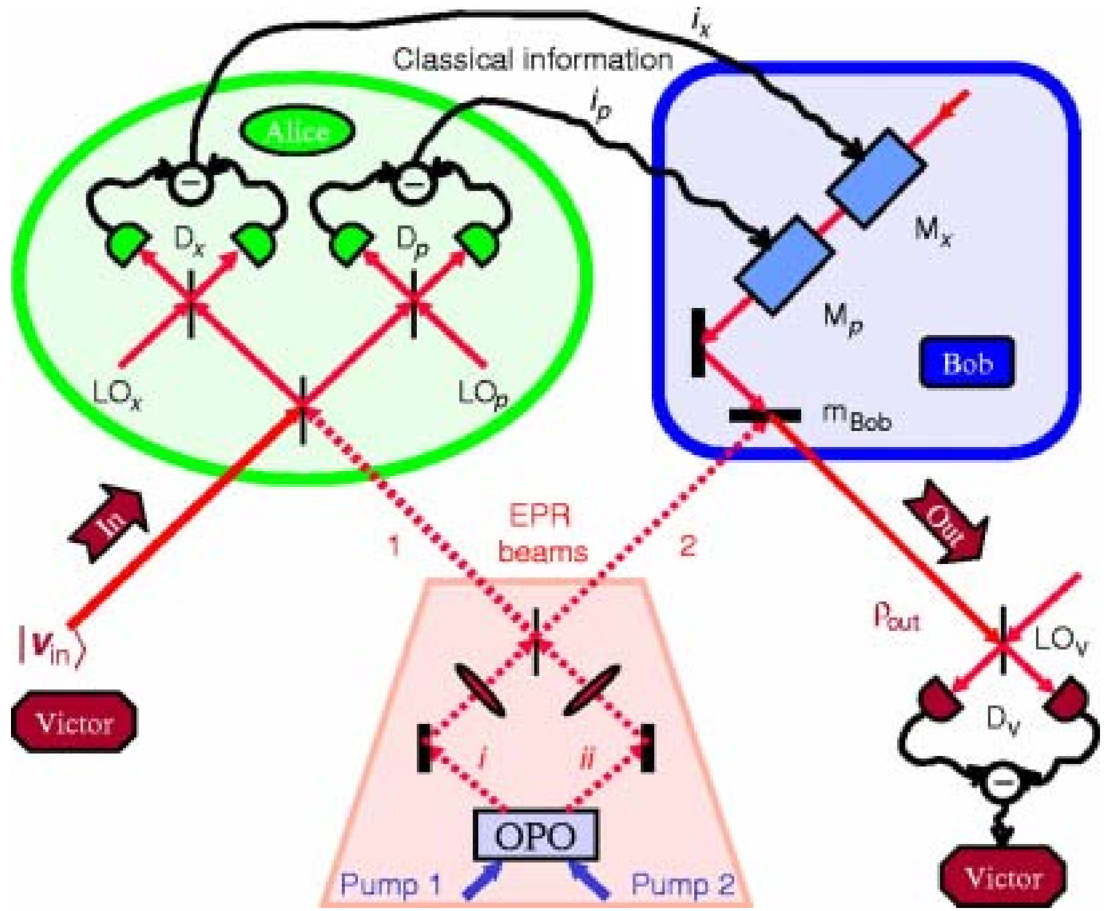
This photo shows Jeremy Caves walking faster than the shutter speed somewhere in Australia.

Where is it?

This photo shows a scene somewhere in New Mexico.

Where is it?

Continuous-variable teleportation



Furasawa et al., Science **282**, 706 (1998) $F = 0.58 \pm 0.02$

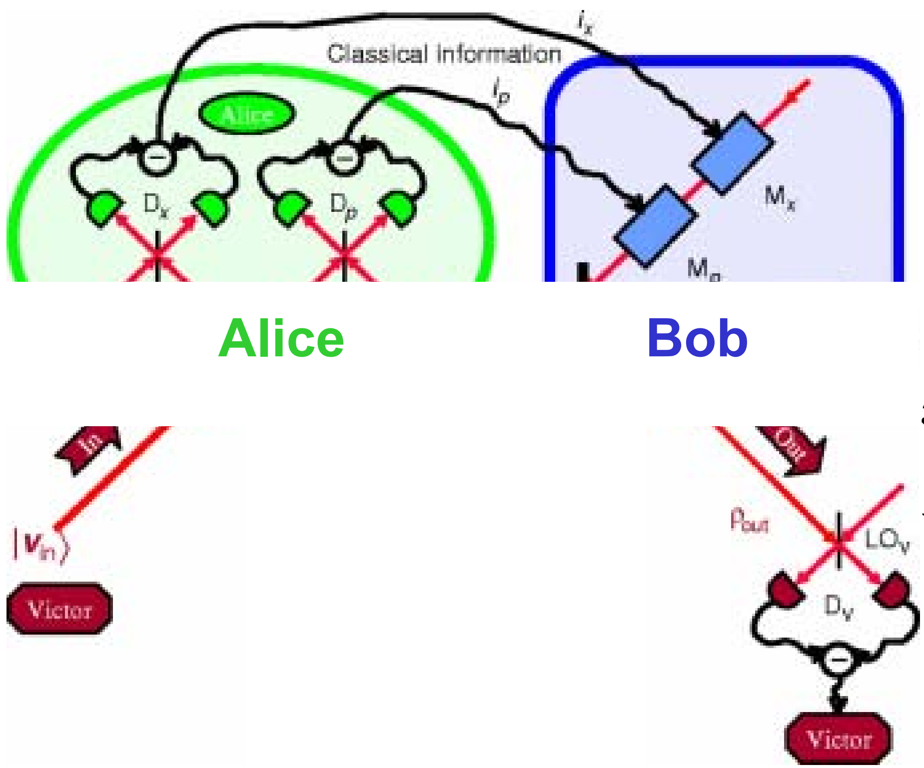
Bowen et al., PRA **67**, 032302 (2003) $F = 0.64 \pm 0.02$

Zhang et al., PRA **67**, 033792 (2003) $F = 0.61 \pm 0.02$

Teleportation of coherent states

Continuous-variable teleportation

Gibberish



Alice

Bob

$x_V\,,\; p_V$

$$\begin{aligned} x_B &= x_V + \sqrt{2}x_C \\ p_B &= p_V - \sqrt{2}p_D \end{aligned}$$

Two units of noise

Uncertainty-principle limits on simultaneous measurements of *x* and *p* require

$$(\Delta x_C)^2, (\Delta p_D)^2 \geq 1/2 \ .$$

Continuous-variable teleportation

$$x_B = -x_A + \sqrt{2}x_C = x_V - \sqrt{2}X + \sqrt{2}x_C$$
$$p_B = p_A - \sqrt{2}p_D = p_V - \sqrt{2}P - \sqrt{2}p_D$$

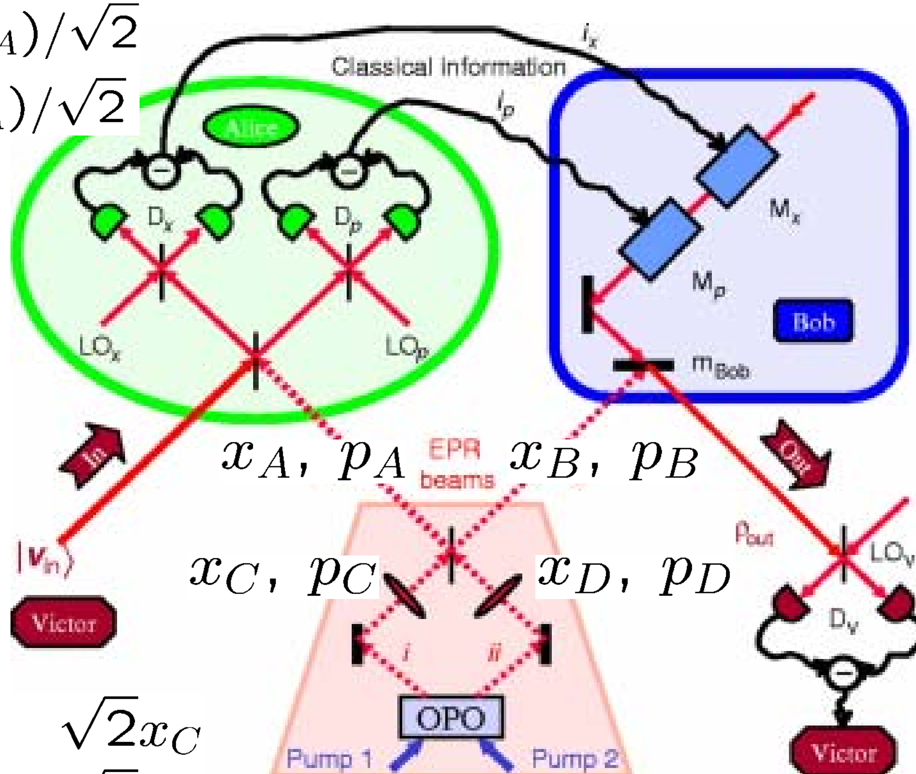
Measure

$$X = (x_V + x_A)/\sqrt{2}$$
$$P = (p_V - p_A)/\sqrt{2}$$

Communicate

$X, \; P$ **Gibberish**

No cloning



$x_A, \; p_A$ $\quad x_B, \; p_B$

$x_V, \; p_V$ $\quad x_C, \; p_C$ $\quad x_D, \; p_D$

Displace

$$x_B = x_V + \sqrt{2}x_C$$
$$p_B = p_V - \sqrt{2}p_D$$

$$x_A + x_B = \sqrt{2}x_C$$
$$p_A + p_B = \sqrt{2}p_C$$
$$x_A - x_B = \sqrt{2}x_D$$
$$p_A - p_B = \sqrt{2}p_D$$

EPR *correlation* when $(\Delta x_C)^2, (\Delta p_D)^2 < 1/2$

Whoa! This is a classical phase-space description (local hidden-variable model)

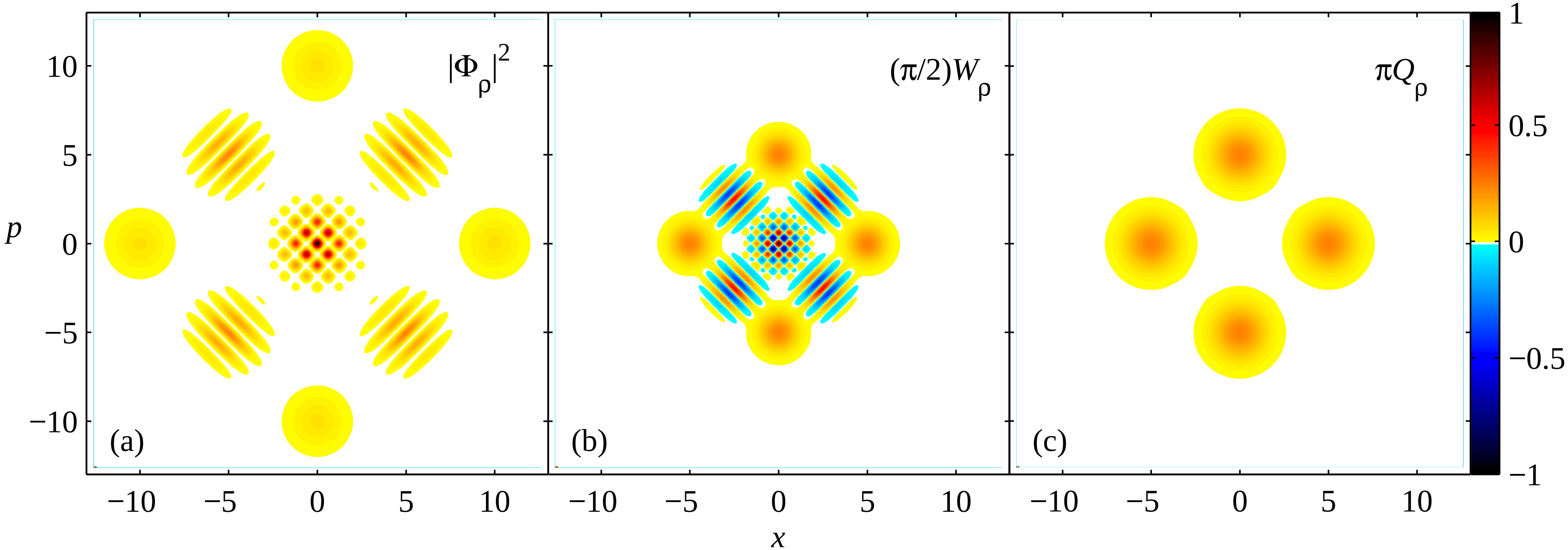
Wigner function: A phase-space quasidistribution

$$\rho \quad \longleftrightarrow \quad W(\alpha) \qquad \alpha = (x + ip)/\sqrt{2}$$

Marginals of the Wigner function give the statistics of measurements of the phase-space variables, *x* or *p*, or any linear combination of *x* and *p*.

$$\Phi(\beta) = \begin{pmatrix} \text{Characteristic} \\ \text{function} \end{pmatrix} = \begin{pmatrix} \text{Fourier transform} \\ \text{of } W(\alpha) \end{pmatrix} = \int d^2\alpha \, W(\alpha) e^{\beta\alpha^* - \beta^*\alpha}$$

Zurek's "compass" state: Superposition of four coherent states



Wigner function: A phase-space quasidistribution

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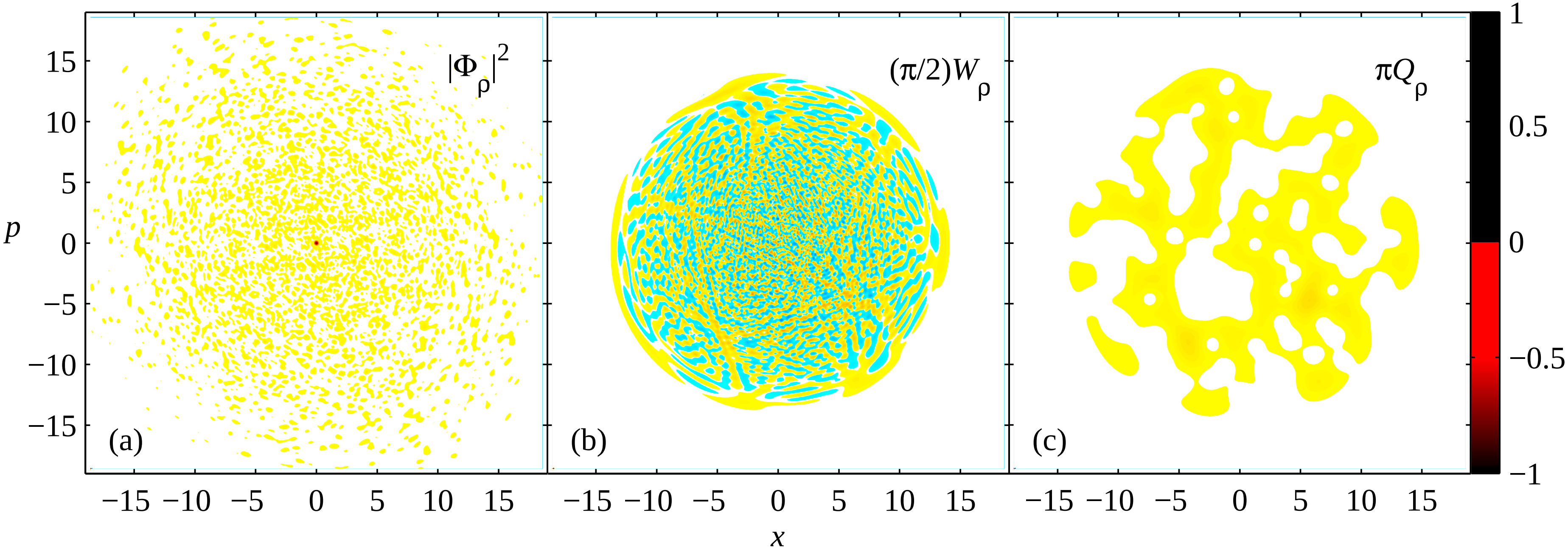
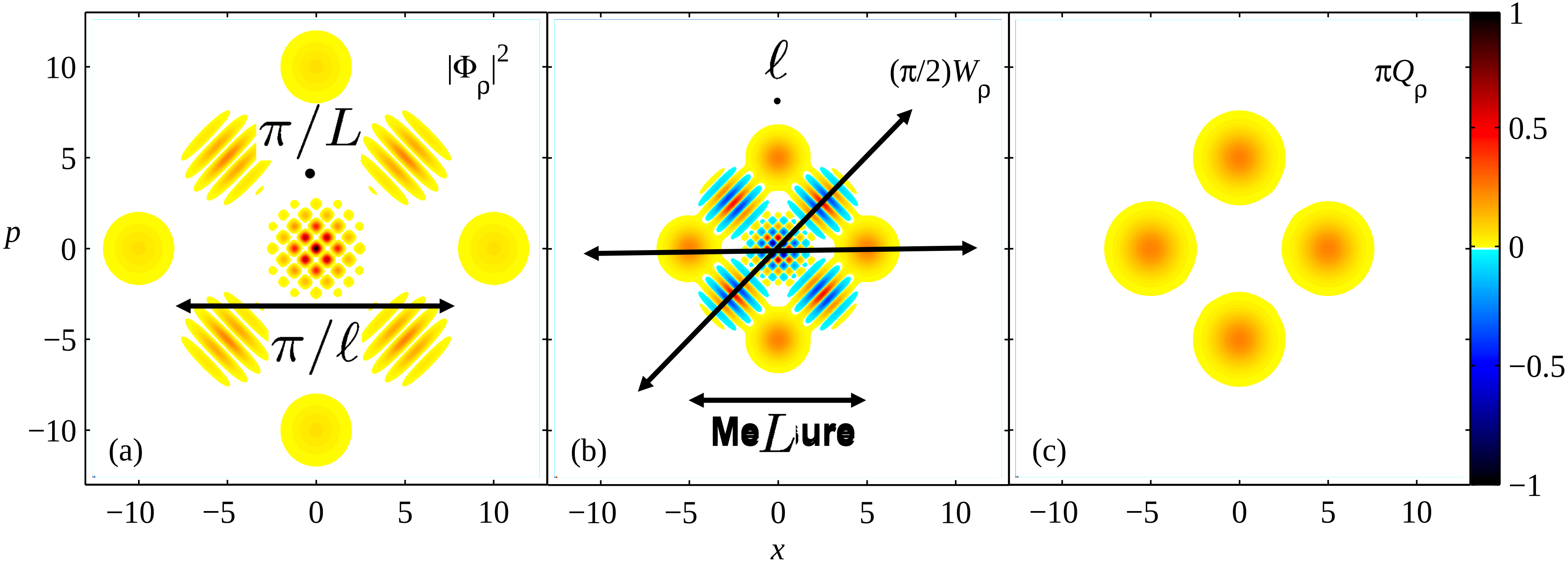
Random superposition of first 100 number states



Sub-Planck structure

Compass state: Superposition of four coherent states



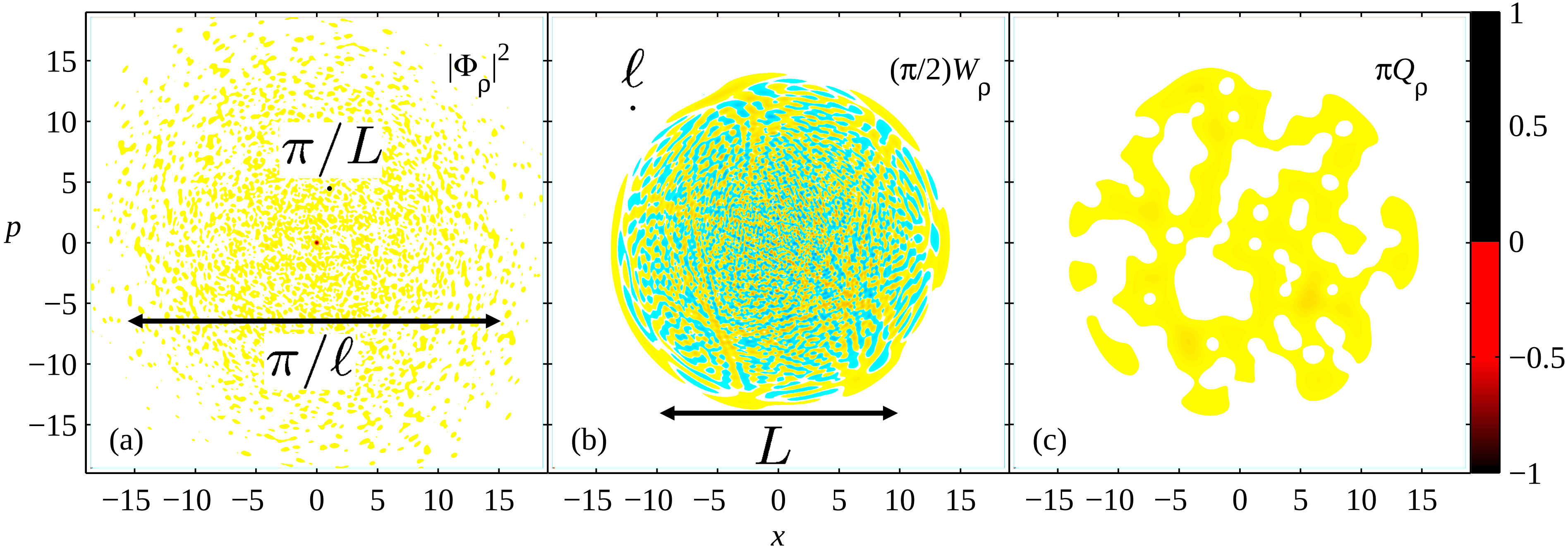
$$\ell = \left(\frac{\pi}{4} \int dx\, dp\, |\nabla W|^2 \right)^{-1/2} = \left(\int \frac{d^2\mu}{\pi} |\mu|^2 |\Phi(\mu)|^2 \right)^{-1/2}$$

$$L = 2\sqrt{(\Delta x)^2 + (\Delta p)^2} = 2 \left(\int d^2\alpha\, d^2\beta\, |\alpha - \beta|^2 W(\alpha) W(\beta) \right)^{1/2}$$

$\ell L = 2$ for pure states.

Sub-Planck structure

Random superposition of first 100 number states



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$\ell L = 2$ for pure states.

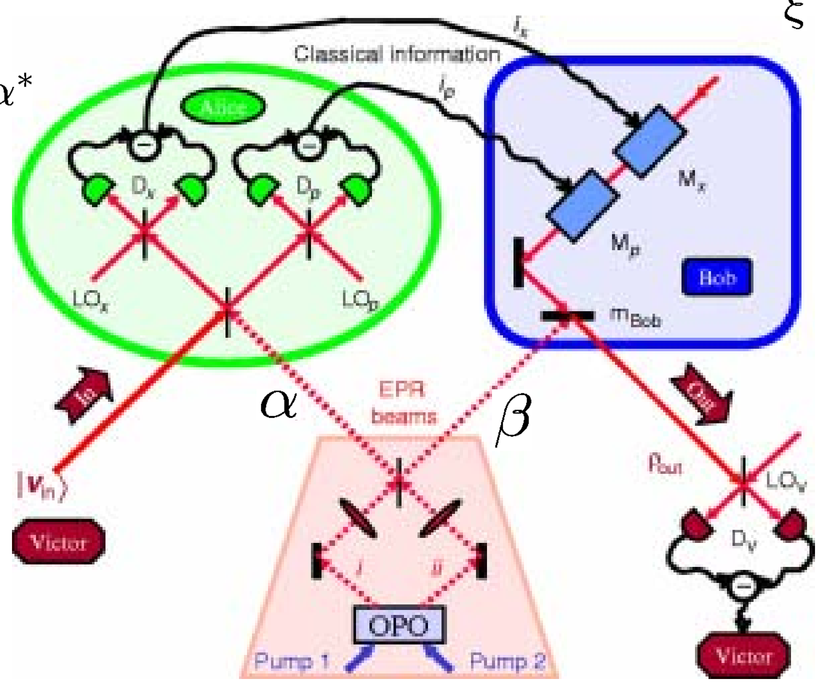
Continuous-variable teleportation: Wigner functions

Measure

$\xi = \nu + \alpha^*$

Communicate

ξ



ν

α

β

$\rho = |\psi\rangle\langle\psi|$

$W_\rho(\nu)$

Displace

ξ

Average output state:

$\bar{\rho}_{\text{out}}$

$W_{\bar{\rho}_{\text{out}}}(\beta)$

Two-mode squeezed state ρ_{AB}

$$W_{AB}(\alpha,\beta) = \frac{4}{\pi^2}\exp\Big(-2(|\alpha|^2 + |\beta|^2)\cosh 2r - 2(\alpha\beta + \alpha^*\beta^*)\sinh 2r\Big)$$

Continuous-variable teleportation: Wigner functions

For Gaussian input states (this includes coherent states), all of which have positive Wigner functions, the Wigner function provides a classical phase-space description (local hidden-variable model). The hidden variables are the phase-space variables for the three modes. The protocol runs on the classical correlations between the phase-space variables, described by the Wigner function.

Yet if Alice and Bob share an unentangled state in modes *A* and *B*, the maximum fidelity for teleporting a coherent state using the standard protocol is 1/2. For teleporting coherent states, a fidelity greater than 1/2 requires a shared resource that quantum mechanics says is entangled, but which is used in a way that can be accounted for by classical correlations of local hidden variables.

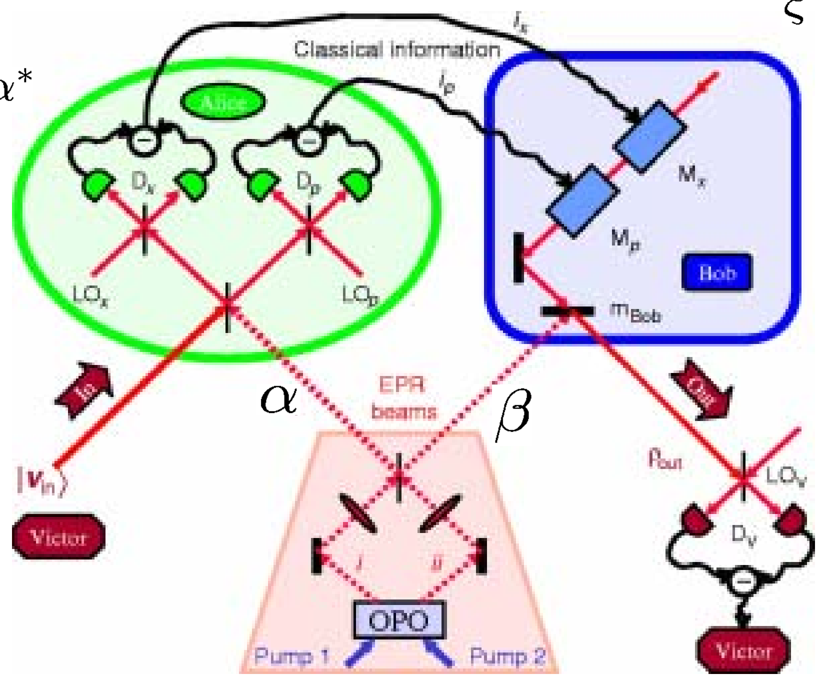
Continuous-variable teleportation: Wigner functions

Measure

$\xi = \nu + \alpha^*$

Communicate

ξ



α β

ν

$\rho = |\psi\rangle\langle\psi|$

$W_\rho(\nu)$

Displace

ξ

Average output state:

$\bar{\rho}_{\text{out}}$

$W_{\bar{\rho}_{\text{out}}}(\beta)$

Two-mode squeezed state ρ_{AB}

$$W_{AB}(\alpha,\beta) = \frac{4}{\pi^2}\exp\Big(-2(|\alpha|^2+|\beta|^2)\cosh 2r - 2(\alpha\beta+\alpha^*\beta^*)\sinh 2r\Big)$$

Continuous-variable teleportation: Input and entanglement

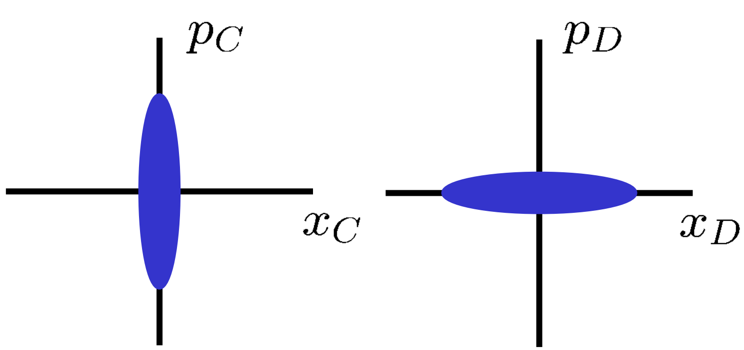
Input pure state: $\rho = |\psi\rangle\langle\psi|$ with Wigner function $W_\rho(\nu)$

Alice and Bob's entangled resource: Two-mode squeezed state ρ_{AB} with Wigner function

$$W_{AB}(\alpha, \beta) = \frac{4}{\pi^2} \exp\Big(-2(|\alpha|^2 + |\beta|^2)\cosh 2r - 2(\alpha\beta + \alpha^*\beta^*)\sinh 2r\Big)$$

Squeeze parameter *r*
Measure of EPR correlations

Use $t = 2e^{-2r}$ instead.
$t = 0$, $r = \infty$: perfect EPR correlation
$t = 1$: will be important (you'll see)
$t = 2$, $r = 0$: no EPR correlation



$$x_B = x_V + \sqrt{2}x_C$$
$$p_B = p_V - \sqrt{2}p_D$$
$$(\Delta x_C)^2 = (\Delta p_D)^2 = \frac{1}{2}e^{-2r}$$
$$(\Delta p_C)^2 = (\Delta x_D)^2 = \frac{1}{2}e^{2r}$$

Continuous-variable teleportation: Output and fidelity

Bob's output state, averaged over measurement results ξ:

$$\bar{\rho}_{\text{out}} = \frac{2}{t}\int \frac{d^2\nu}{\pi}\, e^{-2|\nu|^2/t} D(\nu)\rho D^\dagger(\nu) = \left(\begin{array}{c} \rho \text{ kicked randomly with} \\ \text{"kicking strength" } t \end{array} \right)$$

$$W_{\bar{\rho}_{\text{out}}}(\beta) = \frac{2}{t}\int \frac{d^2\nu}{\pi} W_\rho(\nu) e^{-2|\beta-\nu|^2/t} = \left(\begin{array}{c} \text{Input Wigner function} \\ \text{"smeared out" over a} \\ \text{phase-space scale } \sqrt{t/2} \end{array} \right)$$

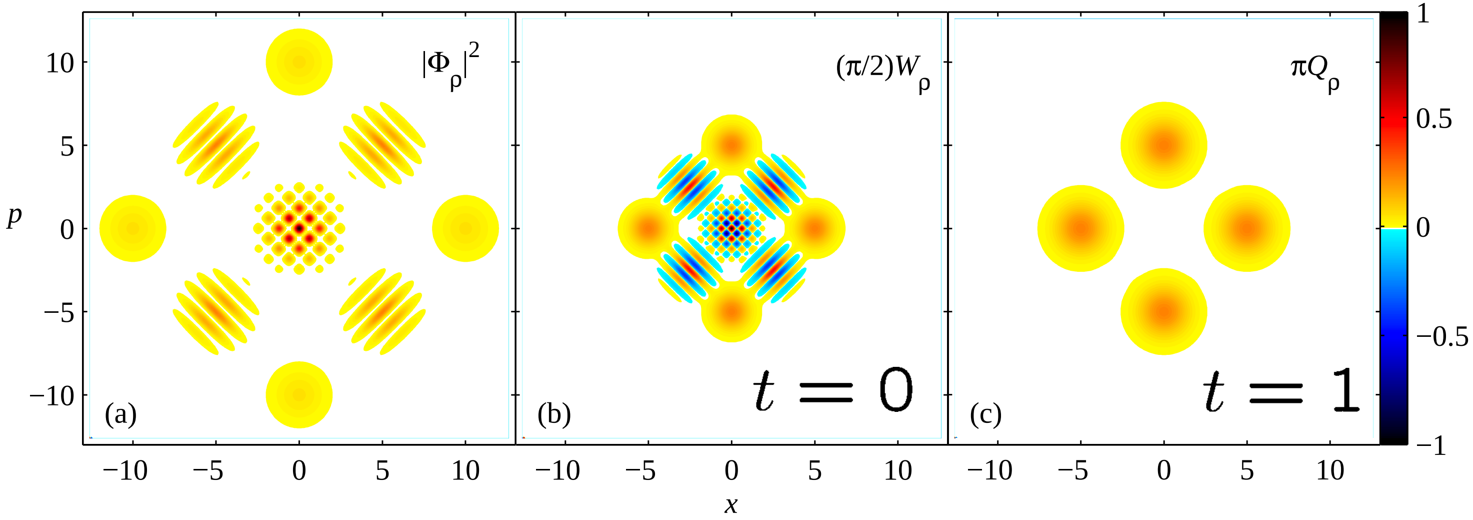
Average teleportation fidelity:

$$\bar{F}_\rho(t) = \langle\psi|\bar{\rho}_{\text{out}}|\psi\rangle = \frac{2}{t}\int d^2\beta\, d^2\nu\, e^{-2|\beta-\nu|^2/t} W_\rho(\beta) W_\rho(\nu)$$

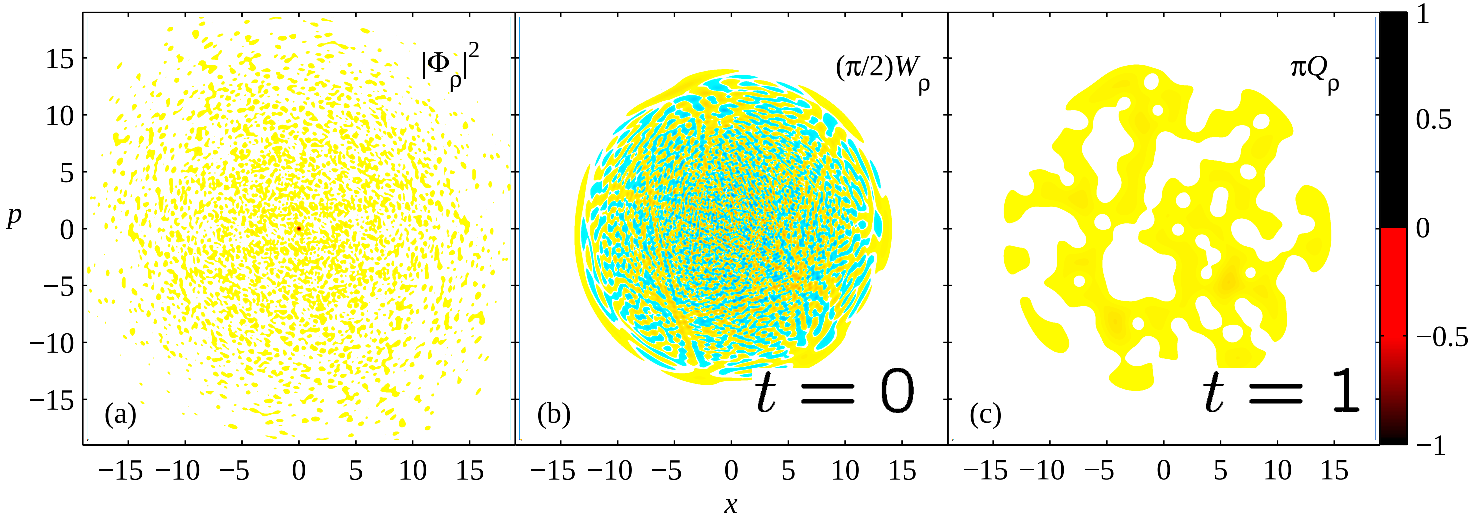
$$\bar{F}_{\text{coh}}(t) = \frac{1}{1+t/2}$$

Teleportation fidelity and sub-Planck structure

Compass state: Superposition of four coherent states

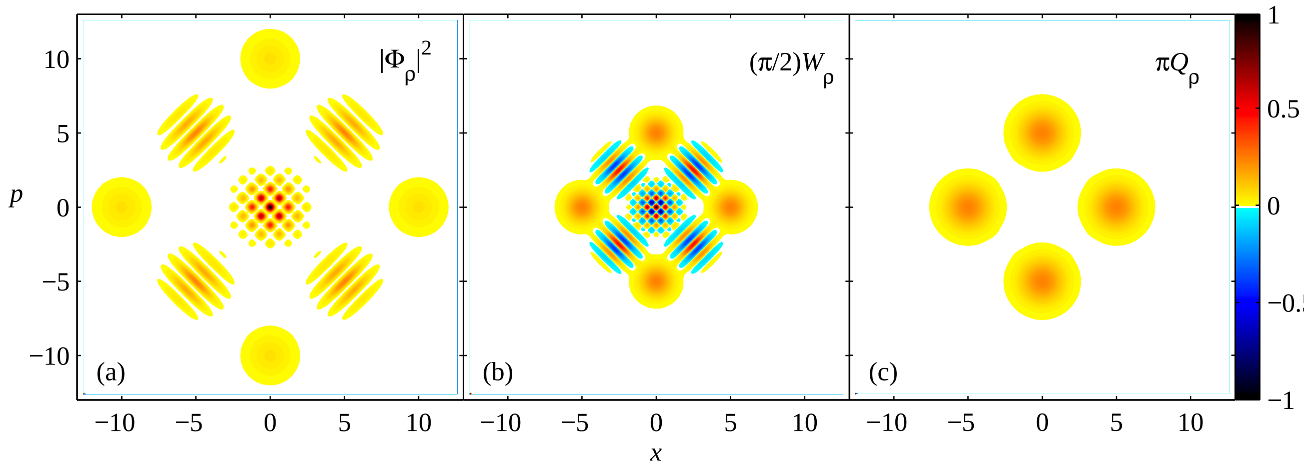


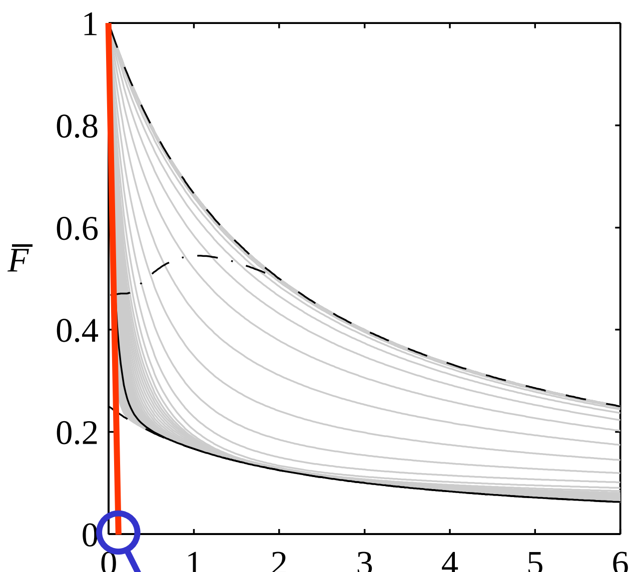
Random superposition of first 100 number states



Teleportation fidelity and sub-Planck structure

Compass state: Superposition of four coherent states





$$t_c = \frac{1}{|d\bar{F}(t)/dt|_{t=0}}$$

$$\ell \equiv \sqrt{t_c/2} = \frac{1}{\sqrt{(\Delta x)^2 + (\Delta p)^2}} = \left(\frac{\pi}{4} \int dx\, dp\, |\nabla W_\rho|^2 \right)^{-1/2}$$

Our measure of small-scale phase-structure finds an operational significance in terms of the difficulty of teleportation.

Teleportation of non-Gaussian pure states

All **non-Gaussian *pure* states have Wigner functions that take on negative values, so they cannot be incorporated in the classical phase-space description.**

"Kicking and cheating" protocol

- **Alice *kicks* the input state randomly with a Gaussian kicking strength *t* chosen so that the kicked state has a positive Wigner function and thus can be incorporated within the classical phase-space description. The minimum required kicking strength is *t* = 1 for all non-Gaussian pure states.**

- **Alice and Bob *cheat* by teleporting the kicked state with perfect fidelity.**

The fidelity achieved by the "kicking and cheating" protocol is the *t* = 1 fidelity within the standard quantum protocol.

Gold standard for continuous variable teleportation of non-Gaussian pure states

$$\begin{pmatrix}\text{Fidelity for teleporting}\\ \rho \text{ within the "kicking}\\ \text{and cheating" protocol}\end{pmatrix} = \overline{F}_\rho(t=1) \;?\!\!<\; \overline{F}_{\text{coh}}(t=1) = 2/3$$

?

$$
\begin{aligned}
\overline{F}_\rho(t) &= \langle\psi|\overline{\rho}_{\text{out}}|\psi\rangle \\
&= \frac{2}{t}\int \frac{d^2\nu}{\pi}\, e^{-2|\nu|^2/t}|\Phi_\rho(\nu)|^2 \\
&= \int d^2\beta\, d^2\nu\, e^{-t|\beta-\nu|^2/2} W_\rho(\beta) W_\rho(\nu) \\
&= \operatorname{tr}\left(\rho\otimes\rho\,\underbrace{\frac{1}{1+t/2}\left(\frac{1-t/2}{1+t/2}\right)^{(b-v)^\dagger(b-v)/2}}_{\equiv A_t}\right) \\
&\le \begin{pmatrix}\text{largest eigenvalue}\\ \text{of } A_t\end{pmatrix} \\
&= \frac{1}{1+t/2}
\end{aligned}
$$

Turn the overlap question for one mode into an expectation value for a pair of modes.

Equality if and only if ρ is a coherent state.

This same technical result shows that if Alice and Bob share an unentangled state, the maximum fidelity for teleporting a coherent state using the standard protocol is 1/2.

The "kicking with added communication" protocol

Alice communicates her actual kick to Bob, who removes it after completing the protocol, thereby achieving the quantum fidelity at the cost of additional classical communication from Alice to Bob.

$$\begin{pmatrix}\text{classical communication}\\ \text{from Alice to Bob}\\ \text{in standard protocol}\end{pmatrix} \sim 2\log_2\left(\frac{e^r}{e^{-r}}\right) = 4r$$

$$\sim 2\log_2\left(\frac{L/2}{\ell}\right) + (\text{a few bits}) \sim 2\log_2 \underbrace{(L/2)^2}_{=D} + (\text{a few bits})$$

$$\begin{pmatrix}\text{additional classical}\\ \text{communication from Alice}\\ \text{to Bob to communicate kick}\end{pmatrix} \sim 2\log_2\left(\frac{1}{e^{-r}}\right) = 2r \sim \begin{pmatrix}\text{50\% of protocol}\\ \text{communication}\end{pmatrix}$$

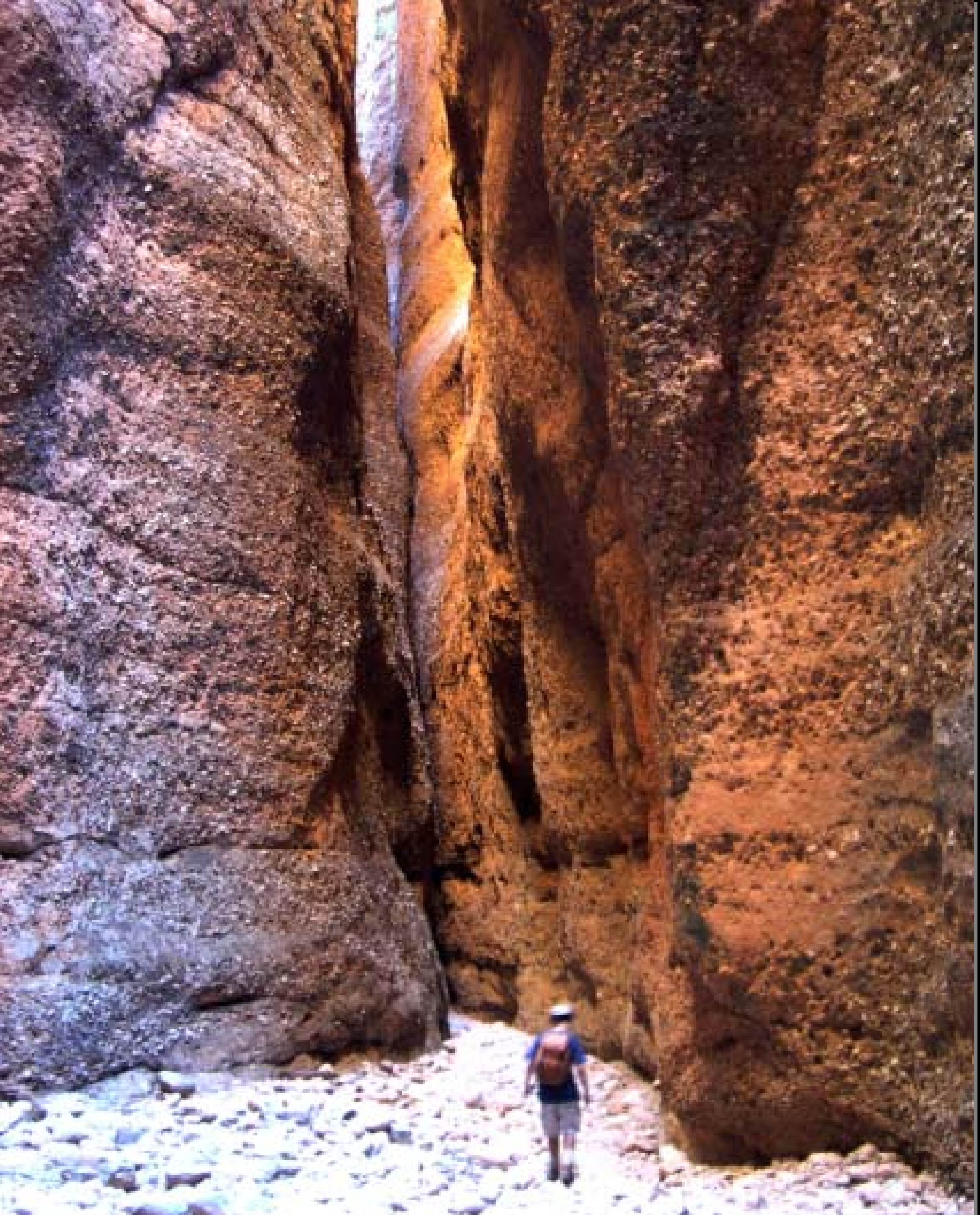
Summary

There is a classical phase-space description (local hidden-variable model) for teleportation, with arbitrary fidelity, of all Gaussian pure states, including coherent states, using the standard protocol.

If Alice and Bob share an unentangled state, they cannot teleport a coherent state using the standard protocol with fidelity greater than 1/2. For teleporting coherent states, a fidelity greater than 1/2 requires a shared resource that quantum mechanics says is entangled, but which is used in a way that can be accounted for by classical correlations of local hidden variables.

There is no classical phase-space description of the teleportation of any non-Gaussian pure state for fidelities greater than or equal to 2/3, even if one allows Alice and Bob to cheat by using a protocol with perfect fidelity.

Alice and Bob can achieve the quantum fidelity within a “kicking with added communication” protocol in which Alice communicates the kick, at the cost of upping the classical communication rate by 50%.



This photo shows Jeremy Caves walking faster than the shutter speed somewhere in Australia.

Where is it?

Echidna Gorge
Bungle Bungle Range
Purnululu NP
Western Australia

2004 June 28

This photo shows a scene somewhere in New Mexico.

Where is it?

Sawtooth Range
West of VLA

2003 August 31