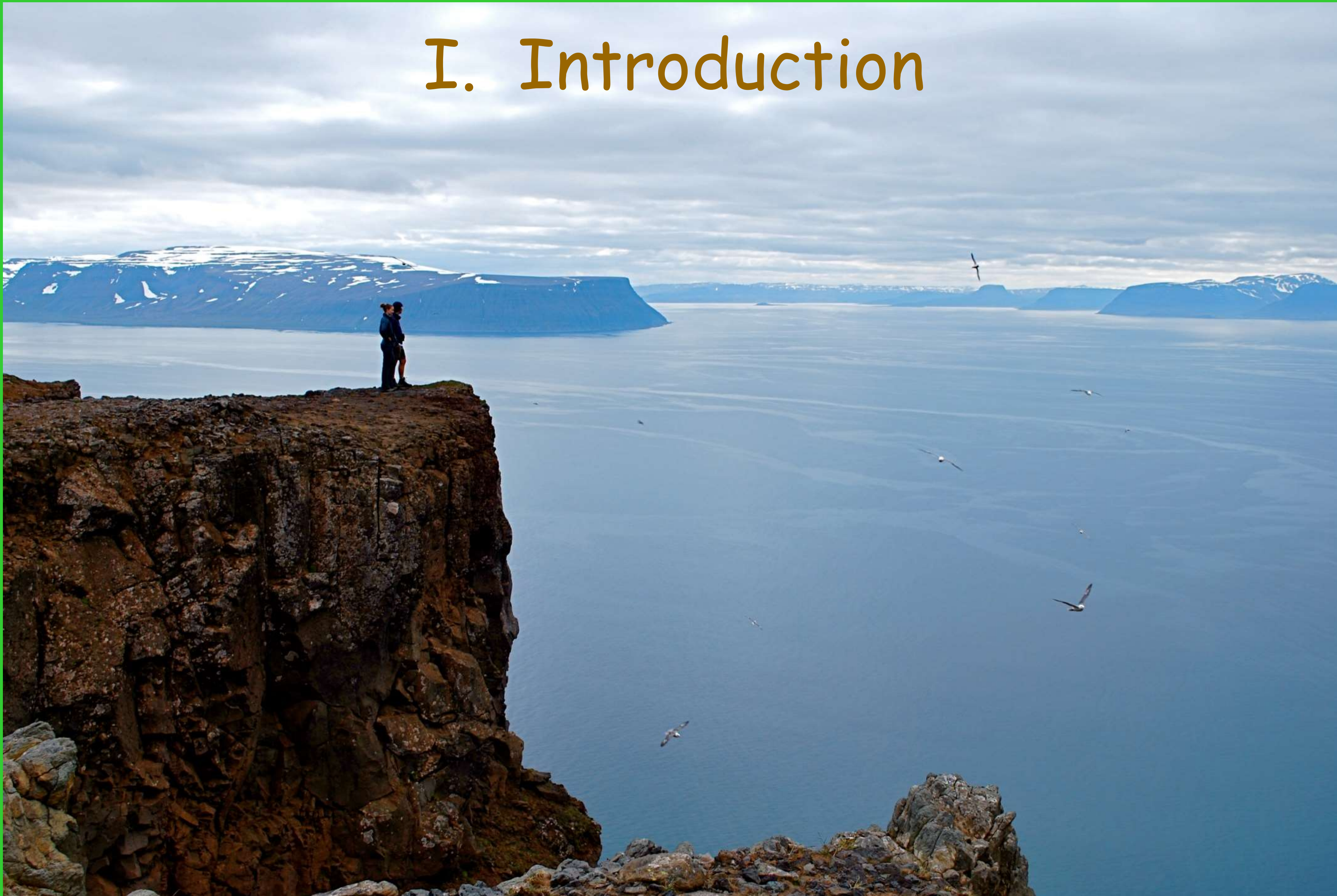
Quantum-limited measurements: One physicist's crooked path from relativity theory to quantum optics to quantum information

I. Introduction
II. Squeezed states and optical interferometry
III. Quantum limits on parameter estimation

Carlton M. Caves
Center for Quantum Information and Control, University of New Mexico

http://info.phys.unm.edu/~caves

I. Introduction



Holstrandir Peninsula overlooking Ísafjarðardjúp
Westfjords, Iceland

Quantum information science

A new way of thinking

Computer science

Computational complexity depends on physical law.

New physics

Quantum mechanics as liberator.

What can be accomplished with quantum systems that can't be done in a classical world?

Explore what quantum systems can do, instead of being satisfied with what Nature hands us.

Quantum engineering

Old physics

Quantum mechanics as nag.

The uncertainty principle restricts what can be done.

Metrology

Taking the measure of things

The heart of physics

Extracting information from physical systems

New physics

Quantum mechanics as liberator.

Explore what quantum systems can do, instead of being satisfied with what Nature hands us.

Quantum engineering

Old physics

Quantum mechanics as nag.

The uncertainty principle restricts what can be done.

Old conflict in new guise

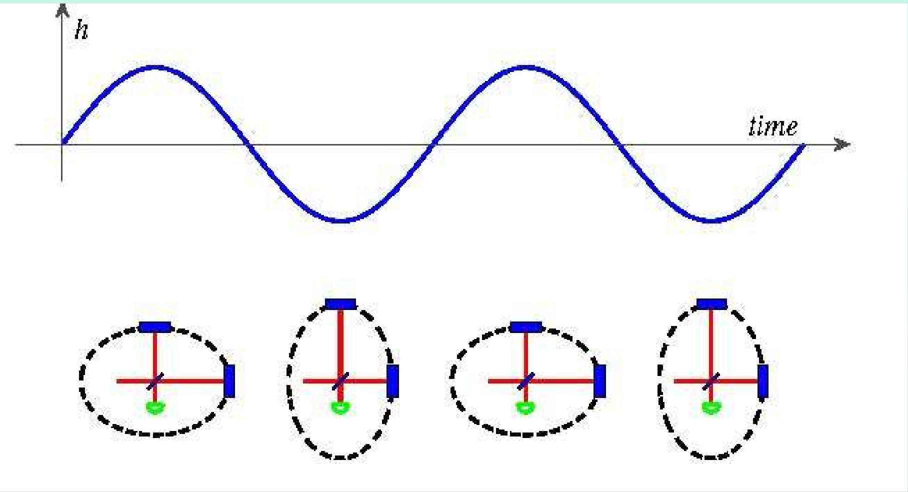
Stories about noise vs. rigorous analytic techniques with proofs

II. Squeezed states and optical interferometry

Tent Rocks
Kasha-Katuwe National Monument
Northern New Mexico

(Absurdly) high-precision interferometry

Hanford, Washington



The LIGO Scientific Collaboration,
Rep. Prog. Phys. 72, 076901 (2009).

Laser Interferometer Gravitational Observatory (LIGO)

Livingston, Louisiana

VIRGO

Cascina, Italy

Digression on gravitational waves

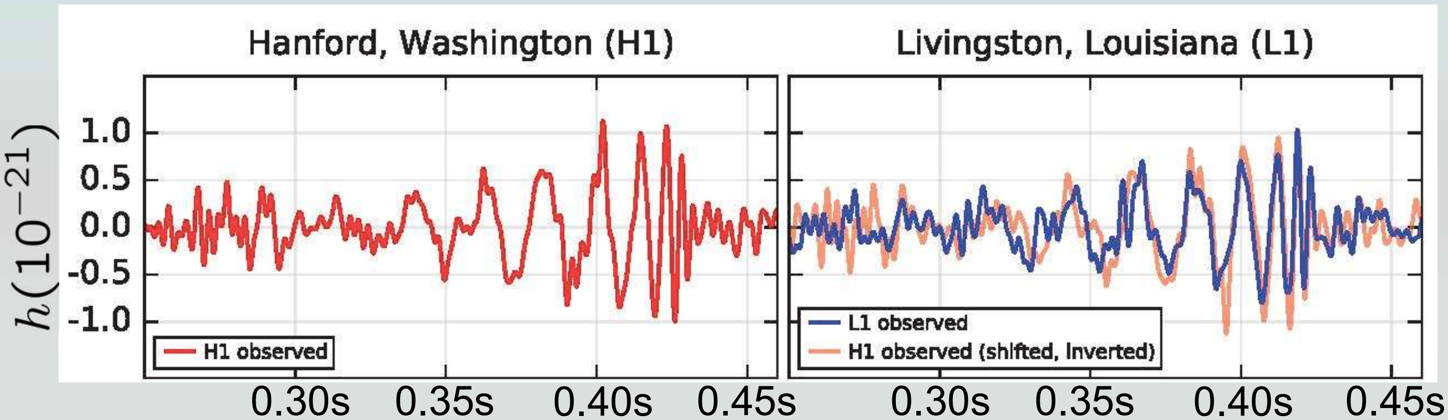
Strong-field burst sources

$$h \sim \frac{G\sqrt{\epsilon}M}{c^2 R} = 1.6 \times 10^{-22} \left(\frac{\epsilon}{0.1}\right)^{1/2} \left(\frac{M}{10 M_\odot}\right) \left(\frac{1\,\mathrm{Gpc}}{R}\right)$$

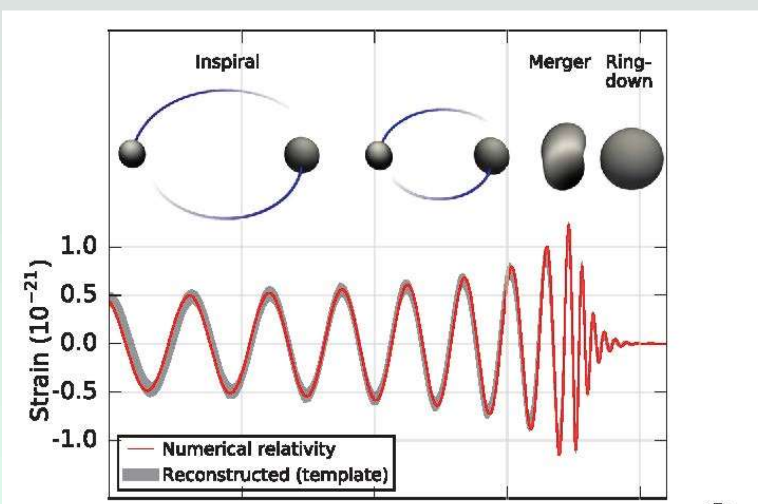
$$\frac{1}{\tau} \sim \delta \frac{c^3}{GM} = 200\,\mathrm{Hz} \left(\frac{\delta}{0.01}\right) \left(\frac{10 M_\odot}{M}\right)$$

Coalescence of two black holes

The LIGO Scientific Collaboration and Virgo Collaboration, PRL 116, 061102 (2016).



$$\begin{aligned} M_1 &\simeq 36^{+5}_{-4}\, M_\odot \\ M_2 &\simeq 29^{+4}_{-4}\, M_\odot \\ M_f &\simeq 62^{+4}_{-4}\, M_\odot \end{aligned}$$



$$\begin{aligned} R &\simeq 410^{+160}_{-180}\,\mathrm{Mpc} \\ E_{\mathrm{gw}} &\simeq 3.0^{+0.5}_{-0.5}\, M_\odot c^2 \\ h_{\mathrm{max}} &\simeq 10^{-21} \\ 1/\tau &\simeq 35\text{–}150\, Hz \end{aligned}$$

(Absurdly) high-precision interferometry

Advanced LIGO

Hanford, Washington

$$\begin{pmatrix}\text{differential}\\ \text{strain}\\ \text{sensitivity}\end{pmatrix} \simeq 10^{-22}$$

$$\begin{pmatrix}\text{differential}\\ \text{displacement}\\ \text{sensitivity}\end{pmatrix} \simeq 4 \times 10^{-19}\,\text{m}$$

from 50 Hz to 2,000 Hz.

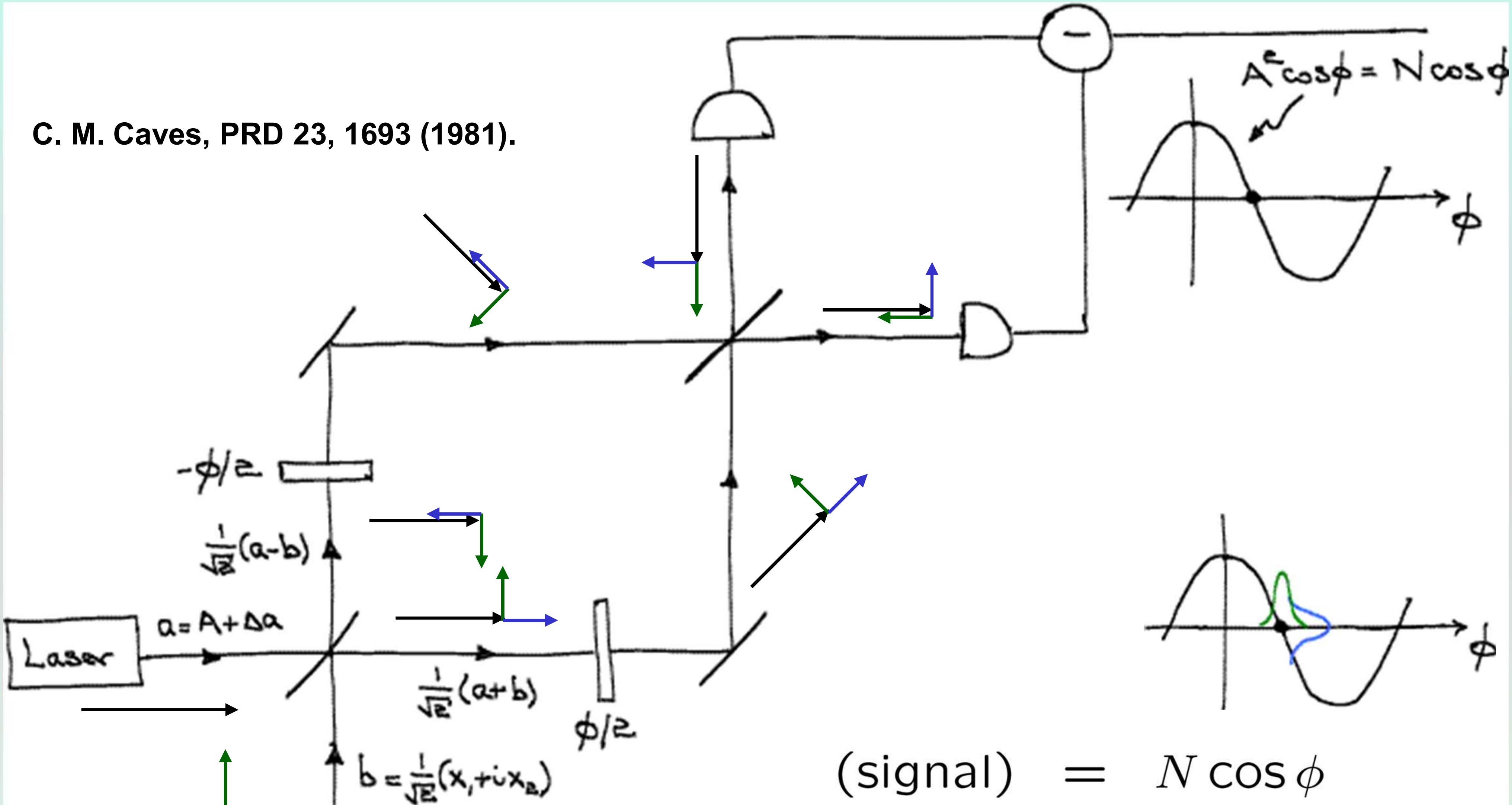
Laser Interferometer Gravitational Observatory (LIGO)

Livingston, Louisiana

High-power, Fabry-Perot Michelson (multipass), power- and signal-recycled, squeezed-light interferometers

Mach-Zehnder interferometer

C. M. Caves, PRD 23, 1693 (1981).



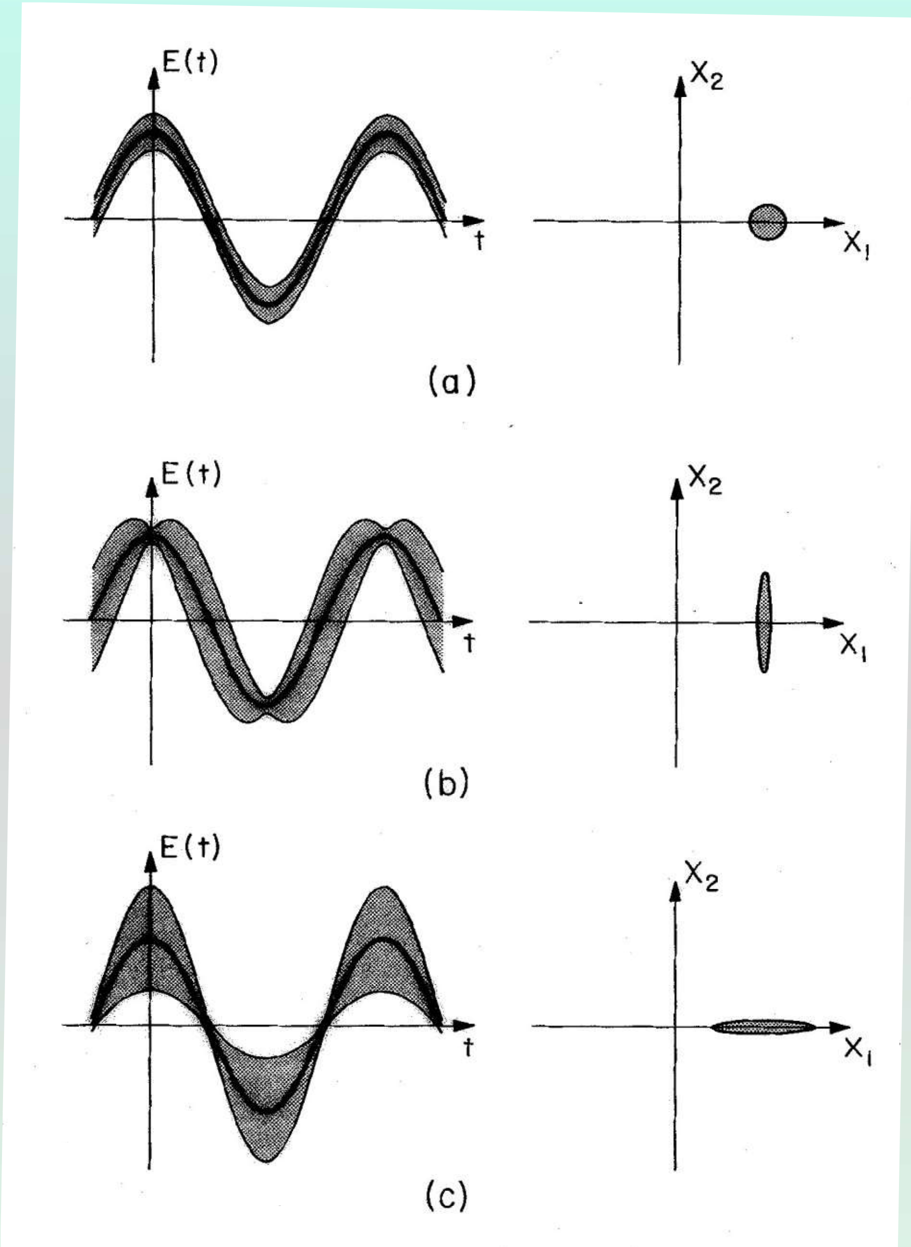
$$\text{(signal)} = N\cos\phi$$

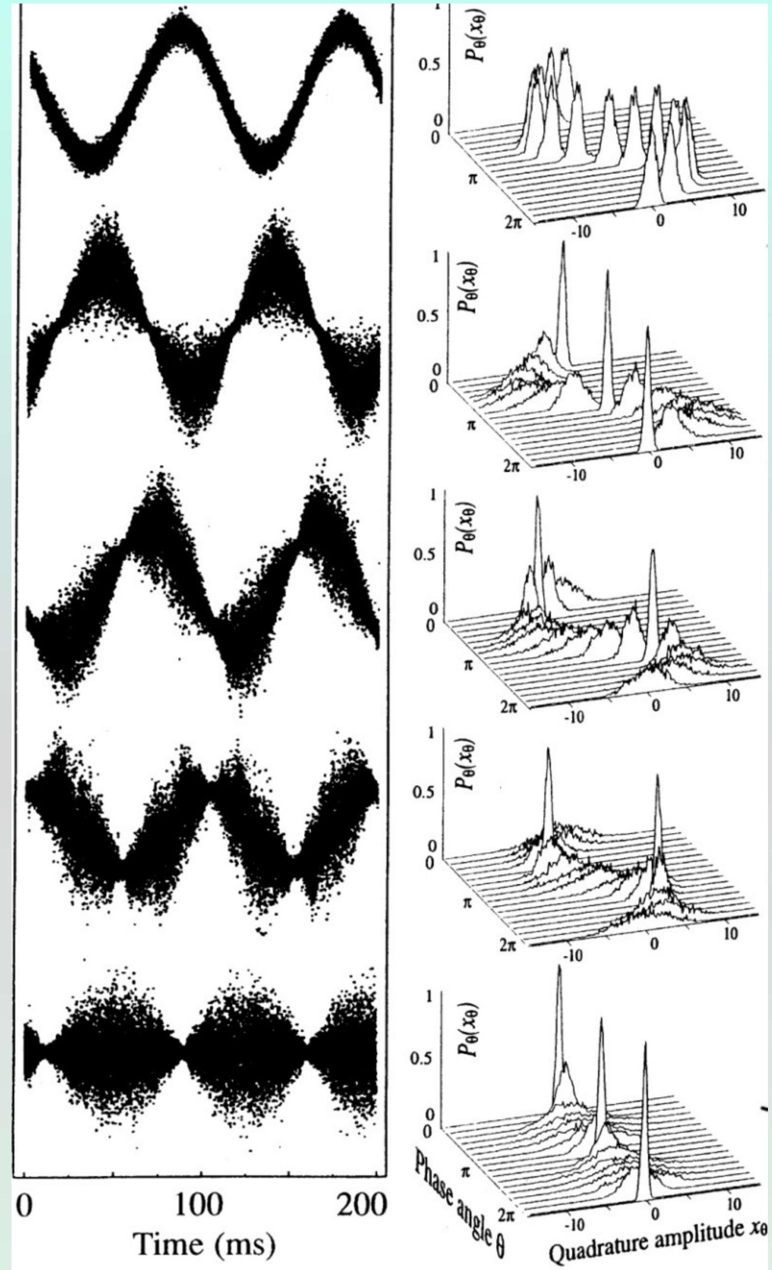
$$\text{(noise)} = \sqrt{2N}\,\Delta x_2 |\sin\phi|$$

Take note: I am not here talking about back action (radiation-pressure noise).

$$\Delta\phi = \frac{\text{(noise)}}{|d\text{(signal)}/d\phi|} = \frac{\sqrt{2}\,\Delta x_2}{\sqrt{N}}$$

Squeezed states of light



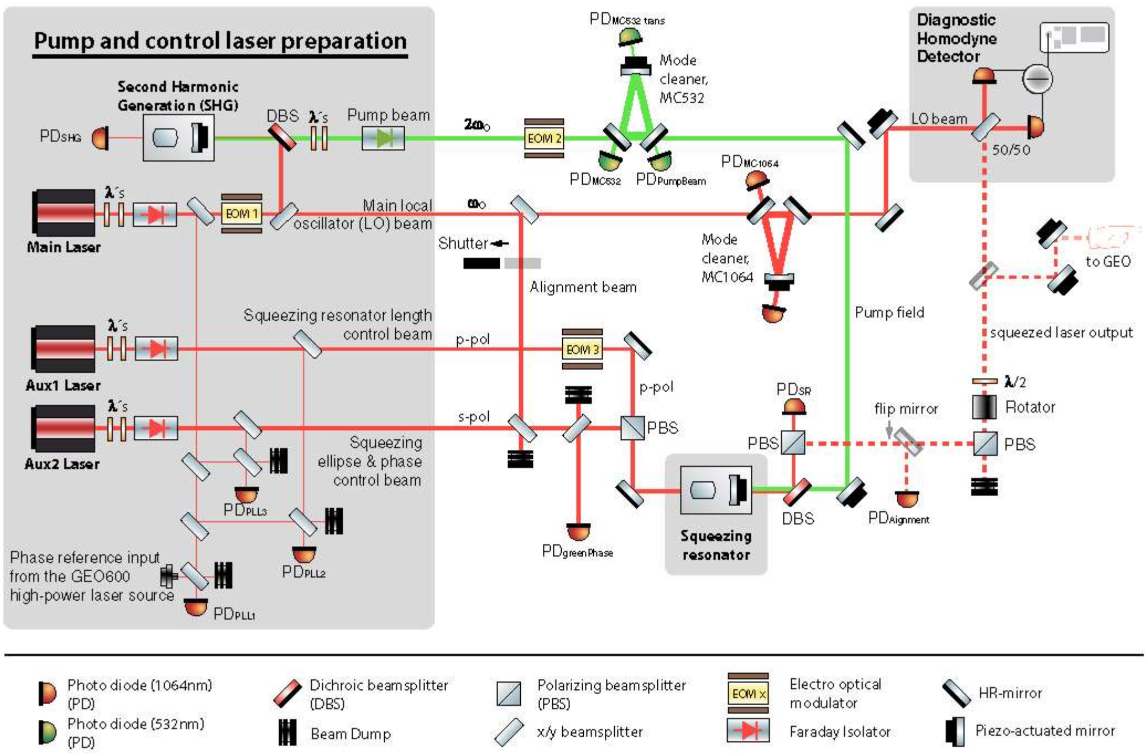


Squeezing by a factor of about 3.5

G. Breitenbach, S. Schiller, and J. Mlynek, Nature 387, 471 (1997).

Groups at ANU, Hannover, Tokyo, and MIT continued to push for greater squeezing (at audio frequencies) for use in Advanced LIGO, VIRGO, and GEO and other quantum metrology and quantum information jobs.

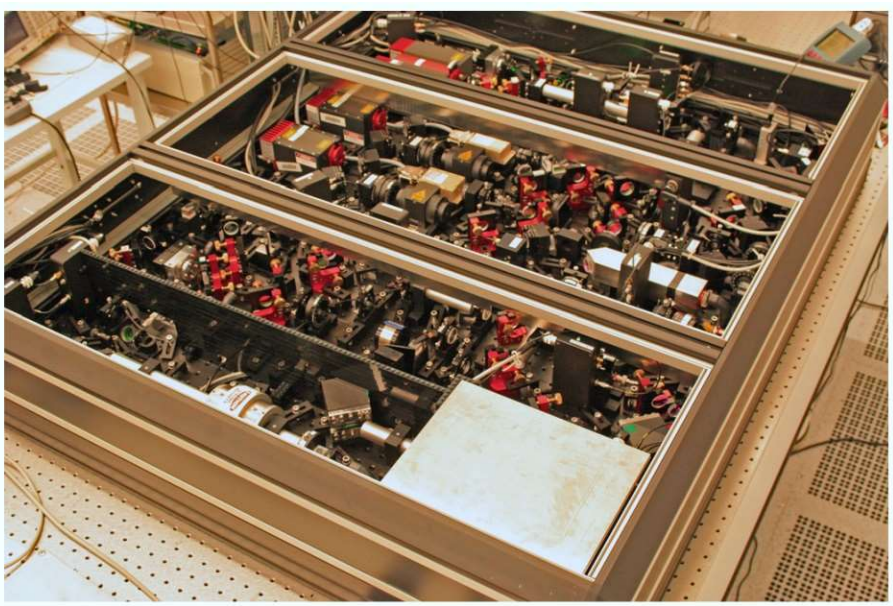
Squeezed states of light

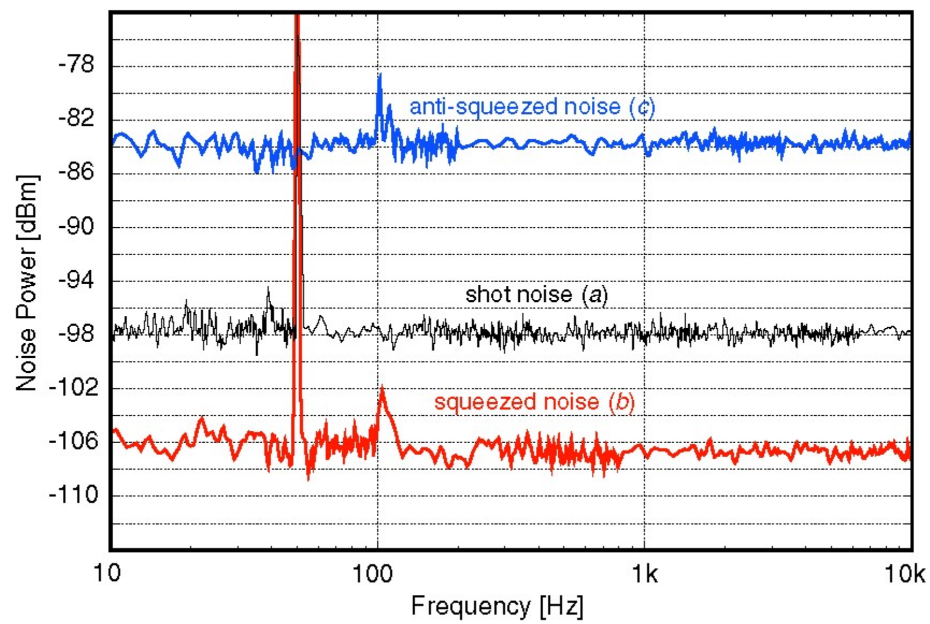


Squeezed light in Hannover

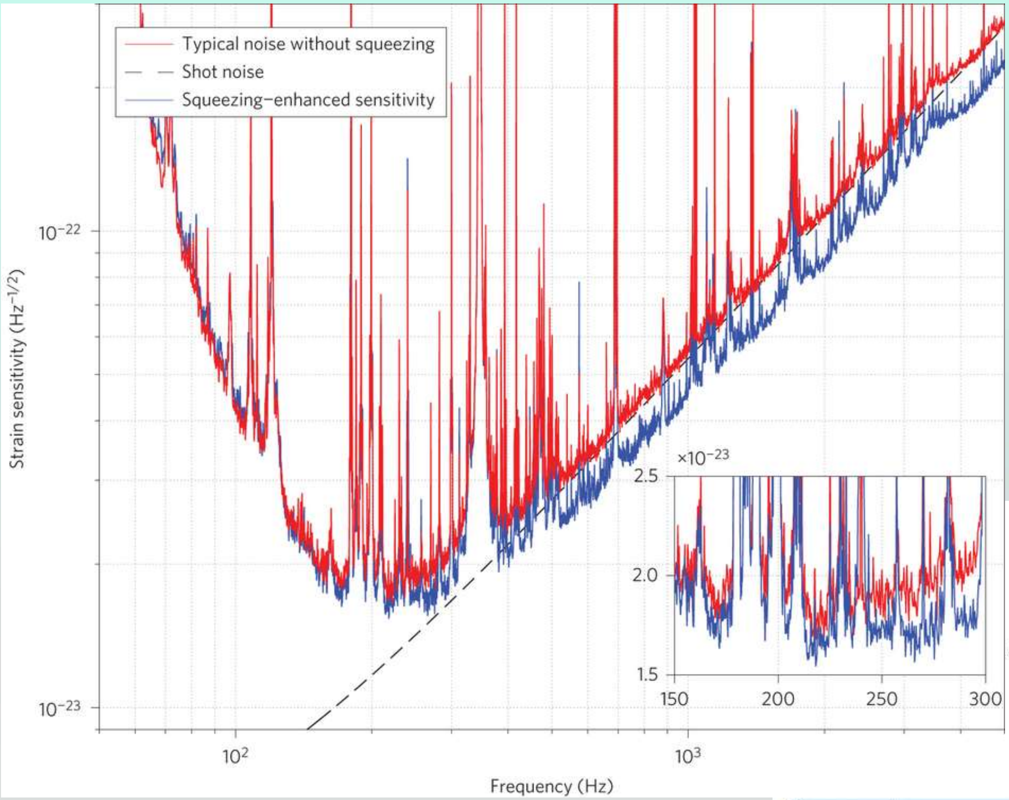
H. Vahlbruch *et al.*, Class. Quant. Grav. 27, 084027 (2010).

8–9 dB below shot noise at audio frequencies



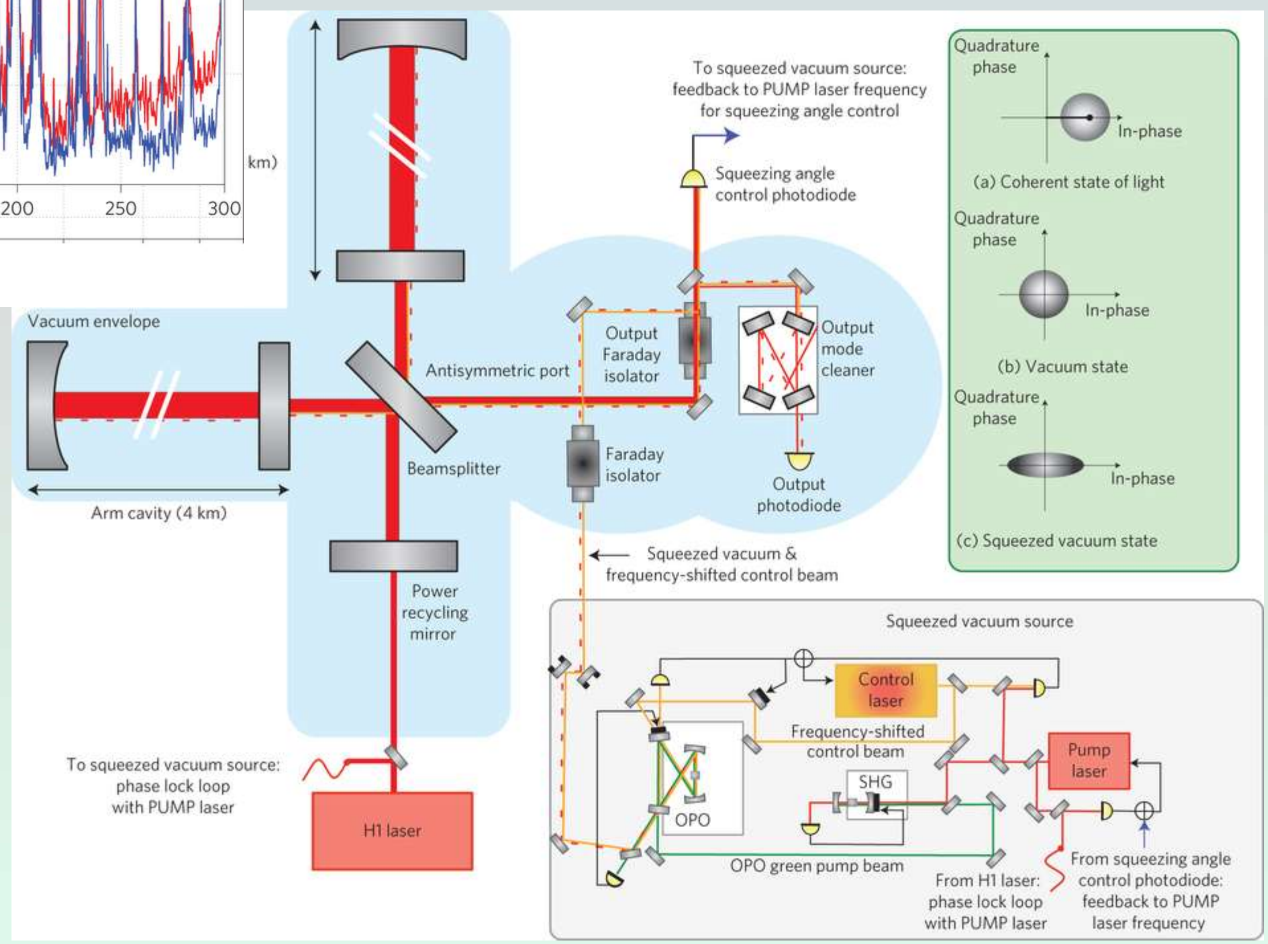


Quantum metrology (nearly) making a difference



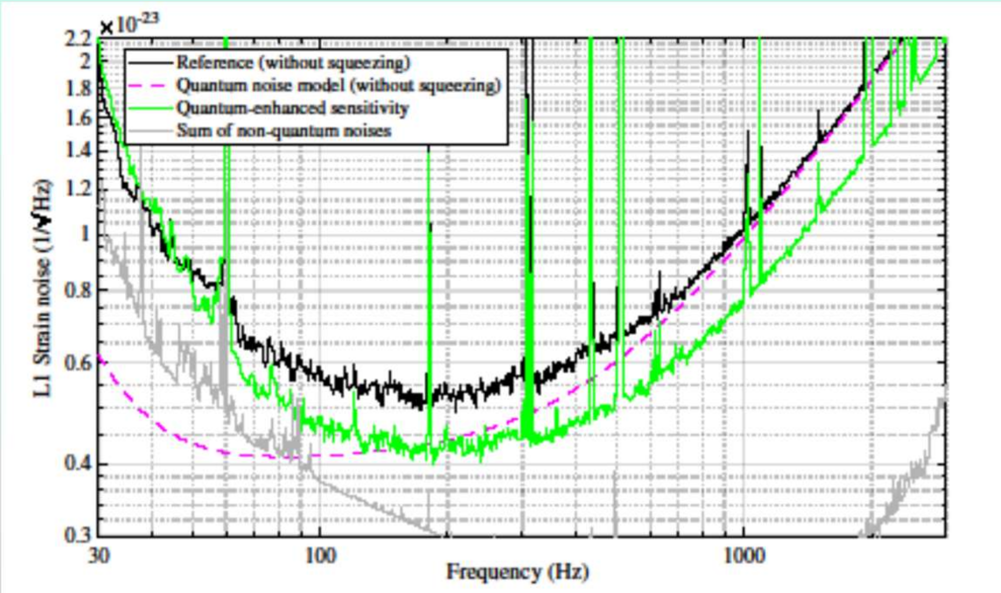
Squeezed light in the LIGO Hanford detector

The LIGO Scientific Collaboration, Nat. Phot. 7, 613 (2013).



~ 2 dB of shot-noise reduction

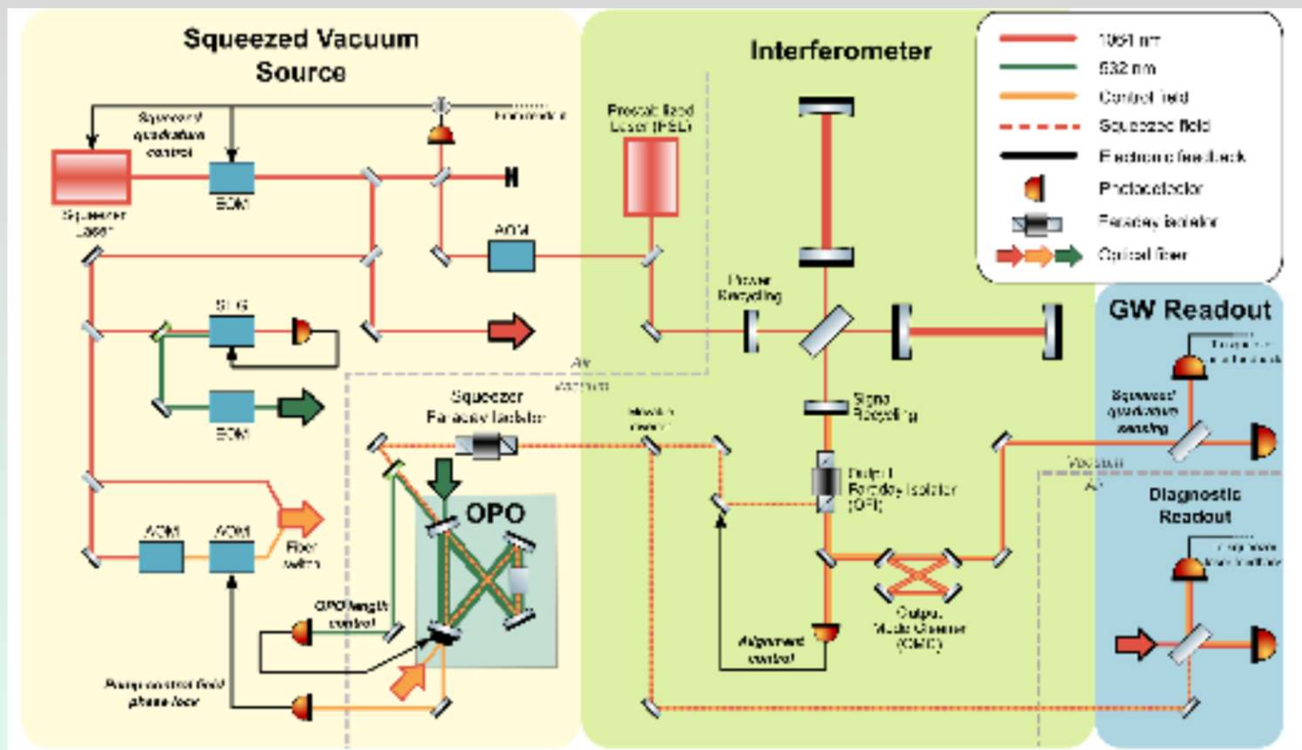
Quantum metrology making a difference



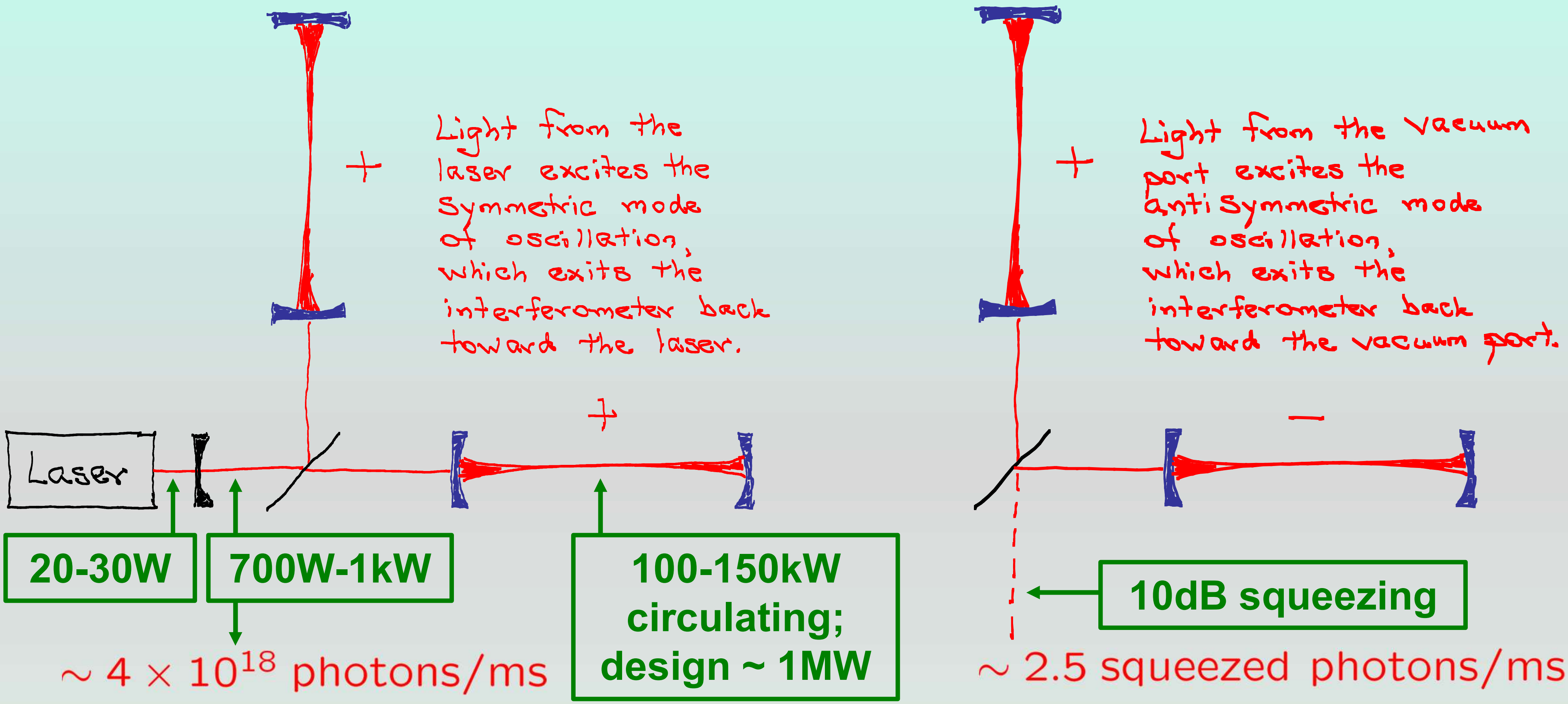
Virgo: F. Acernese *et al.,* PRL 123, 231108 (2019).

LIGO: M. Tse *et al.,* PRL 123, 231107 (2019).

During the ongoing O3 observation run, squeezed states are improving the sensitivity of the LIGO interferometers to signals above 50 Hz by up to 3 dB, thereby increasing the expected detection rate by 40% (H1: Hanford) and 50% (L1: Livingston).



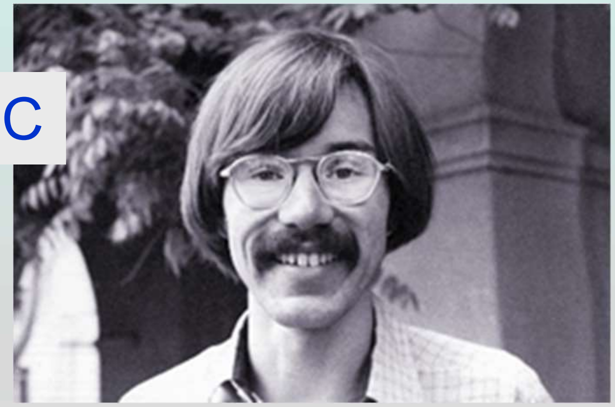
Fabry-Perot Michelson interferometer



Motion of the mirrors produced by a gravitational wave induces a transition from the symmetric mode to the antisymmetric mode; the resulting tiny signal at the vacuum port is contaminated by quantum noise that entered the vacuum port.

Ron Drever

Fabry-Perot Michelson
PDH locking
Nested cavities (recycling)



CMC

Squeezed-light interferometry

When Ron's ideas run out of gas (after 35 years, they have),

Experimenters might then (now) *be forced to learn how to very gently squeeze the vacuum before it can contaminate the light in their interferometers*.

III. Quantum limits on parameter estimation

View from Cape Hauy
Tasman Peninsula, Tasmania

Quantum limits on optical interferometry

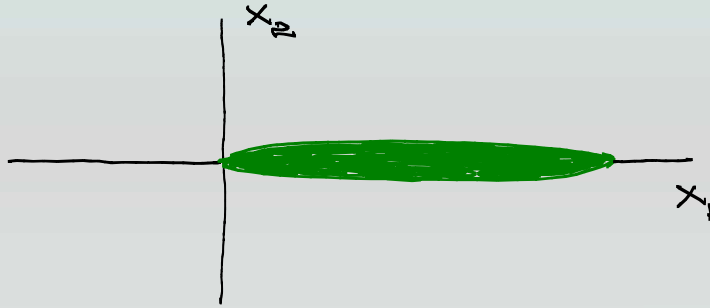
$$\Delta\phi = \frac{\sqrt{2}\Delta x_2}{\sqrt{N}}$$

Reminder: I am not here talking about back action (radiation-pressure noise).

Quantum Noise Limit (Shot-Noise Limit)

$$\Delta x_1 = \Delta x_2 = \frac{1}{\sqrt{2}}\,, \qquad \boxed{\Delta\phi = \frac{1}{\sqrt{N}}}$$

Heisenberg Limit



As much power in the squeezed light as in the main beam

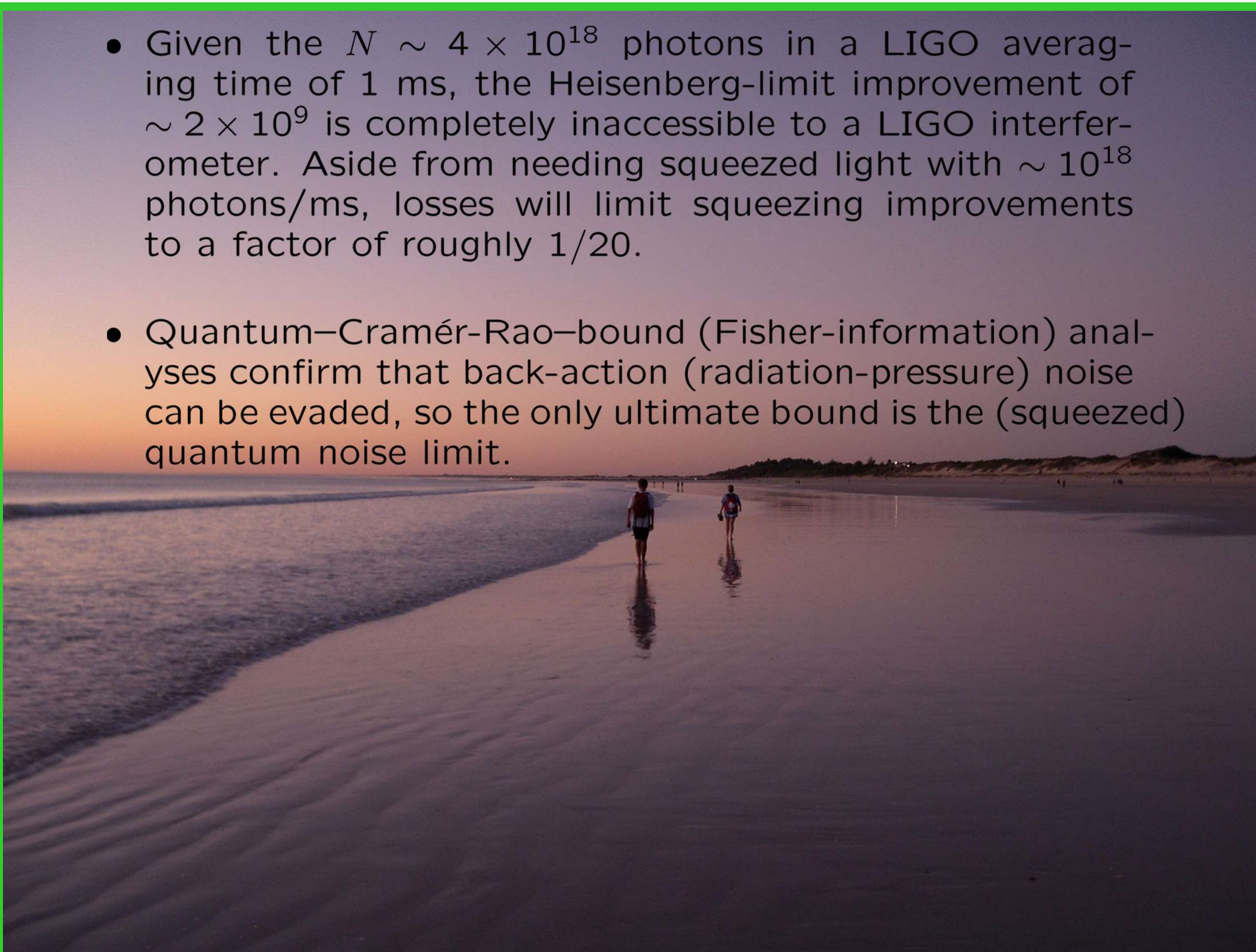
$$\frac{1}{2}(\Delta x_1)^2 \sim N$$

$$\Delta x_2 = \frac{1}{2\Delta x_1} \sim \frac{1}{2\sqrt{2N}}\,, \qquad \boxed{\Delta\phi \sim \frac{1}{2N}}$$

When do these limits hold? Do we think they're limits only because we haven't thought hard enough?

- Given the $N \sim 4 \times 10^{18}$ photons in a LIGO averaging time of 1 ms, the Heisenberg-limit improvement of $\sim 2 \times 10^{9}$ is completely inaccessible to a LIGO interferometer. Aside from needing squeezed light with $\sim 10^{18}$ photons/ms, losses will limit squeezing improvements to a factor of roughly 1/20.

- Quantum–Cramér-Rao–bound (Fisher-information) analyses confirm that back-action (radiation-pressure) noise can be evaded, so the only ultimate bound is the (squeezed) quantum noise limit.



Cable Beach
Western Australia

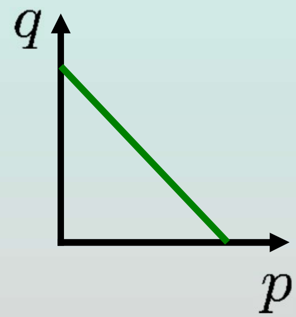
Fisher information

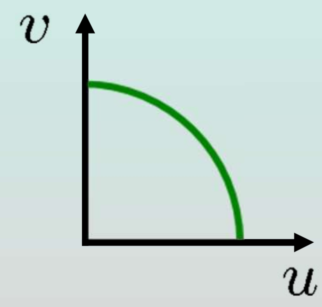
Estimating a probability *p* from *N* trials (random walk, polling)

$$f = \frac{n}{N}$$

$$\bar{f} = \frac{\bar{n}}{N} = p$$

$$(\Delta f)^2 = \frac{pq}{N} \qquad q = 1 - p$$





Measuring the "distance" between neighboring probability distributions in units of their distinguishability

$$\frac{(\delta p)^2}{(\Delta f)^2} = N\frac{(\delta p)^2}{pq} = N\left(\boxed{\frac{(\delta p)^2}{p} + \frac{(\delta q)^2}{q}}\right) = N\left((\delta u)^2 + (\delta v)^2\right)$$

Fisher information

$$u = \sqrt{p}\,, \qquad v = \sqrt{q}$$

Not $|\delta p| + |\delta q|$ or $(\delta p)^2 + (\delta q)^2$ or $(\delta p/p)^2 + (\delta q/q)^2$

(Classical) Cramér-Rao bound

For any parameter ϕ that affects a probability distribution $p_j(\phi)$,

$$\Delta\phi_{\text{est}} \geq \frac{1}{\sqrt{N}}\frac{1}{\sqrt{F}}$$

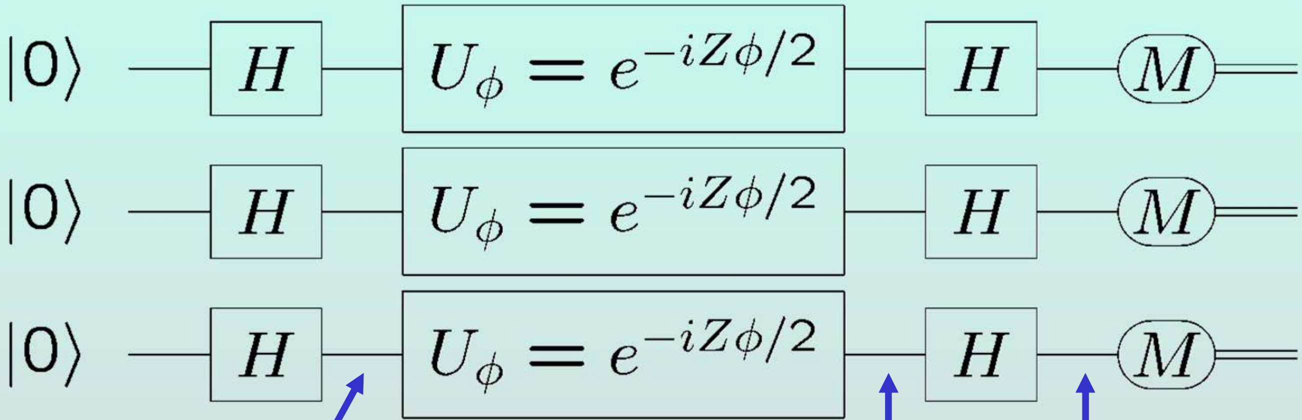
$$F(\phi) = \left(\begin{array}{c}\text{Fisher}\\ \text{information}\end{array}\right) = \sum_j \frac{1}{p_j(\phi)}\left(\frac{dp_j(\phi)}{d\phi}\right)^2$$

Quantum information version of interferometry

Qubits:

Photons with two modes

Atoms with two levels



Quantum noise limit

$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$\frac{1}{\sqrt{2}}(e^{-i\phi/2}|0\rangle + e^{i\phi/2}|1\rangle)$

$\cos(\phi/2)|0\rangle - i\sin(\phi/2)|1\rangle)$

$\frac{1}{\sqrt{2}}(e^{-iN\phi/2}|000\rangle + e^{iN\phi/2}|111\rangle)$

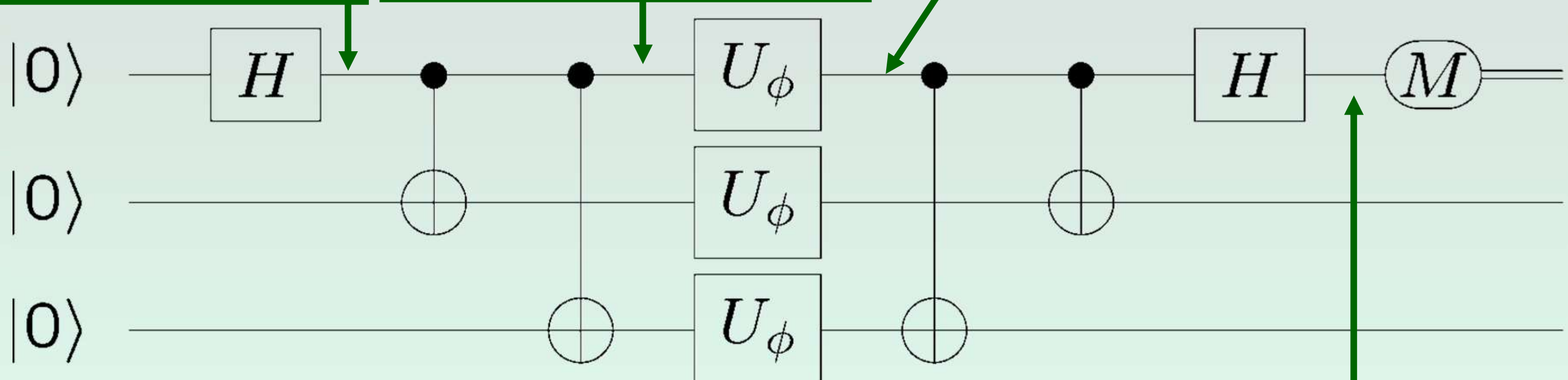
"cat state"

$N = 3$

$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|00\rangle$

$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$

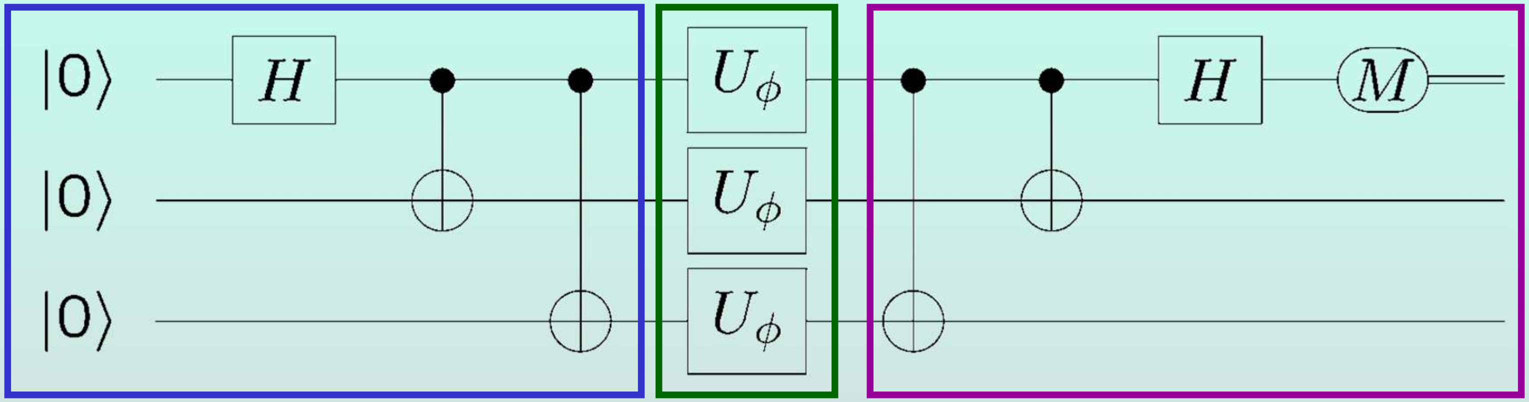
Heisenberg limit



Fringe pattern with period 2π/*N*

$[\cos(N\phi/2)|0\rangle - i\sin(N\phi/2)|1\rangle]|00\rangle$

Cat-state interferometer

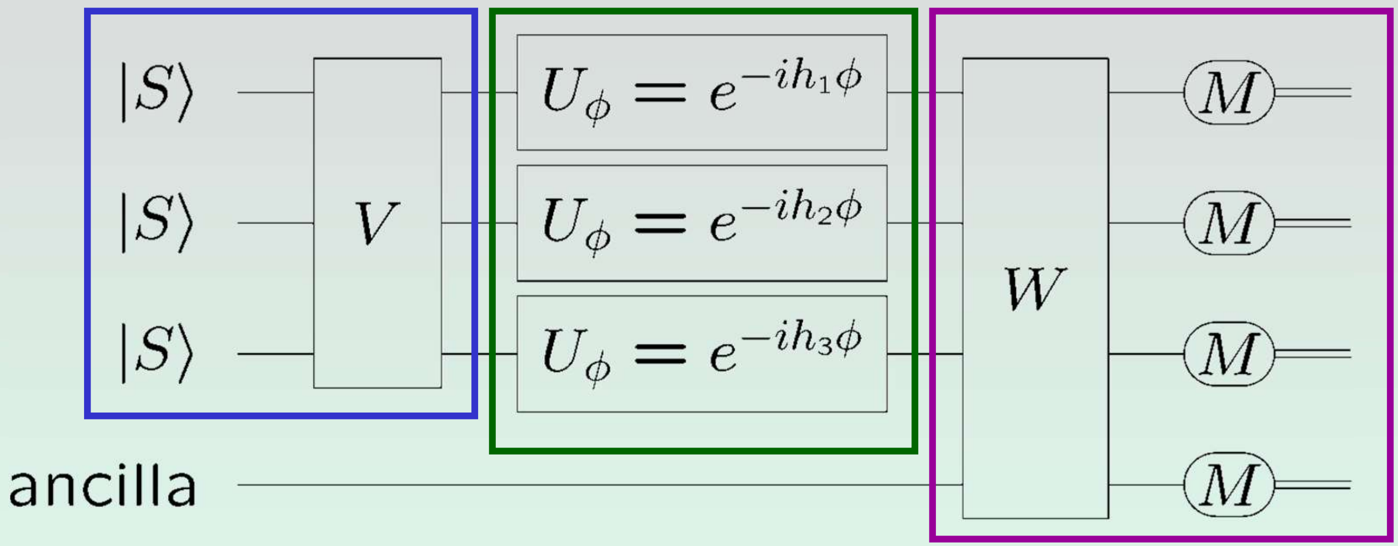


State preparation

Dynamics

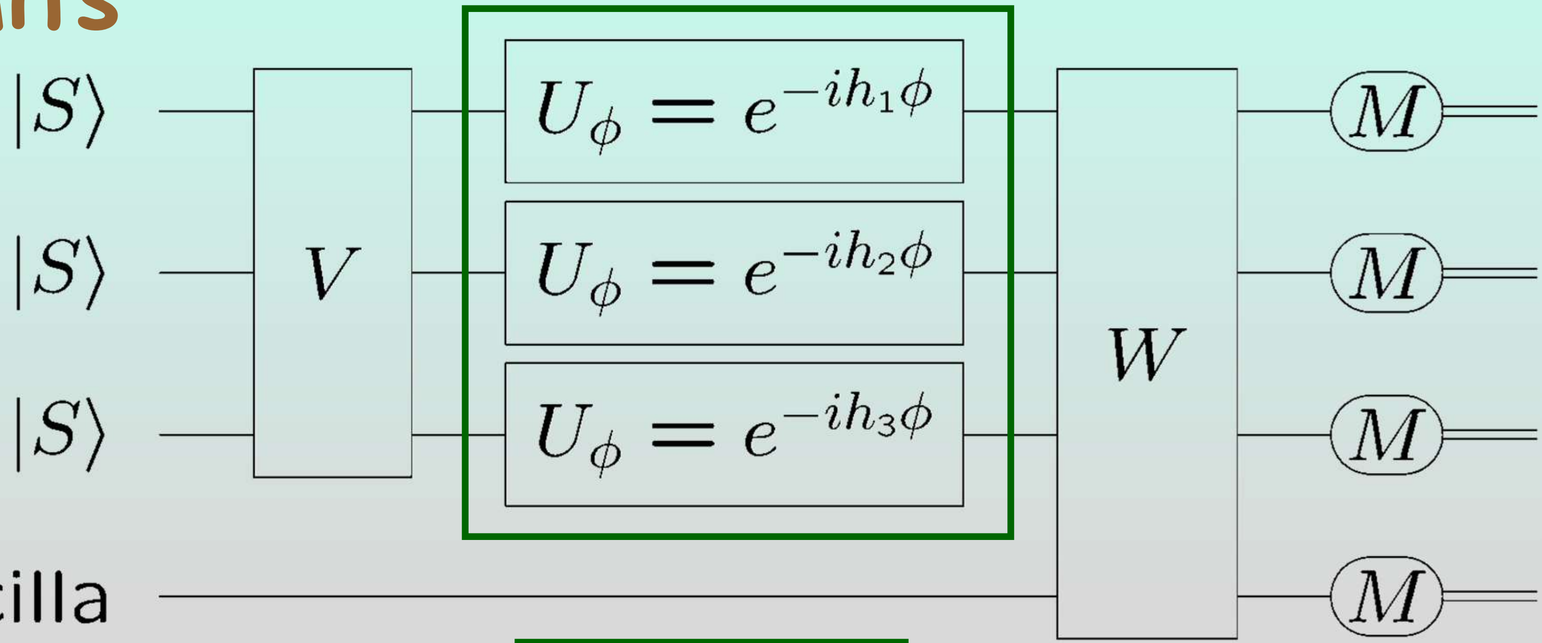
Measurement

Single-parameter estimation



Quantum limits

S. L. Braunstein, C. M. Caves, and G. J. Milburn, Ann. Phys. 247, 135 (1996).
V. Giovannetti, S. Lloyd, and L. Maccone, PRL 96, 041401 (2006).
S. Boixo, S. T. Flammia, C. M. Caves, and JM Geremia, PRL 98, 090401 (2007).



Dynamics $h = \sum_{j=1}^{N} h_j$

$$\Delta\phi_{\text{est}} \geq \frac{1}{2\Delta h} \geq \frac{1}{N(\Lambda - \lambda)}$$

Generalized uncertainty principle
Quantum Cramér-Rao bound

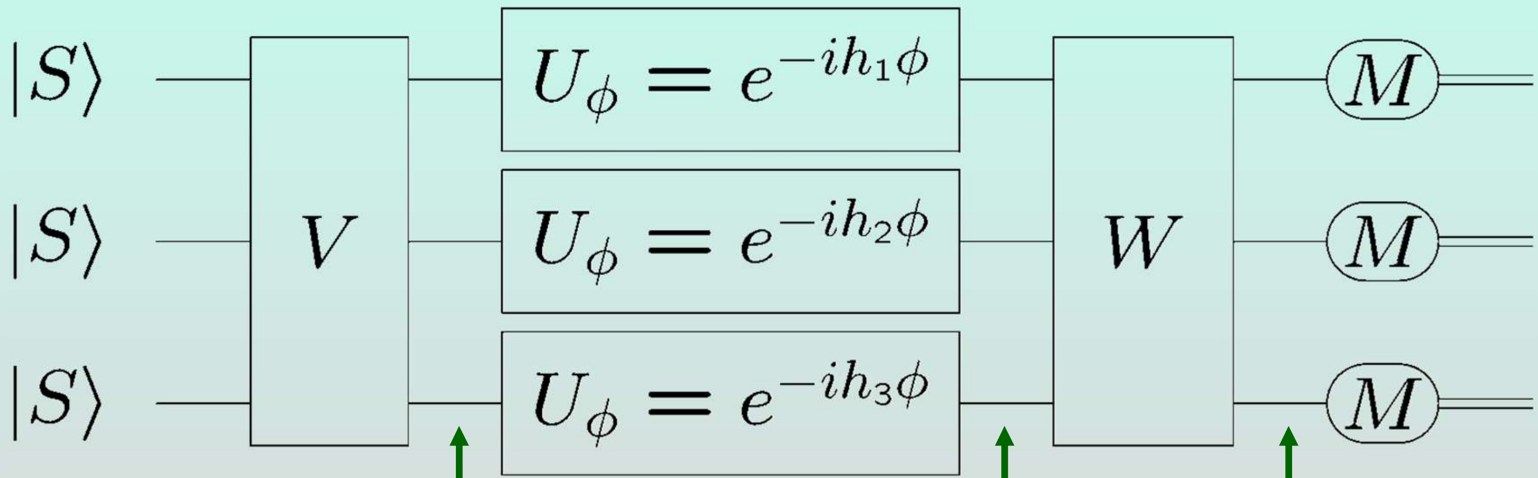
$$\Delta h \leq \frac{1}{2} N(\Lambda - \lambda)$$

Product inputs

$$\Delta h \leq \frac{1}{2}\sqrt{N}(\Lambda - \lambda)$$

$$\Delta \phi \geq \frac{1}{\sqrt{N}(\Lambda - \lambda)}$$

Achieving the Heisenberg limit



cat state $\frac{1}{\sqrt{2}}(|\Lambda,\ldots,\Lambda\rangle + |\lambda,\ldots,\lambda\rangle)$

$$\frac{1}{\sqrt{2}}\left(e^{-iN\Lambda\phi}|\Lambda,\ldots,\Lambda\rangle + e^{-iN\lambda\phi}|\lambda,\ldots,\lambda\rangle\right)$$

$$e^{-iN(\Lambda+\lambda)\phi/2}\left(\cos[N(\Lambda-\lambda)\phi/2]|\Lambda,\ldots,\Lambda\rangle - i\sin[N(\Lambda-\lambda)\phi/2]|\lambda,\ldots,\lambda\rangle\right)$$

Fringe pattern with period $2\pi/N(\Lambda-\lambda)$

$$\Delta\phi = \frac{1}{N(\Lambda-\lambda)}$$

What we really do:
Real-life quantum Cramér-Rao bound

For an optical interferometer powered by a laser, with fractional photon loss $1-\eta=\epsilon$, the optimal strategy is to put squeezed light into the vacuum port, with ultimate sensitivity having shot-noise scaling:

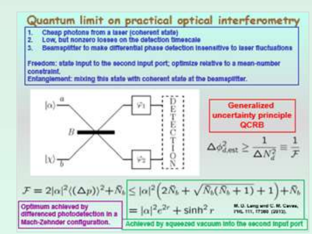
$$\boxed{\Delta\phi \simeq \sqrt{\frac{1-\eta}{\eta N}} = \sqrt{\frac{\epsilon}{(1-\epsilon)N}}\,.}$$

M. D. Lang, UNM PhD dissertation, 2015.
Z. Jiang, PRA 89, 032128 (2014).

Rule of thumb for photon losses, established by many researchers, under many circumstances.

Reaching the Heisenberg limit requires

$$\epsilon N \simeq 1\,.$$



What we really do: Cramér-Rao bound on estimating parameters of classical spacetimes

The generators corresponding to spacetime parameters are stress-energy components of probe fields.

For wideband detection of a nearly planar gravitational wave by a beam of electromagnetic radiation, the Cramér-Rao bound is

$$\Delta h \geq \frac{\hbar}{2}\frac{1}{\Delta \mathcal{X}_1},$$

where $\mathcal{X}_1$ is the wideband quadrature component that is out of phase with the mean field.

T. G. Downes, J. R. van Meter, E. Knill, G. J. Milburn, and C. M. Caves, PRD 96, 105004 (2017).

Telling explanatory stories is what physics is about.

The squeezed-light narrative is about understanding fringe patterns and sources of noise and designing devices to improve phase sensitivity based on this understanding. This is telling stories.

The quantum Cramér-Rao bound is misleading and clueless on practicality, but it verifies whether the stories, always of limited validity, are fooling us.

Which then is better, stories or proofs? You need both, but to design something, you need a story.

Squeezed light into the vacuum (antisymmetric) port is the optimal strategy for optical interferometry.

The (scientific) truth shall make you free.

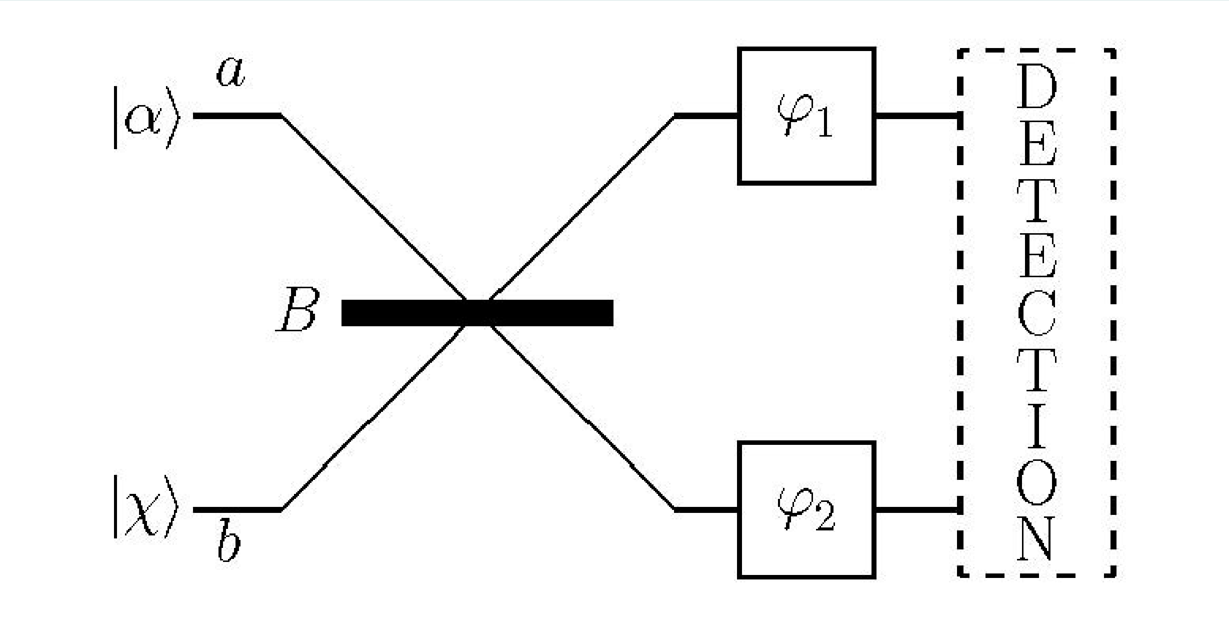
Pinnacles National Park
Central California

Dettifoss
Iceland

Quantum limit on practical optical interferometry

1. Cheap photons from a laser (coherent state)
2. Low, but nonzero losses on the detection timescale
3. Beamsplitter to make differential phase detection insensitive to laser fluctuations

Freedom: state input to the second input port; optimize relative to a mean-number constraint.
Entanglement: mixing this state with coherent state at the beamsplitter.



Generalized uncertainty principle QCRB

$$\Delta\phi^2_{d,\mathrm{est}} \geq \frac{1}{\Delta N_d^2} \equiv \frac{1}{\mathcal{F}}$$

$$\mathcal{F} = 2|\alpha|^2\langle(\Delta p)\rangle^2 + \bar{N}_b \leq |\alpha|^2\left(2\bar{N}_b + \sqrt{\bar{N}_b(\bar{N}_b+1)} + 1\right) + \bar{N}_b$$

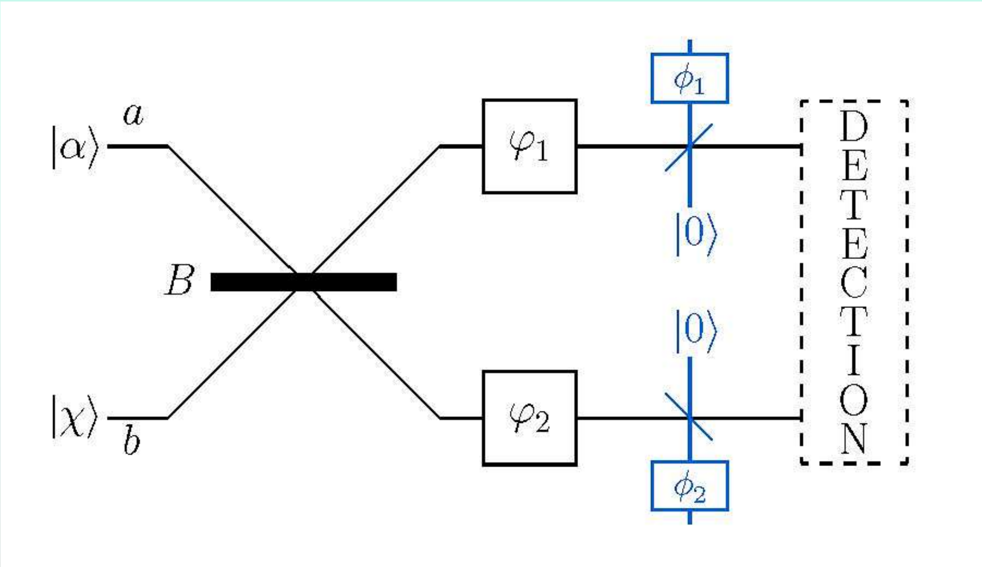
$$= |\alpha|^2 e^{2r} + \sinh^2 r$$

Optimum achieved by differenced photodetection in a Mach-Zehnder configuration.

M. D. Lang and C. M. Caves, PRL 111, 17360 (2013).

Achieved by squeezed vacuum into the second input port

Practical optical interferometry: Photon losses



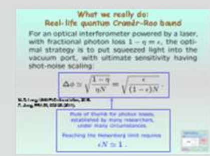
M. D. Lang , UNM PhD dissertation, 2015.

$$1-\eta = \left(\begin{array}{c}\text{fractional loss}\\ \text{in each arm}\end{array}\right)$$

$$\Delta\phi^2_{d,\mathrm{est}} \geq \boxed{\frac{1}{\mathcal{F}_Q} \geq \frac{1}{\mathcal{C}_Q} \geq \frac{1}{I_Q}}$$

B. M. Escher, R. L. de Matos Filho, and L. Davidovich, Nat. Phys. 7, 406–411 (2011).

$$\mathcal{C}_Q = \left(\begin{array}{c}\text{Upper bound on quantum Fisher information}\\ \text{maximized over fake phase shifts } \phi_1 \text{ and } \phi_2\\ \text{and over all states input to second input port}\end{array}\right)$$



$$\mathcal{F}_Q = \left(\begin{array}{c}\text{Quantum Fisher information}\\ \text{for squeezed vacuum}\\ \text{input to second input port}\end{array}\right)$$

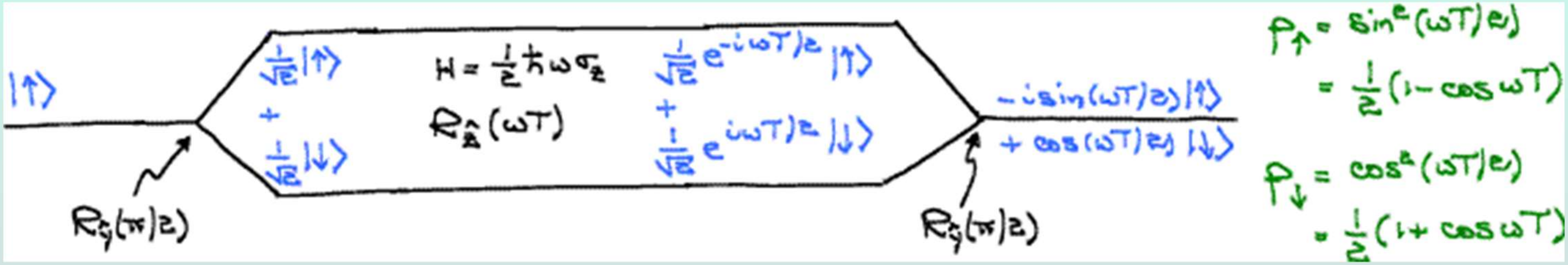
Z. Jiang, PRA 89, 032128 (2014).

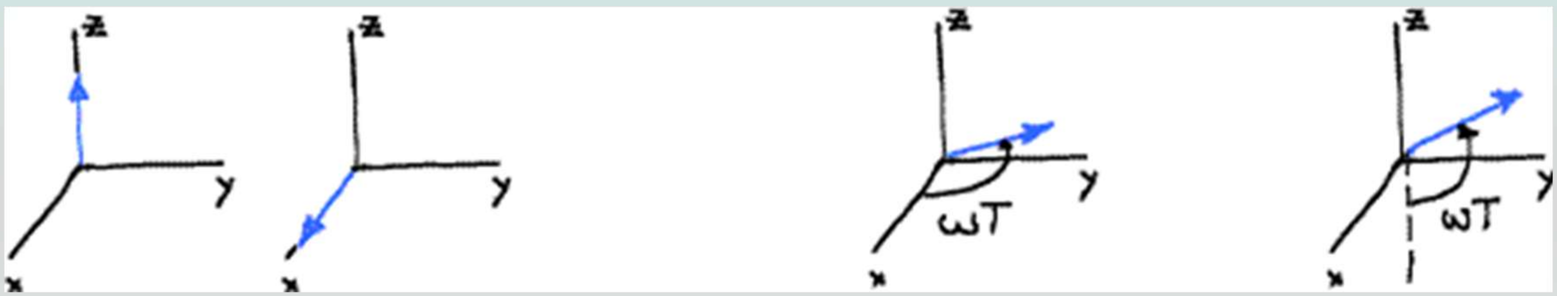
$$I_Q = \boxed{\frac{|\alpha|^2+\bar{N}_b}{\dfrac{1-\eta}{\eta}+\dfrac{1}{2\langle(\Delta p)^2\rangle}} \simeq \frac{\eta}{1-\eta}|\alpha|^2}$$

When $|\alpha|^2 \gg \bar{N}_b$, all agree to within corrections of order $\bar{N}_b/|\alpha|^2$.

Optimum achieved by differenced photodetection in a Mach-Zehnder configuration.

Ramsey (atomic) interferometry





***N* independent "atoms"**

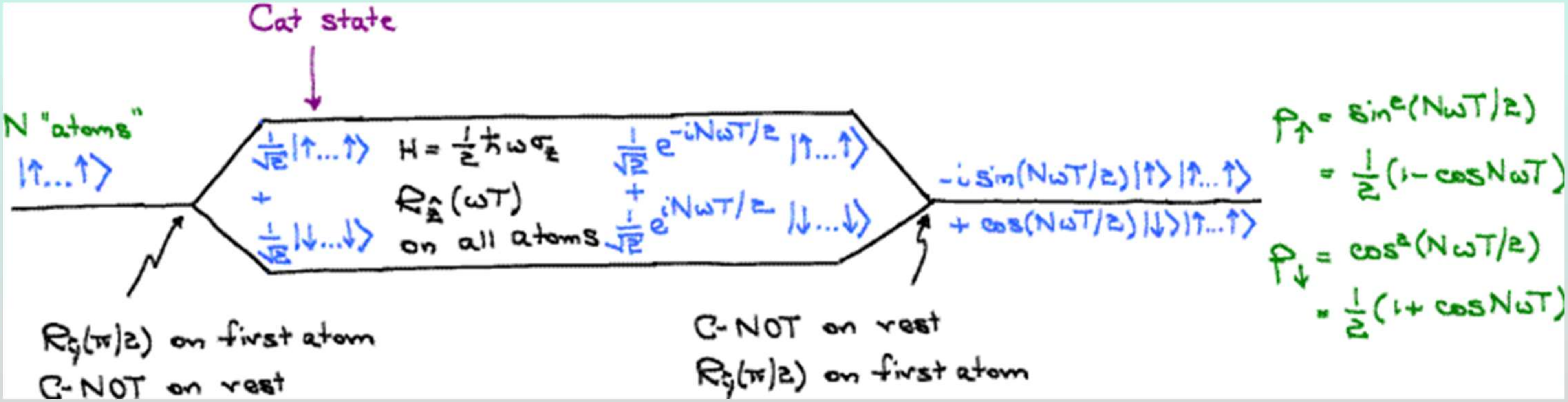
$$
\begin{aligned}
(\text{signal}) &= \langle\sigma_z\rangle = -\cos\omega T \\
(\text{noise}) &= \Delta\sigma_z = \sqrt{1-\cos^2\omega T} = |\sin\omega T|
\end{aligned}
$$

$$\Delta(\omega T) = \boxed{\frac{1}{\sqrt{N}}}\,\frac{(\text{noise})}{|d(\text{signal})/d(\omega T)|} = \frac{1}{\sqrt{N}}$$

Frequency measurement
Time measurement
Clock synchronization

Cat-state Ramsey interferometry

J. J. Bollinger, W. M. Itano, D. J. Wineland, and D. J. Heinzen, Phys. Rev. A 54, R4649 (1996).



Fringe pattern with period 2π/*N*

$$\begin{aligned}
(\text{signal}) &= \langle\sigma_z\rangle = -\cos N\omega T \\
(\text{noise}) &= \Delta\sigma_z = \sqrt{1-\cos^2 N\omega T} = |\sin N\omega T|
\end{aligned}$$

$$\Delta(\omega T) = \boxed{\frac{1}{\sqrt{\nu}}}\frac{(\text{noise})}{|d(\text{signal})/d(\omega T)|} = \frac{1}{\sqrt{\nu}}\boxed{\frac{1}{N}}$$

$\nu = (\text{number of trials})$

***N* cat-state atoms**

The Milieu: Caltech TAPIR late 70s-early 80s

Kip Thorne

Vladimir Braginsky

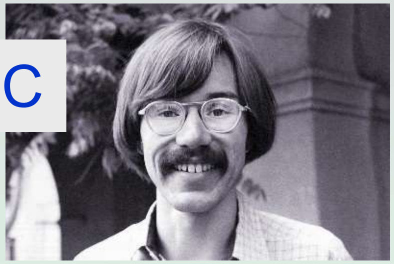
Ron Drever

Quantum nondemolition
Back-action evasion

Fabry-Perot Michelson
PDH locking
Nested cavities (recycling)
Drever interferometer

Vern Sandberg
Mark Zimmermann

CMC



Squeezed-light interferometry

Dana Anderson
Yekta Gürsel
Mark Hereld
Siu-Au Lee
Bob Spero
Stan Whitcomb

Bonny Schumaker