CQuIC Summer Course 2024: Reconstructions and Foils

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July 17, 2024

“Principled” Theories

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- Invariance between inertial reference frames

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Textbook quantum mechanical axioms tend to be somewhat less elegant...

STOP DOING QUANTUM MECHANICS

- PHYSICS WAS NOT MEANT TO BE QUANTIZED
- YEARS OF MEASURING yet NO REAL-WORLD USE FOUND for doing more than STATISTICAL MECHANICS
- Wanted to predict more for a laugh? We had a tool for that: It was called "GUESSING"
- "Yes please give me |0>+i|1>" of something. Please give ρ" - Statements dreamed up by the utterly Deranged

Look at what Physicists have been demanding your Respect for all this time, with all the time & funding we've given them

(This is REAL physics done by REAL physicists):

I. For every state there exists a unique Hermitian, nonnegative, unit-trace operator ρ.	**II**. A Hermitian oprator R on a Hilbert space exists for every physical quantity with $\langle R \rangle = \mathrm{Tr}(\rho R)$.	**III**. States evolve by the action of a central extension of the Galilei group.
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"Hello I would like a $\frac{E_i \rho E_i^\dagger}{\mathrm{Tr}\left(E_i^\dagger E_i \rho\right)}$ photon please"

They have played us for absolute fools

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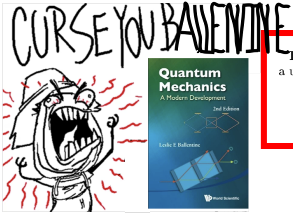

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Classical Mechanics

- States are specifications of distributions of initial positions and momenta $\rho(Q, P)$
- Time evolution by Hamilton's equations
- Physical quantities are functions of $\{Q, P\}$, their expected values are obtained averaging with respect to the distribution: $\langle A \rangle = \int dPdQA(Q, P)\rho(Q, P)$.

Quantum Mechanics

- States are unit-trace nonnegative operators on a Hilbert space $\rho \in \mathcal{B}_1(\mathcal{H})$
- Time evolution by Schrödinger's equation or measurement effects
- Expected values of physical quantities are given by their overlap with the state: $\langle A \rangle = \mathrm{Tr}(A\rho)$.

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Classical analogies still neither explain

- *why* these particular mathematical changes are necessary
- the physical intuition behind these changes

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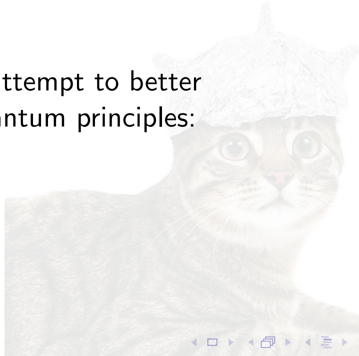

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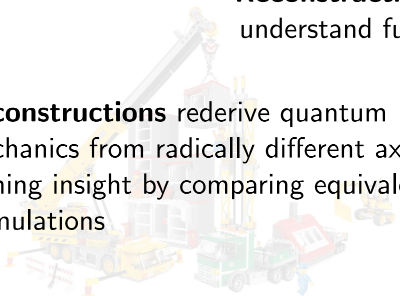
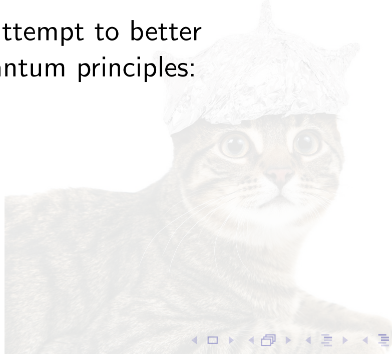


Reconstructions rederive quantum mechanics from radically different axioms, gaining insight by comparing equivalent formulations



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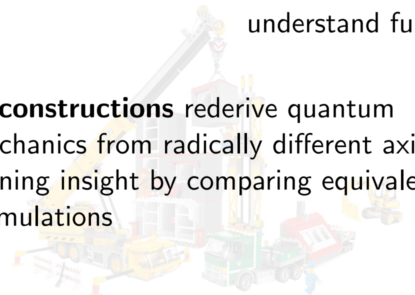

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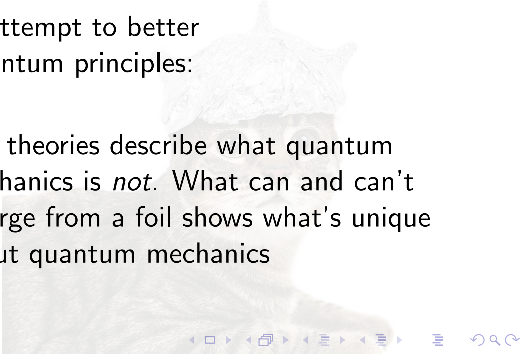

Foil theories describe what quantum mechanics is *not*. What can and can't emerge from a foil shows what's unique about quantum mechanics

What Do We Care About?

A clear axiomitization can hopefully shed light on what advantages quantum mechanics has over other theories

- Probability: to match experiments: we can't always know outcomes in advance
- Causality: weaker determinism enables non-definite predictions
- Locality: successful theories are often local
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We know these principles are at odds within conventional quantum mechanics

Spekkens' Toy Model (Covered Last Time)

Key idea is "**knowledge balance**": amount of knowledge of the **ontic state** of a system is equal to the information lacked

Reproduces several elements of quantum mechanics:

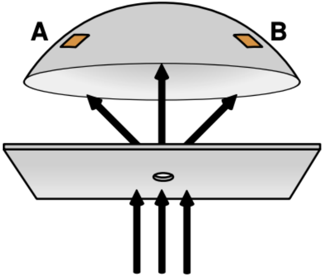
- noncommutativity of measurements
- teleportation
- no-cloning

However, this toy model **does not feature**

- Nonlocality
- Contextuality

Investigating features of nonlocality in models is a common theme of both reconstructions and foils

Einstein 1927: Definite and Indefinite



A single particle diffracting at the hole is associated with some $|\psi\rangle$ such that $p(A|\psi) = p(B|\psi) = |\langle A|\psi\rangle|^2 = |\langle B|\psi\rangle|^2 \neq 0$

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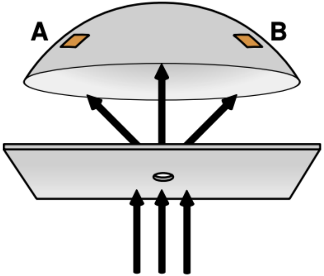

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Question: are these the probabilities that **a** particle is detected at A/B, or that **the** particle is detected?

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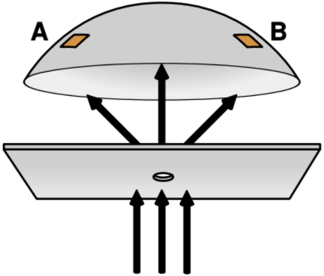

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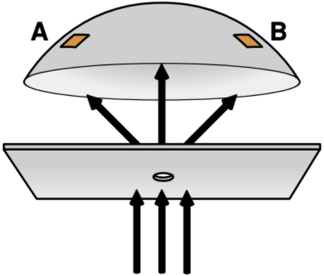

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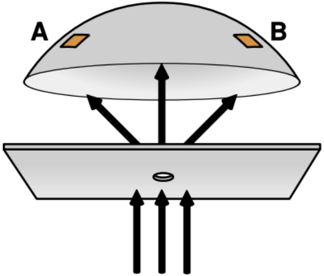

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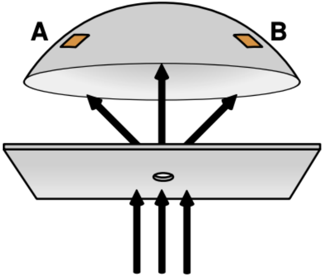

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Since $p(A|\psi) = p(B|\psi) \neq 0$, $p(A\&B|\psi) \neq 0$. **This contradicts experiment**

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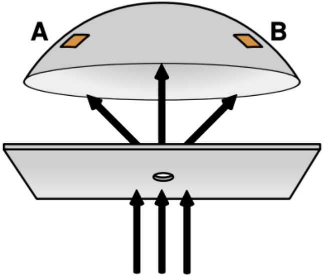

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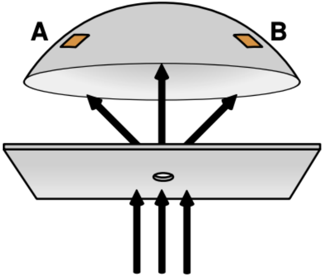

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However, it gives us two flavors to combine:

- **Interpretation of quantum mechanics**
- **(Non)-locality**

Harrigan and Spekkens

Einstein, incompleteness, and the epistemic view of quantum states

Nicholas Harrigan
QOLS, Blackett Laboratory, Imperial College London,
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Robert W. Spekkens
Department of Applied Mathematics and Theoretical Physics,
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(Dated: June 15, 2007)

Does the quantum state represent reality or our knowledge of reality? In making this distinction precise, we are led to a novel classification of hidden variable models of quantum theory. Indeed, representatives of each class can be found among existing constructions for two-dimensional Hilbert spaces. Our approach also provides a fruitful new perspective on arguments for the nonlocality and incompleteness of quantum theory. Specifically, we show that for models wherein the quantum state has the status of something real, the failure of locality can be established through an argument considerably more straightforward than Bell's theorem. The historical significance of this result becomes evident when one recognizes that the same reasoning is present in Einstein's preferred argument for incompleteness, which dates back to 1935. This fact suggests that Einstein was seeking not just *any* completion of quantum theory, but one wherein quantum states are solely representative of our knowledge. Our hypothesis is supported by an analysis of Einstein's attempts to clarify his views on quantum theory and the circumstance of his otherwise puzzling abandonment of an even simpler argument for incompleteness from 1927.

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Primary idea: nonlocality is easier to establish for "ontic" theories

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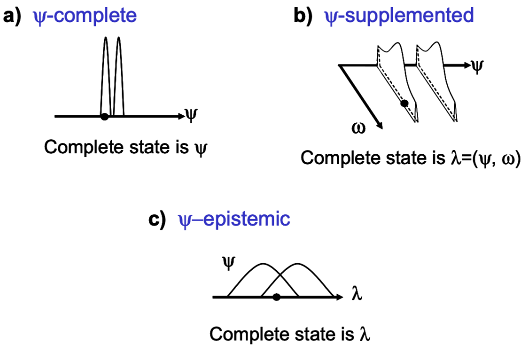
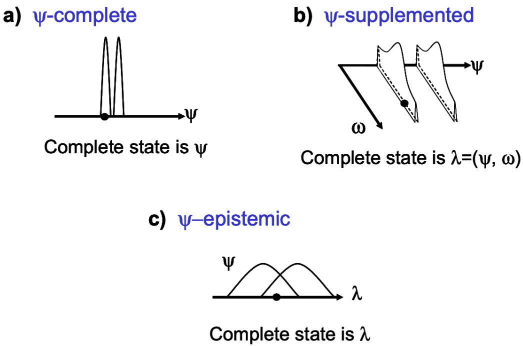
For a preparation P producing the quantum state ψ, the last requires the model reproduce standard quantum probabilities.

Locality and Epistemics: Three Ontological Models



Locality and Epistemics: Three Ontological Models

a) ψ-Complete: ψ fully and uniquely describes reality



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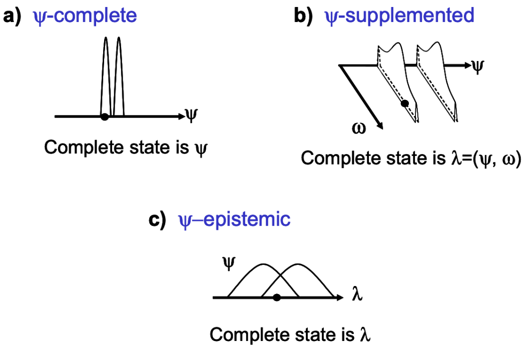

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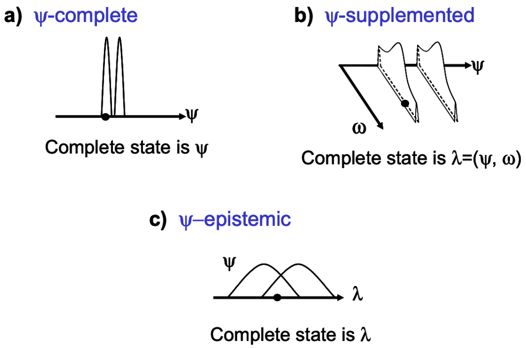
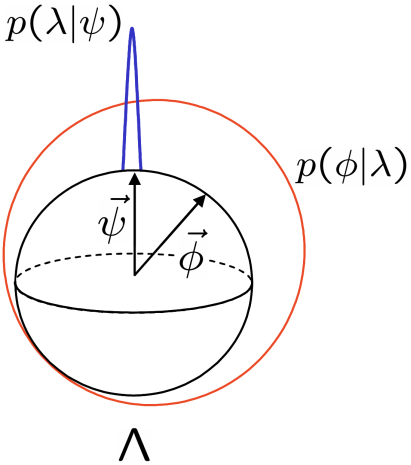
c) ψ-Epistemic: ψ is a heuristic: multiple describe the same reality



ψ-Complete: Beltrametti-Bugajski Model

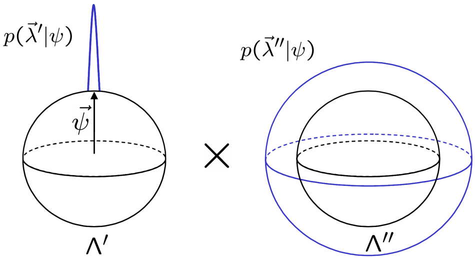
- Recasts "orthodox" spin-$\frac{1}{2}$ quantum mechanics in foundational terms.
- ψ is everything, i.e. **complete**: $\Lambda = P\mathcal{H}$ – "$p(\lambda|\psi) = \delta(\lambda - \psi)$"
- Standard probabilities recovered:

$$p(k|M,\psi) = \int_\Lambda d\lambda p(k|\lambda, M)p(\lambda|\psi), \\ = \int_\Lambda d\lambda \mathrm{tr}\left(|\lambda\rangle\langle\lambda|E_k\right)\delta(\lambda - \psi), \\ = \mathrm{tr}\left(|\psi\rangle\langle\psi|E_k\right).$$



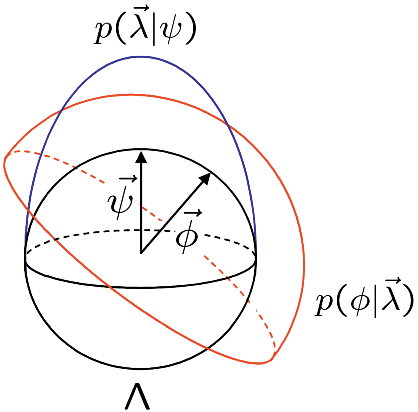
ψ-Supplemental: Bell-Mermin Model

- Ontic space is a product of two Bloch spheres $\Lambda = \Lambda' \times \Lambda''$ – Hilbert space is **supplemented**
- Probability factors: $p(\lambda', \lambda''|\psi) = p(\lambda'|\psi)p(\lambda''|\psi) = \delta(\lambda' - \psi) \times \frac{1}{4\pi}$
- Measure $|\phi\rangle$ and its orthonorm using $p(\phi|\lambda', \lambda'') = \Theta(\phi \cdot (\lambda' + \lambda''))$: 1 if ϕ is pointing in the same direction as λ and λ''.
- Measurement probabilities reproduce normal spin-$\frac{1}{2}$ quantum mechanics



ψ-Epistemic: Kochen-Specker Model

- Ontic space Λ is just the Bloch sphere, but $p(\lambda|\psi) = \frac{1}{\pi}\Theta(\psi \cdot \lambda)\psi \cdot \lambda$.
- Distribution over Λ: $p(\phi|\lambda) = \Theta(\phi \cdot \lambda)$ that is, ψ is **epistemic**
- Measurement outcomes still reproduce "normal" quantum probabilities



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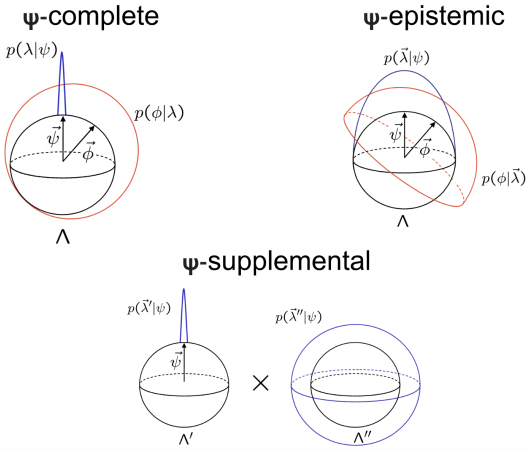

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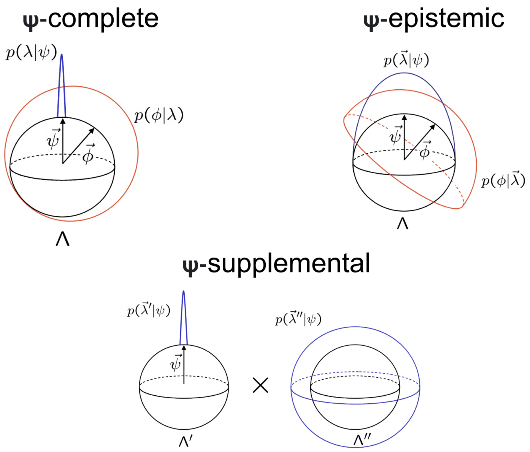

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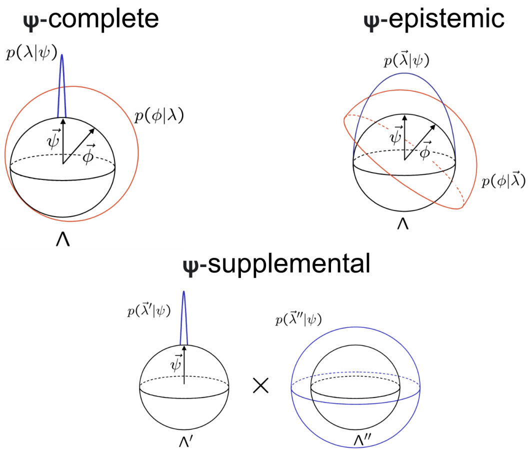

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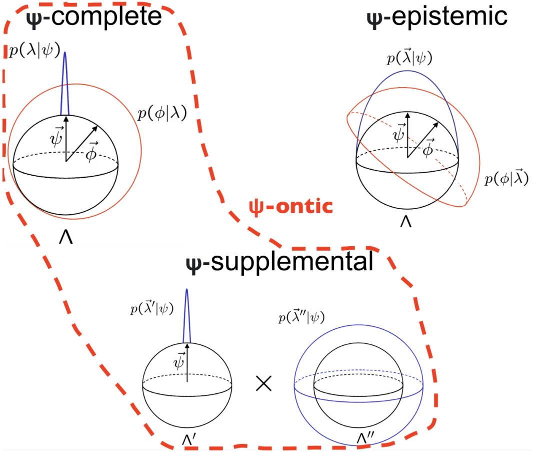
- **Further takeaways from thinking about "ontic" theories...**



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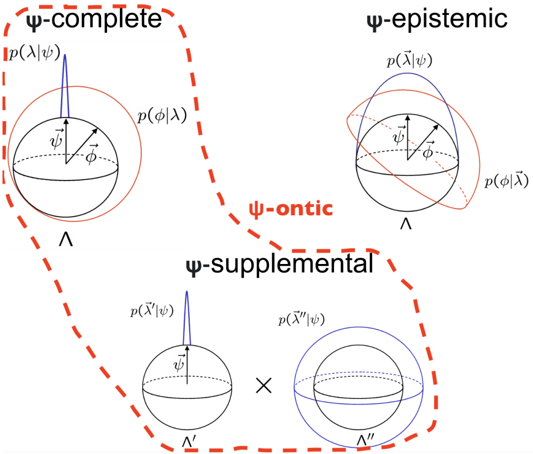
Such interpretations the authors refer to as being ψ-**ontic**



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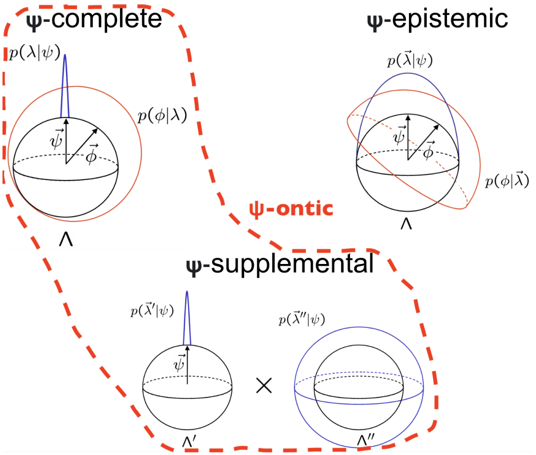
Concretely, **ontic** means that for any distinct preparations P_ψ and P_ϕ leading to quantum states ψ and ϕ, $p(\lambda|P_\psi)p(\lambda|P_\phi) = 0 \ \forall\lambda$.



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- Locally Causal: $p(B|A, \lambda_C) = p(B|\lambda_C)$ for a region C "screening" $J^-(A) \cap J^-(B)$.

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- Locally Causal: $p(B|A, \lambda_C) = p(B|\lambda_C)$ for a region C "screening" $J^-(A) \cap J^-(B)$.

None of these Words Mean What they Should

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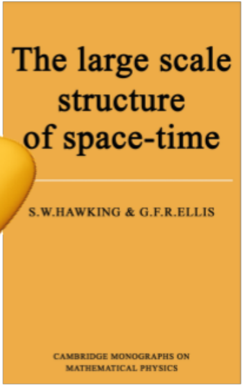
The authors modify another argument of Einstein from 1935 (not EPR) to show that these definitions are at odds with an ontic model



The Argument

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Therefore $p(\lambda|A_\pm)p(\lambda|A_{0/1}) > 0$ for some choice of possibilities, **contradicting the ontic assumption**

Switching the Order

We've seen that indeterminism and epistemics are to an extent interchangeable

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Note that a causal nonlocal theory may be indeterministic

Popescu and Rohrlich

Quantum Nonlocality as an Axiom

Sandu Popescu[1] **and Daniel Rohrlich**[2]

Received July 2, 1993; revised July 19, 1993

In the conventional approach to quantum mechanics, indeterminism is an axiom and nonlocality is a theorem. We consider inverting the logical order, making nonlocality an axiom and indeterminism a theorem. Nonlocal "superquantum" correlations, preserving relativistic causality, can violate the CHSH inequality more strongly than any quantum correlations.

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Exactly what it says on the tin

How to Define

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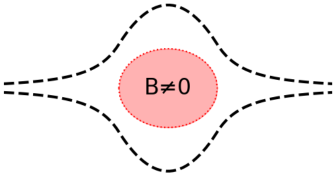
- Nonlocal equations of motion (Aharanov-Bohm effect)



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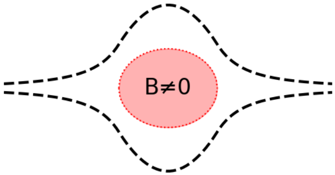

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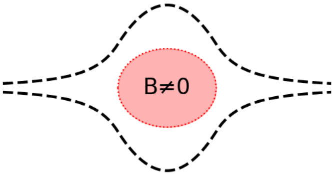
Even assuming just "nonlocal correlations" leads to some surprises...



Recall CHSH

Take a system with two parts far from one another with corresponding pairs of observables A, A' and B, B' taking values ± 1

$$E(A, B) = P_{AB}(1, 1) + P_{AB}(-1, -1) - P_{AB}(-1, 1) - P_{AB}(1, -1)$$

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$$|E(A,B) + E(A,B') + E(A',B) - E(A',B')| \leq 2$$

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If all the correlations were independent the upper bound would be 4 instead

Marginality

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If the two parts are distant, then the choice of measurement basis for one system should not affect the probabilities of the other:

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$$\begin{aligned} P_{AB}(1,1) = P_{AB}(-1,-1) = P_{AB'}(1,1) = P_{AB'}(-1,-1) = P_{A'B}(1,1) \\ = P_{A'B}(-1,-1) = P_{A'B'}(1,-1) = P_{A'B'}(-1,1) = \tfrac{1}{2}, \end{aligned}$$

One can show $E(A,B) + E(A,B') + E(A',B) - E(A',B') = 4$

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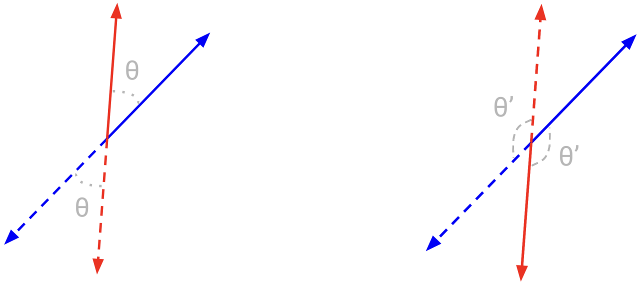

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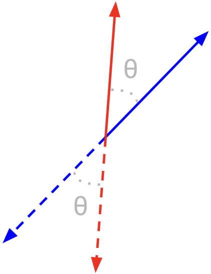
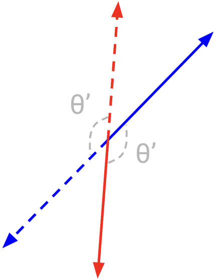
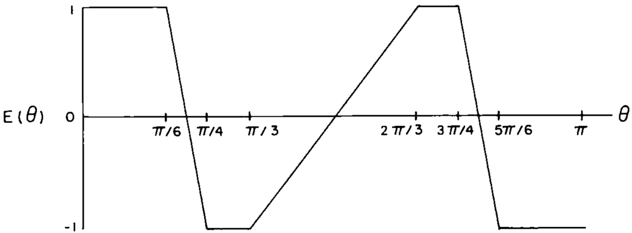
- Rotating by π, $P(\uparrow\uparrow) = P(\downarrow\downarrow)$ and $P(\uparrow\downarrow) = P(\downarrow\uparrow)$
- $\pi - \theta$ corresponds to flipping one of the measurement axes, $E(\pi - \theta) = -E(\theta)$

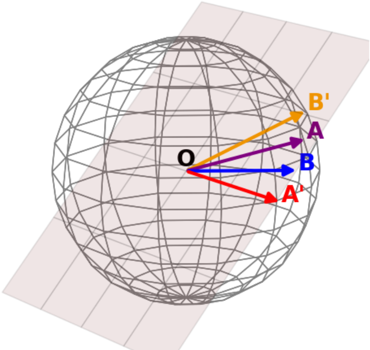




Better Example (continued)

Take the "superquantum" correlation function on the left, and let the measurement axes A', B, A, and B' be separated by $\frac{\pi}{12}$ all in a plane





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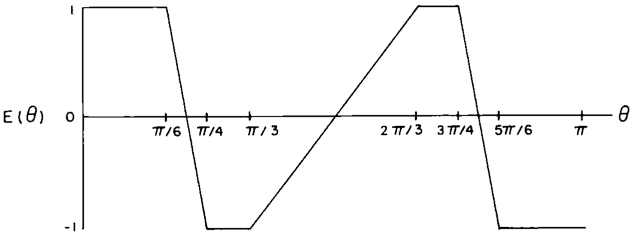

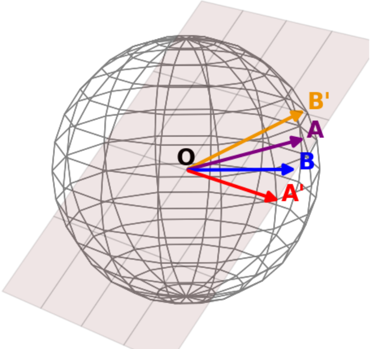

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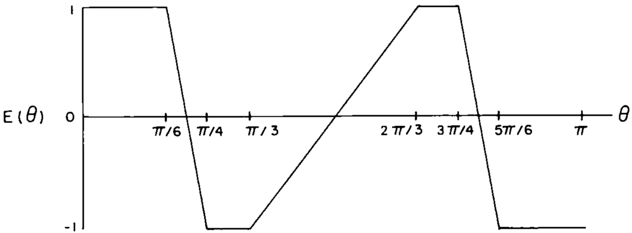
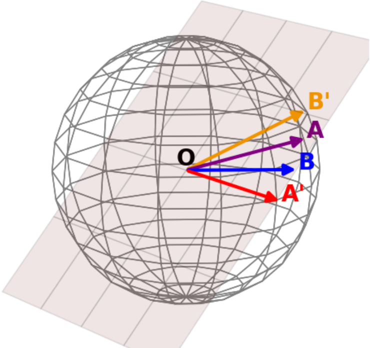
$$E(A,B) + E(A',B) + E(A,B') - E(A',B') = 3E(\tfrac{\pi}{12}) - E(\tfrac{\pi}{4}) = 4.$$

$E(\theta)$ is silly, but it shows that **correlations stronger than quantum mechanics can be imagined**





Takeaway

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More constraints are necessary to recover standard quantum mechanics

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More constraints are necessary to recover standard quantum mechanics

One might envision that superquantum phenomena possess further advantages...

(Pass to Robby)

How Foils and Reconstructions (attempt to) Work

Robert Kramer's follow up to Cole Kelson-Packer

July 17, 2024

Outline

1. Scalar "Quantum" Theory
2. Modal "Quantum" Theory
3. Generalized Probability Theories (GPTs)

Real-Vector Space Quantum Theory

"The principle of generating small amounts of *finite* improbability by simply hooking the logic circuits of a Bambleweeny 57 Sub-Meson Brain to an atomic vector plotter suspended in a strong Brownian Motion producer (say a nice hot cup of tea) were of course well understood" ... "Many respectable physicists said that they weren't going to stand for this, partly because it was a debasement of science" - Douglas Adams

Turning Hilbert Space into A Real-Vector Space

1. A pure state is represented by a unit vector in a Hilbert space over the complex numbers.
2. An ideal repeatable measurement is represented by a set of orthogonal projection operators whose supports span the vector space.
3. A reversible transformation is represented by a unitary operator U
4. A composite system has as its state space the tensor product of the state spaces of its components

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Wooters, 2013 arxiv:quant-ph/1301.2018

The “Real” Schrödinger Equation

The -iH operator in the Schrödinger Equation can be replaced with an antisymmetric real operator S:

$$i\hbar \frac{d}{dt} |\psi\rangle = H |\psi\rangle \rightarrow \frac{d}{dt} |s\rangle = S |s\rangle \tag{1}$$

So for a time-independent S, the state evolves according to

$$|s(t)\rangle = e^{St} |s(0)\rangle \tag{2}$$

Wooters, 2013 arxiv:quant-ph/1301.2018

Optimal Information Transfer

Unlike standard Quantum Mechanics, real vector quantum theory features optimal information transfer.

Consider an initial state $|\psi\rangle$ where Alice performs a unitary to transfer the state into $U|\psi\rangle$ and then sends it to Bob.

For Hilbert spaces of dimensions 3 or greater, the SIC-POVM measurement does not give Bob the maximum information about Alice's unitary.

In real vector quantum theory, Bob always can find a universally optimal measurement regardless of the dimension of the space.

Wooters, 2013 arxiv:quant-ph/1301.2018

Modal Quantum Theory

"such a machine is a *virtual* impossibility, then it must logically be a *finite* improbability. So all I have to do in order to make one is to work out exactly how improbable it is, feed that figure into the finite improbability generator, give it a fresh cup of really hot tea ... and turn it on!" - Douglas Adams

Intro to the Mobit

Unlike the *probabilistic* theory of actual quantum physics (AQP), modal quantum physics (MQP) is a *possibilistic* theory, only able to distinguish between possible and impossible.

The simplest example of a model system is the "mobit," which we can define on a two-dimensional vector space $\mathcal{V}$ on the field $\mathcal{F} = \mathbb{Z}_2$, which has three unit vectors: $|0), |1)$, and $|\sigma) = |0) + |1)$.
The dual space also has three unit vectors, giving the outcomes:

$$
\begin{array}{lll}
(a|0) = 1 & (a|1) = 0 & (a|\sigma) = 1 \\
(b|0) = 0 & (b|1) = 1 & (b|\sigma) = 1 \\
(c|0) = 1 & (c|1) = 1 & (c|\sigma) = 0
\end{array}
$$

Schumacher, Westmoreland, 2012 arxiv:quant-ph/1204.0701

Modal Entanglement

Like AQP, MQP demonstrates entanglement phenomenon in joint systems. Consider the "singlet"

$$|S) = |0,1) - |1,0) \tag{3}$$

for any effect $(e|$

$$(e,e|S) = (e|0)(e|1) - (e|1)(e|0) = 0 \tag{4}$$

Just like the actual singlet state, identical measurement outcomes on the modal singlet are anti-correlated.

Schumacher, Westmoreland, 2012 arxiv:quant-ph/1204.0701

Modal Measurement

Any effect (either time evolution or measurement) is just assigning any subspace M of $\mathcal{V}$ into possible or impossible.

$$E(M) = \begin{cases} \text{posible}, & \text{if } (e|m) \neq 0 \text{ for some } (e| \in E, |m) \in M \\ \text{impossible}, & \text{if } (e|m) = 0 \text{ for all } (e| \in E, |m) \in M \end{cases}$$

For some generalized measurement E_a, we expect at least one result to always occur, so we have the "normalization" condition

$$\langle \bigcup_a E_a \rangle = \mathcal{V}^* \tag{5}$$

Schumacher, Westmoreland, 2012 arxiv:quant-ph/1204.0701

Modal Evolution

In AQP, we can rewrite any evolution (channel) on a quantum state as continuous measurement of some Kraus operators:

$$\mathcal{E}(\rho) = \mathrm{Tr}_{(E)} U(\rho_{(S)} \otimes |0\rangle\langle 0|_{(E)}) U^{\dagger} = \sum_k A_k \rho A_k^{\dagger}$$

The same rewriting is possible in MQP:

$$\mathcal{E}(M_{(S)}) = R_{(E)}(T_{(SE)}(M_{(S)} \otimes M_{0(E)})) = \langle \bigcup_k A_k M_{(S)} \rangle$$

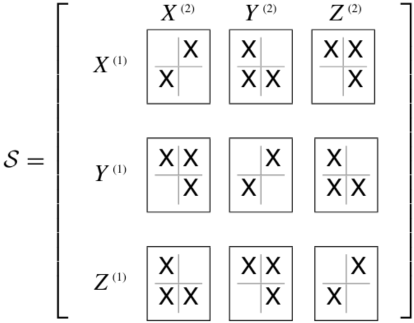
Where

$$A_k|\phi_{(S)}) = (e_{k(E)}|T_{(SE)}|\phi_{(S)}, 0_{(E)}), \quad \bigcap_k \ker A_k = \langle 0 \rangle$$

Schumacher, Westmoreland, 2012 arxiv:quant-ph/1204.0701

Possibility Tables

- Possibility tables illustrate which measurement outcomes are possible on a given modal state.
- **X**'s indicate a possible outcome, while blank squares indicate impossible outcomes

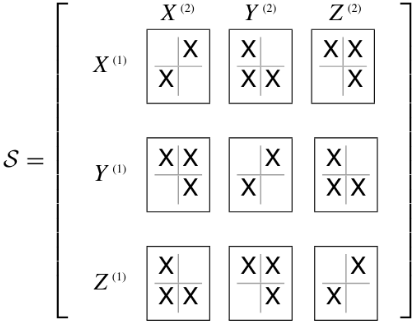


The possibility table for our modal singlet state

Schumacher, Westmoreland, 2012 arxiv:quant-ph/1204.0701

No Signaling

- The measurement performed on one subsystem should not be able to affect the measurement outcomes on the other.
- In this case, that means if an **X** occurs in the table, at least one **X** must occur in corresponding sub-rows and sub-columns.



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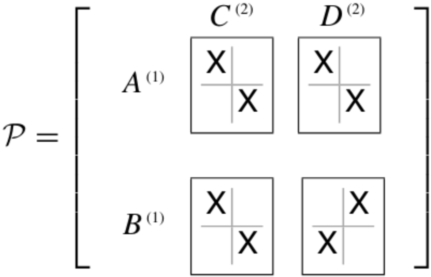
Modal PR Box

- Possibility tables also feature a PR box that agrees with no-signaling.
- However, one can prove that no mobit state produces this possibility table.



The possibility table for the mobit PR box. (**X**'s are replaced with 1/2 in the probabilistic setting)

Schumacher, Westmoreland, 2012 arxiv:quant-ph/1204.0701

MQP Summary

MQP does not have

- Probabilities
- Continuous sets of states and observables
- Inner products, outer products, orthogonality
- Convexity
- Hermitian conjugation
- Density operators
- Effect operators
- CP maps
- Unextendable product bases

But MQP does have:

- Distinct "classical" and "quantum" theories
- Superposition, interference
- Complementary measurements
- Entanglement
- No local hidden variables
- Superdense coding, teleportation
- Mixed states, generalized effects, generalized evolution maps
- No cloning theorem
- Nonclassical computation models

Schumacher, Westmoreland, 2012 arxiv:quant-ph/1204.0701

Generalized Probability Theories

"He did this, and was rather startled to discover that he had managed to create the long-sought-after golden Infinite Improbability generator out of thin air. It startled him even more when just after he was awarded the Galactic Institute's Prize for Extreme Cleverness he got lynched by a rampaging mob of respectable physicists who had finally realized that the one thing they really couldn't stand was a smart-ass" - Douglas Adams

How does one Generalize Probability?

- One can think about GPTs as the convex-operational approach to the foundations of quantum mechanics.
- The key difference between GPTs and standard probability theory is we do not assume the ability to perform simultaneous measurements.
- "The resulting post-classical probability theory, while in some respects rather conservative, presents a vast, poorly explored, and rather wild landscape, within which even quantum probability theory seems rather tame." [1]

[1] Barnum, Wilce. 2012 arxiv:quant-ph/1205.3833

Axioms of State Space

State space is

- a set of points
- convex
- closed in some physically motivated topology
- bounded
- a subset of a real, finite-dimensional vector space with Euclidean topology.

Plávala. 2021. arxiv: quant-ph/2103.07469

State Space

- Let x be a state in the state space K. x is a *pure state* if $x = \lambda y + (1 - \lambda) z \implies y = z$. $(y, z \in K)$.
- Let x be a state in the state space K. if x is not a pure state (i.e. $x = \lambda y + (1 - \lambda) z$ for some $y \neq z$), then it is a mixed state
- Theorem: For a state space K, say $\text{ext}(K)$ is the set of pure states of K. Then $K = \text{conv}(\text{ext}(K))$

Plávala. 2021. arxiv: quant-ph/2103.07469

Effect algebra

- Let K be a state space, then the *effect algebra* over K will be denoted $E(K)$. $E(K)$ is the set of all affine functions $f : K \to [0,1]$:

$$0 \leq f(x) \leq 1, \quad f(\lambda x + (1-\lambda)y) = \lambda f(x) + (1-\lambda)f(y) \tag{6}$$

- $1_K \in E(K)$ is the constant function giving $1_K(x) = 1$ for all $x \in K$
- Proposition: Let $A(K)$ be the vector space of affine functions on K and let $A(K)^+$ be the cone of positive affine functions on K.
  - Then $A(K) = \text{span}(E(K))$ and $A(K)^+ = \text{cone}(E(K))$

Plávala. 2021. arxiv: quant-ph/2103.07469

Classical Theory

A classical theory is one containing n independent pure states, each a distribution over the probabilities $(p_1, ..., p_n)$. Of course, $\sum_i p_i = 1$.

This tells us that the state space of a classical theory is a simplex S_n:

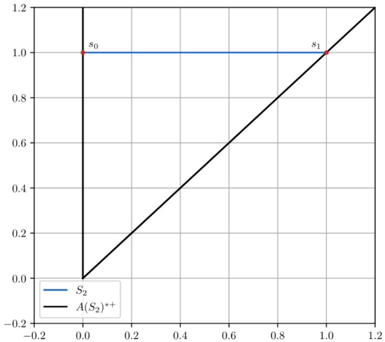
$$S_n = \mathrm{conv}(\{s_1, ..., s_n\}) \tag{7}$$

The effect algebra $E(S_n)$ can be generated by the functions $b_1, ..., b_n$ such that $\langle s_i, b_j \rangle = \delta_{ij}$. It is easy to see:

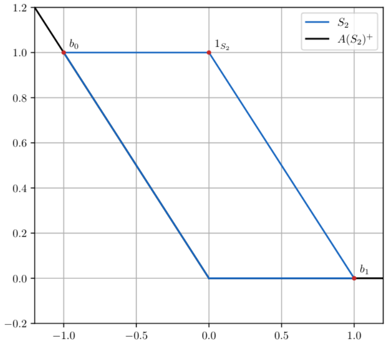
$$1_{S_n} = \sum_{i=1}^{n} b_i \tag{8}$$

Plávala. 2021. arxiv: quant-ph/2103.07469

S_n for $n = 2$



(a) Picture of the state space S_2 as subset of $A(S_2)^*$. The red points are the pure states s_0 and s_1, the blue line is the state space S_2, and the black lines are the boundary of the positive cone $A(S_2)^{*+}$.

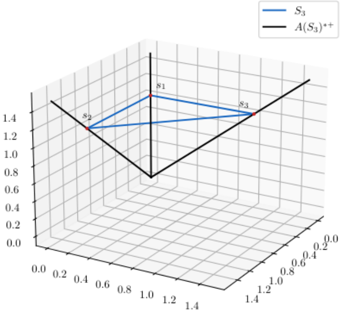


(b) Picture of the effect algebra $E(S_2)$ as subset of $A(S_2)$. The red points are the effects b_0, b_1, 1_{S_2}, the blue lines are the boundary of the effect algebra $E(S_2)$ and the black lines are the boundary of the positive cone $A(S_2)^+$.

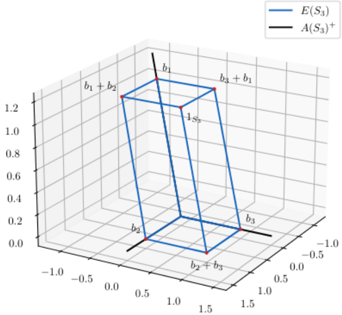
S_2 and the effect algebra $E(S_2)$

Plávala. 2021. arxiv: quant-ph/2103.07469

S_n for $n = 3$



(a) Picture of the state space S_3 as subset of $A(S_3)^*$. The red points are the pure states s_1, s_2 and s_3, the blue lines are the edges of the state space S_3, and the black lines are the edges of the positive cone $A(S_3)^{*+}$.



(b) Picture of the effect algebra $E(S_3)$ as subset of $A(S_3)$. The red points are the effects b_1, b_2, b_3, $b_1 + b_2$, $b_2 + b_3$, $b_3 + b_1$, 1_{S_2}, blue lines are the edges of the effect algebra $E(S_3)$, and the black lines are the edges of the positive cone $A(S_3)^+$.

S_3 and the effect algebra $E(S_3)$

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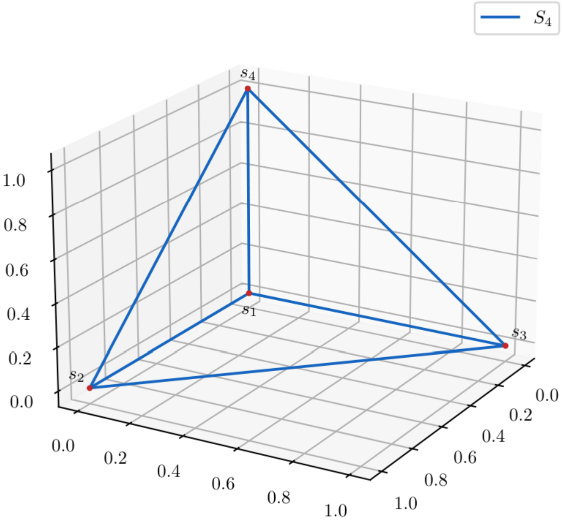
S_n for $n = 4$



S_4 on the w=1 plane

Boxworld Theory

Consider the boxes with one input and one output bit s_{ij} with the following input-output relations:

	$1 \mapsto 0$	$1 \mapsto 1$
$0 \mapsto 0$	s_{00}	s_{01}
$0 \mapsto 1$	s_{10}	s_{11}

The extreme points of the simplest boxworld state space

s_{ij} are not affinely independent, but we have:

$$s_{00} + s_{11} = s_{01} + s_{10} \tag{9}$$

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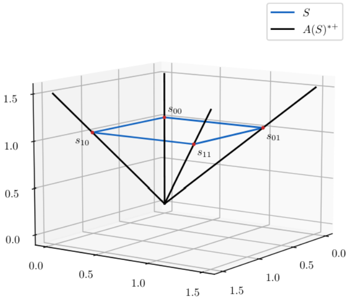
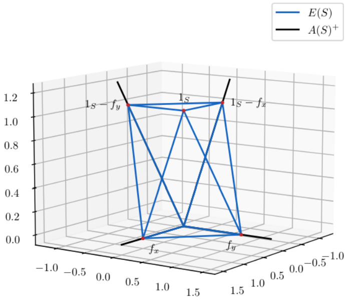
Boxworld State Space and Effect Algebra



(a) Picture of the state space S as a subset of $A(S)^*$. The red points are the pure states s_{00}, s_{10}, s_{01}, and s_{11}, the blue lines are the edges of the state space S, and the black lines are the edges of the positive cone $A(S)^{*+}$.



(b) Picture of the effect algebra $E(S)$ as a subset of $A(S)$. The red points are the effects f_x, $1_S - f_x$, f_y, $1_S - f_y$, and 1_S, the blue lines are the edges of the effect algebra $E(S)$, and the black lines are the edges of the positive cone $A(S)^+$.

The state space S and effect algebra $E(S)$ for the scenario presented in the last slide

Plávala. 2021. arxiv: quant-ph/2103.07469

Final Thoughts + Bibliography

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- Almost Quantum Theory. Benjamin Schumacher, Michael D. Westmoreland. 2012. *Quantum Theory: Informational Foundations and Foils*
- Post-Classical Probability Theory. Howard Barnum, Alexander Wilce. 2012. *Quantum Theory: Informational Foundations and Foils*
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"'Ford!' he said, 'there's an infinite number of [pseudo-quantum theories][2] outside who want to talk to us about this script for [adjacent realities][3] they've worked out.'" - Douglas Adams

[2] monkeys

[3] Hamlet