Reference Frames

Jalan Ziyad

Does the quantum state mean anything “by itself”?

- Is the universe a computer?
- Can everything be represented by bits?
- Does QM describe everything?

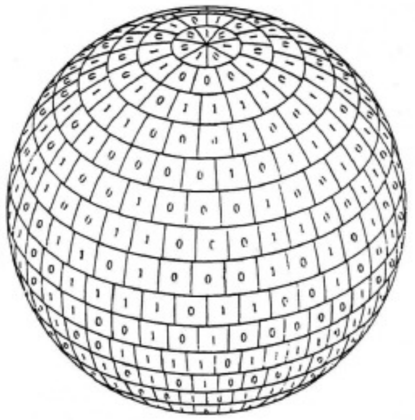


Fig. 19.1. Symbolic representation of the “telephone number” of the particular one of the 2^N conceivable, but by now indistinguishable, configurations out of which this particular blackhole, of Bekenstein number N and horizon area $4N\hbar \log_e 2$, was put together. Symbol, also, in a broader sense, of the theme that *every* physical entity, every it, derives from bits. Reproduced from JGST, p.220.

Two types of information

- "Speakable"-Peres
  - Information where the choice of representation doesn't matter
  - Qubits: spin ½ states, any two-level quantum system
  - Bits: two voltage levels , etc.
- "Unspeakable"
  - Choice of representation does mater
  - Reference frames: Direction in space, time information.
    - Stored as physical systems: Gyroscopes, clocks, measurement devices etc.

Asher Peres

Peres, Asher, and Petra F Scudo. "Unspeakable Quantum Information." *ArXiv Preprint Quant-Ph/0201017*, 2002.

Reference frames have consequences for our interpretations of physical theories:

Can the universe be a computer? Not likely

- All physical states are defined with reference to an external frame
- You need frame information to interpret physical theories
  - Ben Schumacher's Quantum Grue-Bleen problem

Reference frames are intimately connected to decoherence

- Lacking a RF limits the operational tasks you can perform on a physical system→ Super-selection rules/decoherence
- How can you align a reference frame? →Calibration/phase estimation

Many Worlds: The universe as a wavefunction

- "The mathematical formalism of the quantum theory is capable of yielding its own interpretation" –Bryce DeWitt
- Ingredients of a many worlds universe:
  - The universal wave function and "measurements"

Before Measurement

$$|\psi_0\rangle \otimes |0\rangle \otimes |\text{“ready”}\rangle \longrightarrow \left(\alpha\,|\uparrow\rangle \otimes \left|+\tfrac{\hbar}{2}\right\rangle + \beta\,|\downarrow\rangle \otimes \left|-\tfrac{\hbar}{2}\right\rangle\right) \otimes |\text{“ready”}\rangle$$

After Measurement

$$\alpha\,|\uparrow\rangle \otimes \left|+\tfrac{\hbar}{2}\right\rangle \otimes |\text{“up”}\rangle + \beta\,|\downarrow\rangle \otimes \left|-\tfrac{\hbar}{2}\right\rangle \otimes |\text{“down”}\rangle$$

The measurement events can be inferred from the expectation values of observables

$$\boldsymbol{C} = |\uparrow\rangle\langle\uparrow| \otimes |\text{“up”}\rangle\langle\text{“up”}| + |\downarrow\rangle\langle\downarrow| \otimes |\text{“down”}\rangle\langle\text{“down”}|$$

$$\langle C\rangle = 0$$

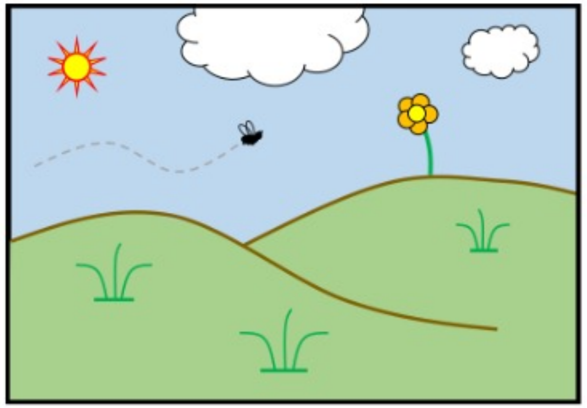
Uncorrelated

$$\langle C\rangle = 1$$

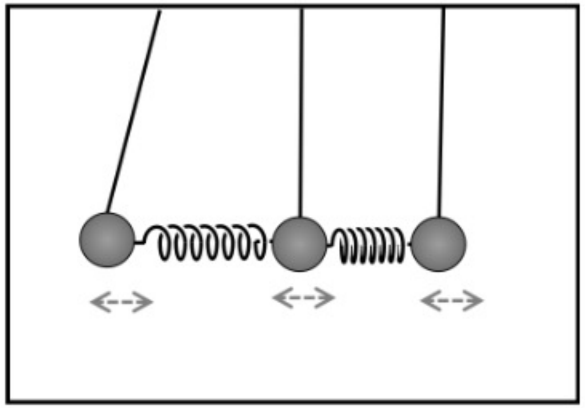
Correlated

Schumacher, Benjamin, and Michael D Westmoreland. "Interpretation of Quantum Theory: The Quantum 'Grue-Bleen' Problem." *Entropy* 24, no. 9 (2022): 1268.

Two worlds, two equivalent pictures



Q



Q'

$$|\Psi(t)\rangle = \boldsymbol{U}(t)\,|\Psi_0\rangle \qquad |\Psi(t)\rangle = \boldsymbol{V}(t)|\Psi_0\rangle$$

$$\tilde{\boldsymbol{B}}_k = \left(\boldsymbol{U}(t)\boldsymbol{V}^\dagger(t)\right) \boldsymbol{B}_k \left(\boldsymbol{V}(t)\boldsymbol{U}^\dagger(t)\right)$$

“Hilbert space is featureless”

Without a way to restrict the state symmetries of a system, there are no non-trivial interpretational propositions in the theory.

Reversible theory schema:

- $\mathcal{S}$: a set of allowed states
- $\mathcal{K}$: a group of kinematically allowed maps
- U : a set of similarities with property **S**

Property S. Both of these are true of V:

- V is a bijection.
- $VDV^{-1} \in \mathcal{K}$ if and only if $D \in \mathcal{K}$.

All kinematically allowed transformations are allowed similarities $\mathcal{K} \subseteq U$

Instances and interpretations

- An initial state and a sequence of dynamical maps is an instance of the theory

$$(x_0, \vec{D})$$

Initial state

$$\vec{D} = (D_{1,0}, D_{2,1}, \ldots, D_{N,N-1})$$

A time-ordered sequence of dynamical maps

$$D_{k+1,k} \in \mathcal{K}$$

Each map is a kinematically allowed transformation

- An interpretation includes propositions (either true or false) that only depend on the instance of the theory. Equivalent instances yield equivalent propositions:

$$P(x_0, \vec{D}) \Leftrightarrow P(\tilde{x}_0, \tilde{\vec{D}})$$

- The set of kinematic maps is transitive if

$$\forall x, y \in \mathcal{S} \quad \exists V \in \mathcal{K} \quad x = Vy$$

- **Theorem**: A reversible theory with a transitive $\mathcal{K}$, has no state-dependent interpretational propositions

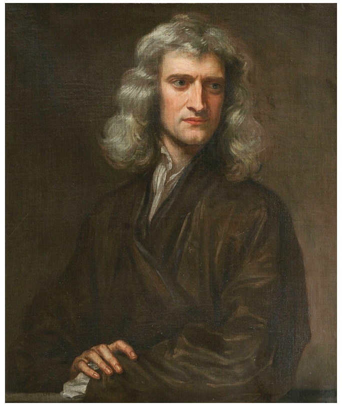
$$P(x) \Leftrightarrow P(Vx) = P(y)$$

WAIT… CLASSICAL PHYSICS IS REVERSIBLE!

Descartes, Newton, and Leibnitz

Problems of RFs were debated by the pioneers of mathematical physics. Descartes' defined true motion as a relational quantity, but didn't account for inertial effects. This lead Newton and Leibnitz to develop their own theories of motion vis a vis the existence or lack of an *absolute* frame.

Rene Descartes



Isaac Newton

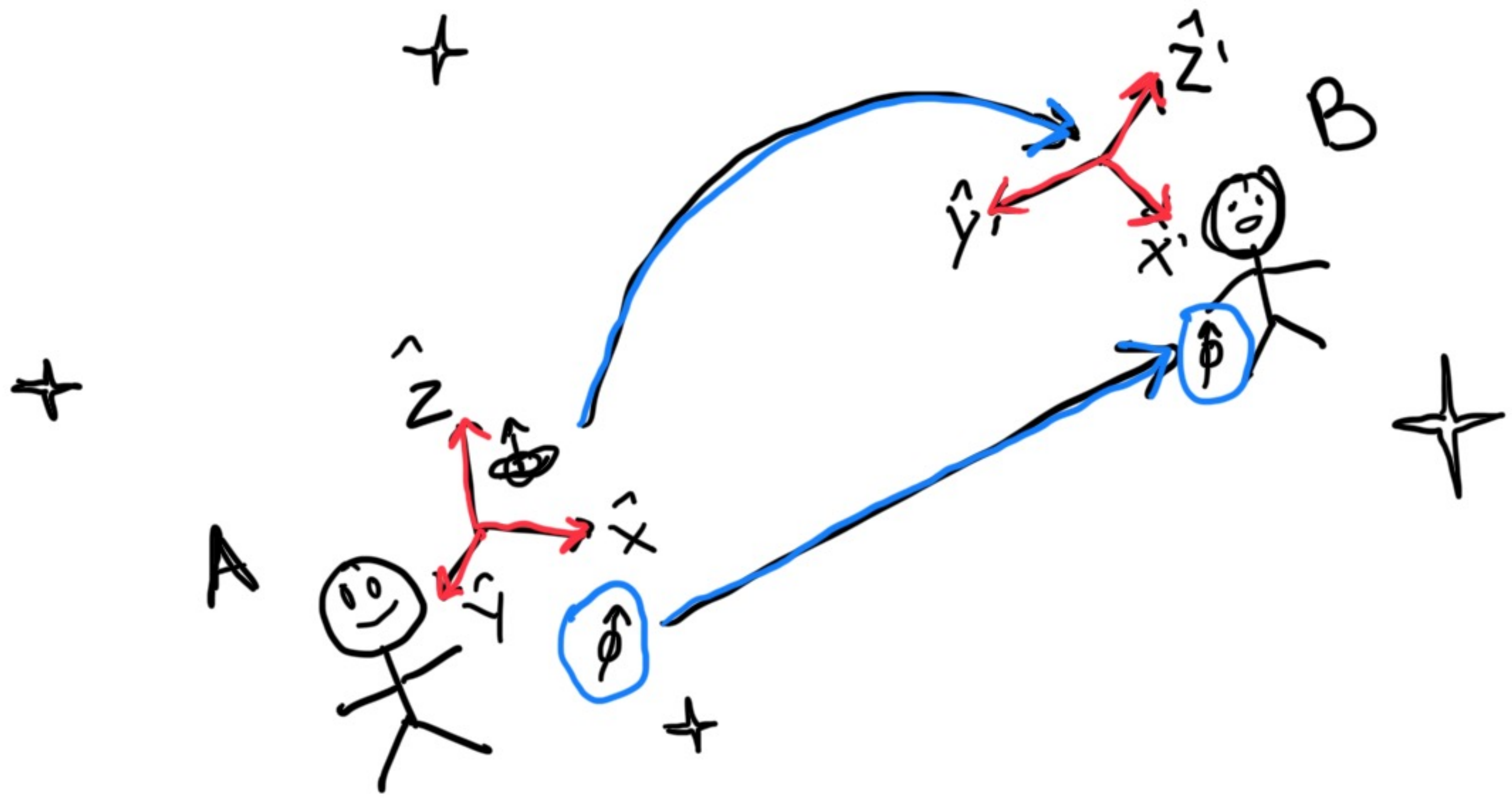
Motion exists relative to a fixed background RF for the universe (absolute space). Allows for the treatment of individual systems and causality. *Metaphysically controversial but practical.*

G. W. Leibnitz

Only relational quantities exist, so physical systems cannot be treated as separate → can only consider the universe as a whole! *Less metaphysical commitments but weird*

Lake, Matthew J, and Marek Miller. "Quantum Reference Frames, Revisited." *ArXiv Preprint ArXiv:2312.03811*, 2023.
Belkind, Ori. "Leibniz and Newton on Space." *Foundations of Science* 18, no. 3 (2013): 467–97. https://doi.org/10.1007/s10699-011-9280-5.

Reference frames: a picture



Theory of Reference frames

- Treat RF's group theoretically

$\mathcal{H}$ Hilbert space

G A compact group: Assuming a finite or a Lie group. (Ex: SO(3), SU(2), U(1), Pauli group etc.)

→ Has a Haar measure dg that acts on $\mathcal{H}$ via a unitary representation $\mathcal{T}$

Bartlett, Stephen D., Terry Rudolph, and Robert W. Spekkens. "Reference Frames, Superselection Rules, and Quantum Information." Rev. Mod. Phys. 79, no. 2 (April 2007): 555–609. https://doi.org/10.1103/RevModPhys.79.555.

Theory of Reference frames

Alice sends a state to Bob: What is the state relative to his reference frame?

- If the RF transformation is known
  - Apply the transformation to the state (active transformation) or the frame (passive transformation)

$$g \in G \qquad \mathcal{T}(g)\rho\mathcal{T}^\dagger(g)$$

- If the relationship between RFs is completely unknown/uncorrelated:
  - G -twirling: Average over all transformations→ Super-selection rules

Leonard Susskind

States

$$\tilde{\rho} = \int_G dg \mathcal{T}(g)\rho\mathcal{T}^\dagger(g) \equiv \mathcal{G}[\rho]$$

$$[\tilde{\rho}, \mathcal{T}(g)] = 0 \quad \forall g \in G$$

Operations

$$\mathcal{E} : B(\mathcal{H}) \to B(\mathcal{H})$$

$$\tilde{\mathcal{E}}[\rho] = \int_G dg \mathcal{T}(g)\mathcal{E}[\mathcal{T}^\dagger(g)\rho\mathcal{T}(g)]\mathcal{T}^\dagger(g)$$

Measurements

$$\{E_k\}$$

$$\tilde{E}_k = \mathcal{G}[E_k]$$

Aharonov, Yakir, and Leonard Susskind. "Charge Superselection Rule." *Phys. Rev.* 155, no. 5 (March 1967): 1428–31. https://doi.org/10.1103/PhysRev.155.1428.

Example: Phase reference in quantum optics

Consider k modes of the EM field relative to a phase reference (e.g. a local oscillator)

$\mathcal{H}^{(k)}$

Hilbert space

The basis states are

$|n_1, ..., n_k\rangle$

where n_i is the number of excitations in mode i.

Number operators

$\hat{N}_i|n_1, ..., n_k\rangle = n_i|n_1, ..., n_k\rangle$

$\hat{N}_{tot} = \sum \hat{N}_i$

The phase operator forms a unitary representation of $U(1)$

$$U(\phi)|\psi\rangle = e^{i\phi \hat{N}_{tot}}|\psi\rangle = \sum e^{in\phi}\Pi_n|\psi\rangle$$

Projection onto a subspace with n excitations

Alice sends her state to Bob where the reference frame is unknown→ **Total photon number super-selection rule**

$$\mathcal{U}[|\psi\rangle\langle\psi|] \equiv \int_0^{2\pi} \frac{\mathrm{d}\phi}{2\pi} U(\phi)|\psi\rangle\langle\psi|U(\phi)^\dagger = \sum_n \Pi_n \rho \Pi_n$$

$$\mathcal{H}^{(k)} = \bigoplus_n \mathcal{H}_n$$

Phase twirling leads to decoherence of number states

Example: Spin ½ particle relative to a phase reference

If the phase reference

- Is known
  - Related by an element of U(1)

- Is completely unknown
  - Complete dephasing

- Fluctuates stochastically
  - Uncorrelated in time
    - Markovian dephasing
  - Correlated in time
    - Non-Markovian dephasing

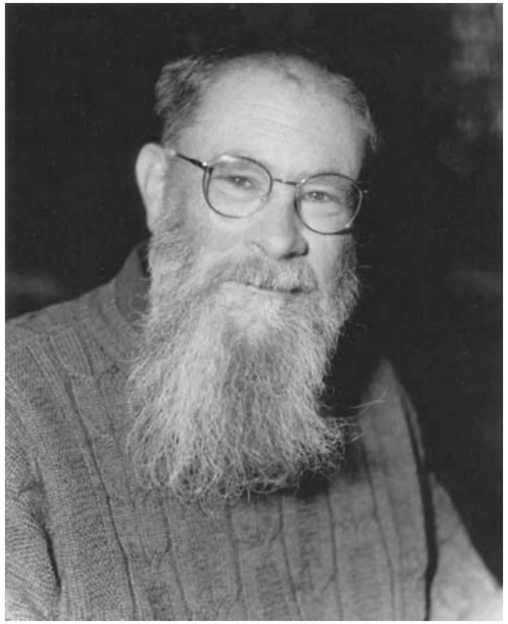
Subverting SSR's: Quantizing the reference frame

- Treat the RF "internally". Attach a quantum system, "R", to serve as a quantum reference frame.
- Ex: Photon number SSR

$$|m\rangle \rightarrow |n-m\rangle_R |m\rangle$$

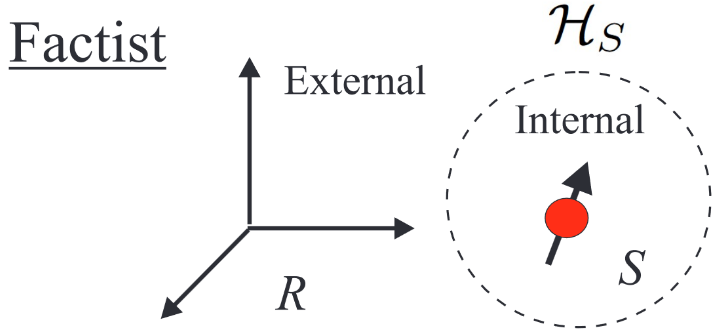
- This is essentially encoding your state in a decoherence free subspace

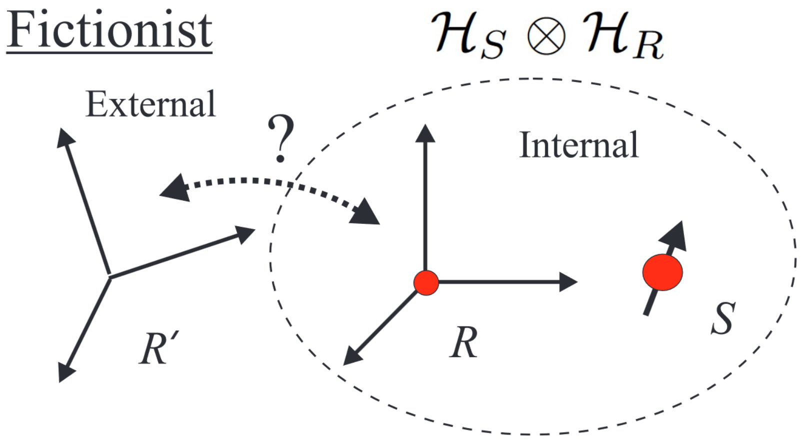
“The intrinsic properties of something depend only on that thing; whereas the extrinsic properties of something may depend, wholly or partly, on something else”



David Lewis

Is the coherence of light an intrinsic property?





$$|\psi_0\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$$

$$|\psi_0\rangle = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1|$$

- The coherence describes a systems relation to an external phase→ coherence is an extrinsic property

Bartlett, Stephen D, Terry Rudolph, and Robert W Spekkens. "Dialogue Concerning Two Views on Quantum Coherence: Factist and Fictionist." *International Journal of Quantum Information* 4, no. 01 (2006): 17–43.

Protocols for aligning reference frames

- Task: What reference frame states and measurements are optimal for aligning reference frames?
  - Related to calibration/phase estimation
  - Ex:Alice and Bob wish to align a directional RF using two spins. What is the optimal state/probability of success?

- Aligning RFs in the presence of noise
  - Equivalent to covariant error correction

$$U_1 \otimes U_2 \otimes U_3 \, \mathcal{E}(U_{\text{in}}^\dagger \rho_{\text{in}} \, U_{\text{in}}) U_1^\dagger \otimes U_2^\dagger \otimes U_3^\dagger = \mathcal{E}(\rho_{\text{in}})$$

  - No go theorems:
    - A finite code cannot perfectly correct a RF for a Lie group
    - (In)finite codes can correct for (in)finite groups
  - The Approximate Eastin-Knill theorems
    - An infinite (or large) code can perfectly (or approximately) circumvent the E-K theorem. (e.g. Attaching an ideal clock to any code allows for universal transversal gates → requires infinite energy)

Hayden, Patrick, Sepehr Nezami, Sandu Popescu, and Grant Salton. "Error Correction of Quantum Reference Frame Information." *PRX Quantum* 2, no. 1 (2021): 010326.

Conclusion

- Essential for understanding the application/interpretation of physical theories
  - No frame info→ no non-trivial interpretations
    - True in classical physics as well
  - All quantum states have extrinsic properties
- RF’s limit operational tasks
  - Super selection rules. These can be circumvented by an appropriately large QRF.
  - Useful for understanding dephasing/non-Markovian noise.

References:

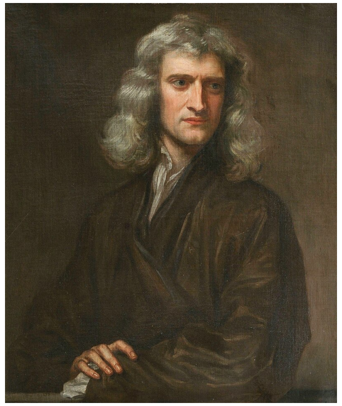
- Aharonov, Yakir, and Leonard Susskind. “Charge Superselection Rule.” *Phys. Rev.* 155, no. 5 (March 1967): 1428–31. https://doi.org/10.1103/PhysRev.155.1428.
- Bartlett, Stephen D, Terry Rudolph, and Robert W Spekkens. “Dialogue Concerning Two Views on Quantum Coherence: Factist and Fictionist.” *International Journal of Quantum Information* 4, no. 01 (2006): 17–43.
- Bartlett, Stephen D., Terry Rudolph, and Robert W. Spekkens. “Reference Frames, Superselection Rules, and Quantum Information.” *Rev. Mod. Phys.* 79, no. 2 (April 2007): 555–609. https://doi.org/10.1103/RevModPhys.79.555.
- Belkind, Ori. “Leibniz and Newton on Space.” *Foundations of Science* 18, no. 3 (2013): 467–97. https://doi.org/10.1007/s10699-011-9280-5.
- Eastin, Bryan, and Emanuel Knill. “Restrictions on Transversal Encoded Quantum Gate Sets.” *Physical Review Letters* 102, no. 11 (2009): 110502.
- Faist, Philippe, Sepehr Nezami, Victor V Albert, Grant Salton, Fernando Pastawski, Patrick Hayden, and John Preskill. “Continuous Symmetries and Approximate Quantum Error Correction.” *Physical Review X* 10, no. 4 (2020): 041018.
- Hayden, Patrick, Sepehr Nezami, Sandu Popescu, and Grant Salton. “Error Correction of Quantum Reference Frame Information.” *PRX Quantum* 2, no. 1 (2021): 010326.
- Lake, Matthew J, and Marek Miller. “Quantum Reference Frames, Revisited.” *ArXiv Preprint ArXiv:2312.03811*, 2023.
- Peres, Asher, and Petra F Scudo. “Unspeakable Quantum Information.” *ArXiv Preprint Quant-Ph/0201017*, 2002.
- Schumacher, Benjamin, and Michael D Westmoreland. “Interpretation of Quantum Theory: The Quantum ‘Grue-Bleen’ Problem.” *Entropy* 24, no. 9 (2022): 1268.
- Woods, Mischa P., and Álvaro M. Alhambra. “Continuous Groups of Transversal Gates for Quantum Error Correcting Codes from Finite Clock Reference Frames.” *Quantum* 4 (March 2020): 245. https://doi.org/10.22331/q-2020-03-23-245.

Descartes, Newton, and Leibnitz

Rene Descartes

"…if we consider what we should understand by motion, not in common usage but in accordance with the truth of the matter, and if our aim is to assign a determinate nature to it, we may say that motion is the transfer of one piece of matter, or one body, from the vicinity of the other bodies which are in immediate contact with it, and which are regarded as being at rest, to the vicinity of other bodies."

Descartes, Newton, and Leibnitz



Isaac Newton

Def1 Place is defined as the part of space a body occupies within the larger space that contains it.

Def2 A property of quantity of motion is that the quantity of motion of the whole is the sum of the quantities of motion of the parts.

P1 If all places are movable, then quantity of motion is indeterminable for individual bodies.

a. To determine the quantity of motion of body A, one needs to determine the motion of a body relative to its (movable) place A′.
b. But the quantity of motion of a body is only part of the quantity of motion attributed to the composite body B containing both the body A and its place A′.
c. To determine the quantity of motion of the composite body B, one needs to determine its motion relative to its (movable) place B′, and so on...

P2 A determinable quantity of motion is necessary for a science of motion.

P3 Either all places are movable or some are im-movable.

C There are immovable places, i.e., there is true motion defined relative to immovable places.

Descartes, Newton, and Leibniz

G. W. Leibniz

*"**For Leibniz, there are no genuine causal relations and no genuine transfers of force**. By conservation of force Leibniz means that the inner force which increases for body A is correlated with a decrease in inner force in body B. The preestablished harmony with which God created substances allows us to analyze the change in A's living force mv2 to be correlated with the change in B's living force. However, while there are no genuine causal relations, **Leibniz argues that all natural phenomena are reducible to apparent chains of causes and effects, corresponding to God's preestablished harmony between substances.**"*

"A remarkable thing: motion is something relative, and one cannot distinguish which of the bodies is moving. And so, if motion is an affection, its subject will not be any individual body, but the whole world."

Question: Do purely operational interpretations of physical theories, which only refer to relational quantities, allow for causal reasoning?