

Physics 405: Lecture 19

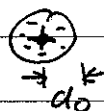
Introduction to Dielectrics

Our study of electrostatics, so far, has been restricted to a "microscopic" description, with imagined charge distributions in vacuum. In macroscopic situations, we typically manipulate charges in the presence of other matter, be it air or other materials.

As discussed in Lect. 11, materials generally fall into two categories: conductors and insulators (also known as dielectrics), nearly ideal in each case. For conductors, there are free charges that can flow almost unimpeded. In a dielectric, positive and negative charges are bound together. Thus, except on a very microscopic scale (of order $\sim \text{\AA}$), the charge density is neutral.

Microscopic picture of matter

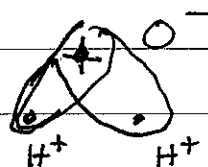
Atoms



$$d_0 = 0.5 \text{\AA}$$

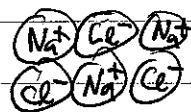
Molecules

Eg. H_2O



Solids

Eg. NaCl

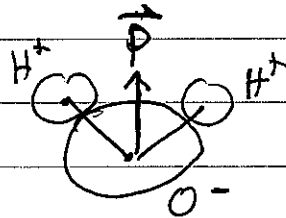


What happens when an atomic vapor, water, or salt crystal is placed in an external \vec{E} -field?

Let us first consider a single atom, molecule, or unit cell of a crystal.

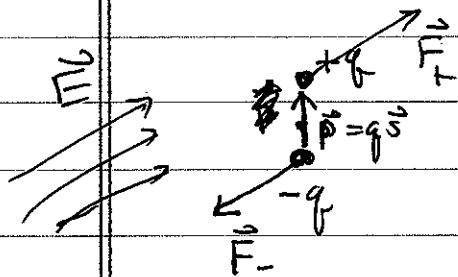
In general the microscopic response is complicated (need QM). We can get a good approximation by studying the multipole moments of matter.

E.g. Water molecule



Water molecule has

a permanent dipole moment. What happens when in a external \vec{E} -field?

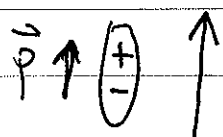


The field exerts a torque on the dipole.

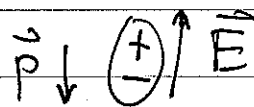
$$\begin{aligned} \vec{\tau} &= \vec{r}_+ \times \vec{F}_+ + \vec{r}_- \times \vec{F}_- \\ &= \frac{s}{2} \times (q\vec{E}) + \left(-\frac{s}{2} \times (-q\vec{E})\right) \\ &= (qs) \times \vec{E} \end{aligned}$$

$$\Rightarrow \boxed{\vec{\tau} = \vec{p} \times \vec{E}}$$

Electric dipoles want to align with the local field



Stable equilibrium



Unstable equilibrium

If the field varies as a function of position, there is also a force on the center of mass

$$\begin{aligned}\vec{F} &= q_+ \vec{E}(\vec{r}_+) - q_- \vec{E}(\vec{r}_-) \\ &= q \left(\vec{E}(\vec{r}_0 + \frac{\vec{d}}{2}) \right) - q \left(\vec{E}(\vec{r}_0 - \frac{\vec{d}}{2}) \right) \\ &\approx q \left[\vec{E}(\vec{r}_0) + \frac{\vec{d}}{2} \cdot \nabla \vec{E} \Big|_{\vec{r}_0} \right] - q \left[\vec{E}(\vec{r}_0) - \frac{\vec{d}}{2} \cdot \nabla \vec{E} \Big|_{\vec{r}_0} \right]\end{aligned}$$

$$\boxed{\vec{F} = (\vec{p} \cdot \nabla) \vec{E}(\vec{r}_0)}$$

For a "permanent" dipole moment \vec{p} , we can write this force as the gradient of a potential

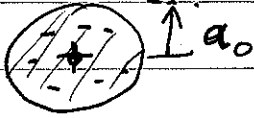
$$\boxed{U(\vec{r}) = -\vec{p} \cdot \vec{E}(\vec{r}_0)}$$

Potential energy of \vec{p} in an external field

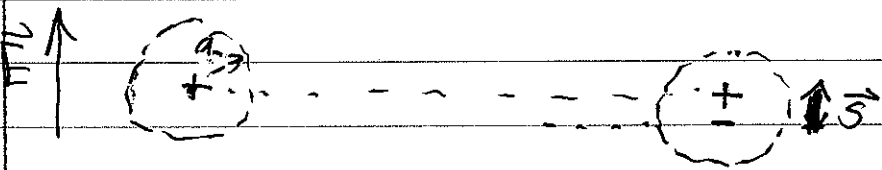
Eg: Atomic response

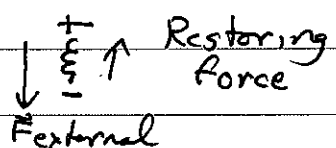
Symmetric molecules or monoatomics have no permanent dipole moments (electron clouds equally distributed about nucleus). However when placed in an external field, positive and negative charges move apart (slightly) leading to an induced electric dipole moment. For moderate fields (small compare to fields inside the atom) the ~~field~~ induced dipole will be proportional to the external field.

E.g. Hydrogen atom: Electron cloud in ground state is spherical symmetric

Classical model  $\vec{p} = 0$

Place in external field. To first order the electron moves, nucleus fixed. Approximate electron cloud as still spherically symmetric

 electron cloud shifts down.

Like a charge on a spring 

Equilibrium position:

$$|\vec{E}_{\text{internal}}| = |\vec{E}_{\text{external}}| \Rightarrow \frac{1}{4\pi\epsilon_0} \frac{e\bar{s}}{a_0^3} = E_{\text{ext}}$$

$$\Rightarrow |\vec{p}_{\text{induced}}| = e\bar{s} = 4\pi\epsilon_0 a_0^3 \vec{E}_{\text{ext}}$$

$$= \alpha |\vec{E}_{\text{ext}}|$$

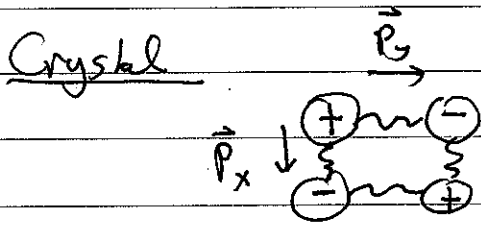
↑
atomic "polarizability"

$$\alpha = 4\pi\epsilon_0 a_0^3$$

Potential energy in a polarizable particle:

$$\vec{F} = \vec{p} \cdot \vec{\nabla} \vec{E} = \alpha \vec{E} \cdot \vec{\nabla} \vec{E} = -\frac{1}{2} \alpha \vec{\nabla} |\vec{E}|^2$$

$$\Rightarrow \boxed{U = -\frac{1}{2} \alpha |\vec{E}|^2}$$



Generally polarizability is a "tensor"

$$\vec{p} = \underline{\alpha} \cdot \vec{E}$$

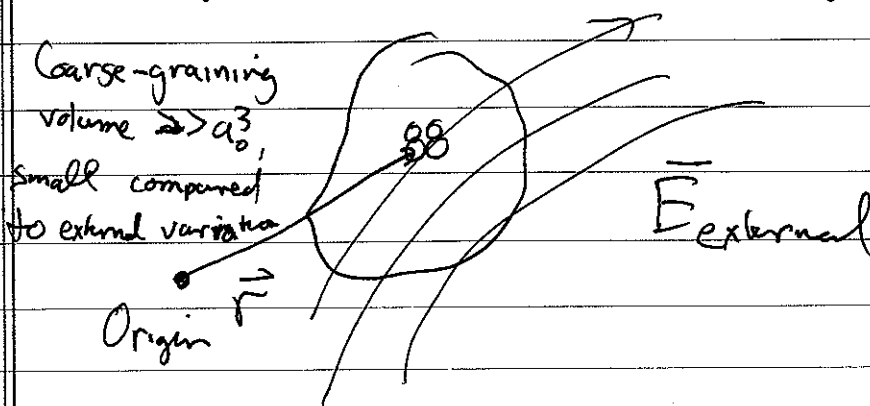
$$p_x = \alpha_{xx} E_x + \alpha_{xy} E_y$$

$$p_y = \alpha_{yx} E_x + \alpha_{yy} E_y$$

Macroscopic Matter

The microscopic fields around atoms is quite complex.

For macroscopic applications, these fields are rarely of importance. Instead, we see the average effects of Avogadro's Number of atoms/molecules. We thus "coarse grain" over a small volume such that the average field doesn't vary much (where as on "fine grains" $\sim \text{\AA}$, the fields vary wildly)



Inside a coarse-grain

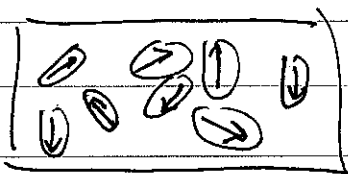
$$\vec{p}_{\text{total}} = \sum_i \vec{p}_i$$

The fundamental object for dielectrics is the coarse-grained electric-dipole density (sometimes known as the "polarization").

$$\vec{P}(\vec{r}) = \frac{\sum_i \vec{p}_i(\vec{r})}{\delta V}, \quad \delta V = \text{volume of coarse-grain}$$

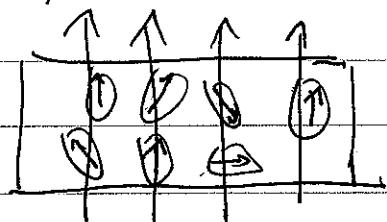
For most materials $\vec{P}(\vec{r}) = 0$ in the absence of an external field.

E.g. Water



Thermal fluctuations
 \Rightarrow randomly oriented dipoles

In an external field



Dipoles try to align with external field

$\vec{E}_{\text{external}}$

$\Rightarrow \vec{P}(\vec{r}) \neq 0$

Again, for moderate fields $\vec{P} \sim \vec{E}_{\text{external}}$

Define: $\vec{P} = \epsilon_0 \vec{\chi} \cdot \vec{E}$

\uparrow electric "susceptibility"

For dilute gas (e.g. atomic vapor)

$$\vec{P} = \overset{\leftarrow \text{atomic density}}{N} \vec{p} = N \vec{\alpha} \cdot \vec{E} \Rightarrow \vec{\chi} = \frac{N \vec{\alpha}}{\epsilon_0}$$

(No "local field" correction)