Physics 405: Lecture 28

Magnetic field in Matter

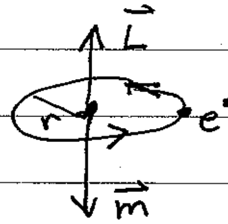
Magnetic fields in materials is analogous to, but much more complicated than, electric field in materials, because the nature of magnetic fields is ~~relat~~ intrinsically quantum mechanical in nature. There are two contributions to the magnetic ~~moment~~ fields of materials

- "Orbital motion" of electrons around nuclei
- The instrinsic magnetic moment of elementary particles (electron and nuclear spin)

~~Spin angular momentum and intrinsi~~

– Gyromagnetic Ratio and Orbital Angular Momentum

Consider a classical picture of an electron orbit around the nucleus



$$\vec{m} = \pi r^2 I \ \hat{z}$$

The current $I = -\frac{e}{T}$ ← period of orbit

$$\Rightarrow T = \frac{2\pi}{\omega} \quad \Rightarrow \quad I = -\frac{e\omega}{2\pi}$$

$$\Rightarrow \vec{m} = -\frac{e}{2}\omega r^2 \hat{z} = \frac{-e}{2m_e}\underbrace{(m\omega^2 r)}_{\vec{L} \text{ anglar momentum}}\hat{z}$$

Thus $\vec{m} = \underbrace{\left(-\frac{e}{2m_e}\right)}_{\text{"Gyromagnetic ratio"}} \vec{L}$

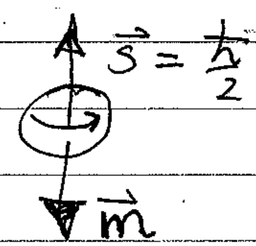
(Generally, give a "body" of mass M, with total charge q, $\vec{m} = \frac{q}{2M}\vec{L}$)

[Aside: For atoms, the characteristic unit of angular momentum is $\hbar$, Plancks constant / 2π. The characteristic unit of magnetic moment of an electron is the

$$\mu_B \equiv \frac{e\hbar}{2m_e} \equiv \text{Bohr magneton}$$
]

– Spin Angular Momentum and intrinsic magnetic moment

Elementary particles have "intrinsic properties" such as charge and mass and spin = intrinsic angular momentum (not orbital motion of "spinning charge")

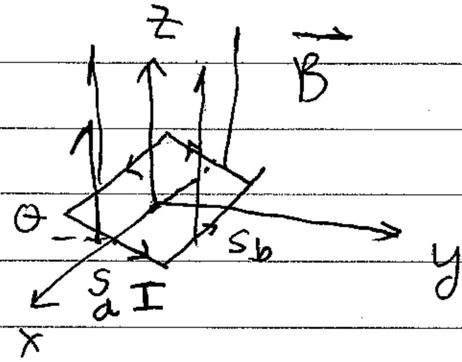
electron 

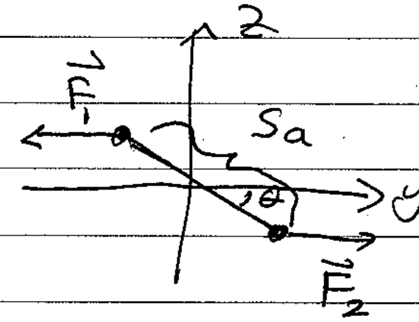
$$\vec{m} = -\underbrace{2}_{\text{"g-factor"}}\mu_B\left(\vec{S}/\hbar\right)$$

Nuclei, made from protons and neutrons, also have intrinsic moments. This depends on the isotope.

Magnetic Moments in External $\vec{B}$-fields

- Consider ~~a~~ of fixed rectangular current loop in uniform $\vec{B}$



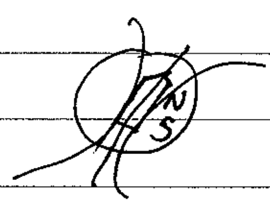


$|\vec{F}_1| = |\vec{F}_2| = I s_b B$ Torque: $\vec{\tau} = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2$

$$\Rightarrow \vec{\tau} = 2\vec{r}_1 \times \vec{F}_1 = I s_a s_b B \sin\theta \, \hat{x}$$

$$\Rightarrow \boxed{\vec{\tau} = |\vec{m}|\,|\vec{B}| \sin\theta \, \hat{x} = \vec{m} \times \vec{B}}$$

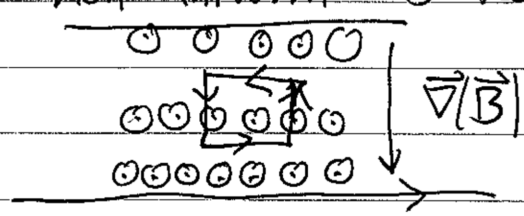
Magnetic moment wants to align with $\vec{B}$



Compass!

- In a non-uniform $\vec{B}$-field

e.g.



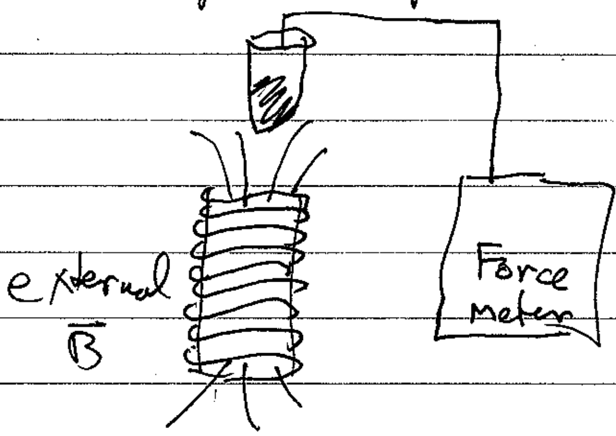
$$\boxed{\vec{F} = \vec{\nabla}(\vec{m} \cdot \vec{B})}$$

P.S. #10

Potential energy of $\vec{m}$ in external $\vec{B}$

$$\boxed{U = -\vec{m} \cdot \vec{B}}$$

Macroscopic Response to external field



Weakly repelled (Diamagnetic)

H_2O, Cu, NaCl ...

Weakly attracted (Paramagnetic)

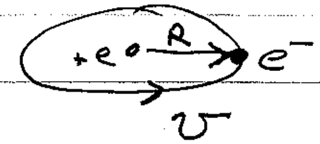
Al, Na, ...

Strongly attracted (10^3–10^5 more)
Ferromagnetic

Fe, Ni, Co ...

These very different responses arise from different source of magnetic moments in matter.

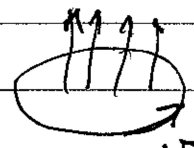
- Orbital : Classical picture of hydrogen



$$m_e \frac{v^2}{R} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{R}$$

Coulomb force sets radius

Now add an external $\vec{B}$-field $\perp$ to orbit



additional centripetal force from Lorentz $-e\vec{v}\times\vec{B}$

$\Rightarrow$ New velocity $\bar{v} = v + \Delta v$

$$m_e\left(\frac{\bar{v}^2}{R}\right) = \frac{1}{4\pi\epsilon_0}\frac{e^2}{R^2} + e\bar{v}B$$

$$\Rightarrow e\bar{v}B = m_e\left(\frac{\bar{v}^2 - v^2}{R}\right) = \frac{m_e}{R}(\bar{v}+v)(\bar{v}-v)$$

$$e v B \approx 2\frac{m_e}{R} v \Delta v$$

$\Rightarrow$ Change in velocity $\Delta v \approx \dfrac{eRB}{2m_e}$

As the electron speeds up, the magnetic moment increases in the direction opposite to $\vec{B}$

$$\Delta \vec{m} = -\frac{1}{2} e (\Delta v) R \hat{z} = -\left(\frac{e^2 R^2}{4 m_e}\right) \vec{B}$$

$\Rightarrow$ Material repelled by increasing $\vec{B}$

Diamagnetism

- Spin:

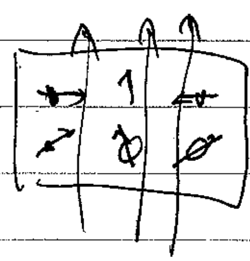
In addition to the orbital contribution, we have the spin contribution.

E.g. Na vapor, one valence electron, no orbital in ground state:

$$\text{Na} \quad \vec{m} = \gamma \vec{S}$$

$\gamma \leftarrow$ gyromagnetic ration

In presence of $\vec{B}$, torque tries to align $\vec{m}$

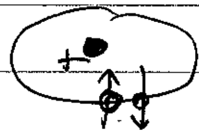


Weakly induced $\vec{m}$ along $\vec{B}$ $\Rightarrow$ Lower energy in increase $\vec{B}$

$\Rightarrow$ Paramagnetic

– Chemistry, Pauli exclusion, and magnetism

Electron fill "orbitals" with opposite spins

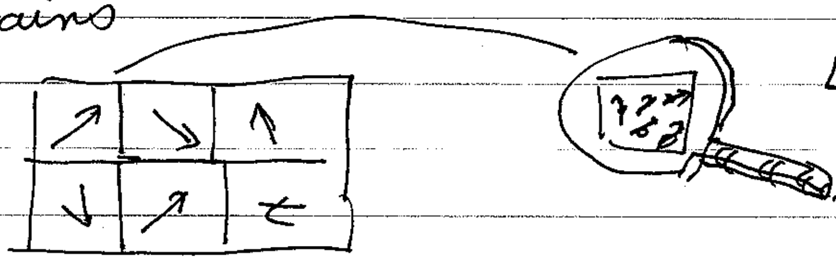


e.g. Helium

$\Rightarrow$ Diagmagnetic

<u>Ferromagnetism</u>

In some materials, the spin-spin interactions can "self assemble" into macroscopic domains



Large average $\vec{m}$ in some direction

Now, if an external field alligns each domain, this is a much <u>larger</u> <u>effect</u>

<u>Moreover</u>, ~~[illegible]~~ in the proper environment, a <u>phase-transition</u> can occur which leaves a whole macroscopic sample with well define $\langle \vec{m} \rangle$

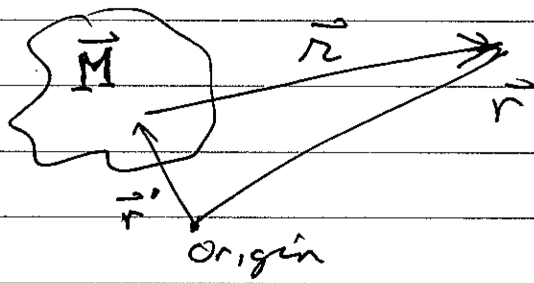
$\Rightarrow$ <u>Permanent magnets</u>

Magnetization

As in a dielectric, let us define

$$\vec{M}(\vec{r}) = \frac{\text{Coarse grain magnetic dipole moment}}{\text{Volume}}$$

Field of a magnetized object



$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int d^3r' \, \vec{M}(\vec{r}') \times \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$= \frac{\mu_0}{4\pi}\int d^3r' \, \vec{M}(\vec{r}') \times \left(\vec{\nabla}' \frac{1}{\mathscr{r}}\right)$$

Integration by parts

$$\Rightarrow \vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\left\{\int d^3r' \, \frac{\vec{\nabla}' \times \vec{M}}{\mathscr{r}} - \int \vec{\nabla}' \times \left(\frac{\vec{M}(\vec{r}')}{\mathscr{r}}\right) d^3r'\right.$$

$$\Rightarrow \boxed{\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{1}{\mathscr{r}}\left(\vec{\nabla}' \times \vec{M}(\vec{r}')\right) + \frac{\mu_0}{4\pi}\oint_S \frac{1}{\mathscr{r}}\left(\vec{M}(\vec{r}') \times \hat{n}\right) da'}$$

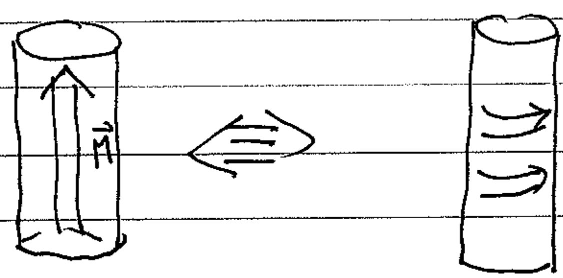
The field generated by a magnetized object can thus be understood as arised from <u>bound current</u>

$\vec{J}_{bound}(\vec{r}) \equiv \vec{\nabla} \times \vec{M}$: Bound current density inside material

$\vec{K}_{bound} = \vec{M} \times \hat{n}$: Bound surface current density

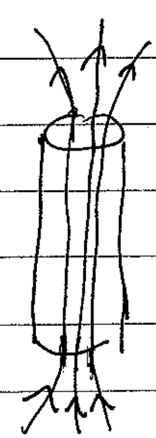
$\Rightarrow$ If we find the bound current distribution we can use our prior methods to determine $\vec{B}$.

Example: Cylindrical "bar magnet", uniform magnetization $\vec{M} = M_0 \hat{z}$



$\vec{K} = M_0 \hat{\phi}$

$\Rightarrow$ Magnetic field of a solenoid

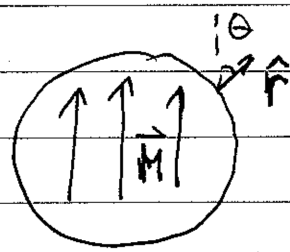


$\Rightarrow$ Inside

$|\vec{B}| \approx \mu_0 |\vec{K}| = \mu_0 M$

Example: Field of a Uniformly Magnetized Sphere

$\vec{M} = M_0 \hat{z}$



$$\vec{J}_{bound} = 0$$

$$\vec{K}_{bound} = \vec{M} \times \hat{n} = M_0 (\hat{z} \times \hat{r}) = M_0 \sin\theta\, \hat{\phi}$$

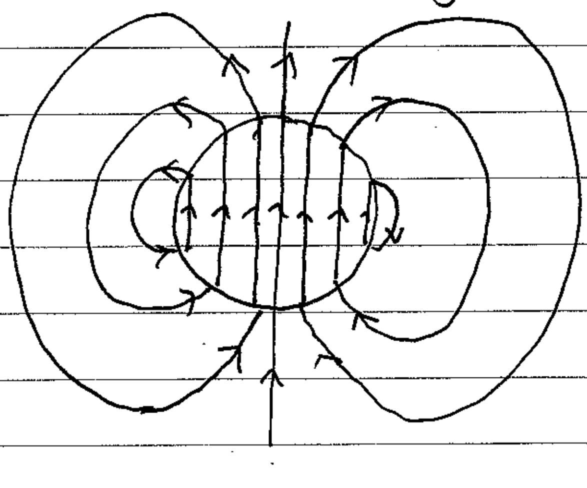
This is the current distribution we studied in Lecture 26 for a rotating sphere with uniform surface charge density σ

There, $\vec{K} = (\sigma \omega R) \sin\theta\, \hat{\phi}$

Using those results, we find

$$\vec{B} = \begin{cases} \mu_0 \frac{2}{3} \vec{M} & \text{inside} \quad r < R \\ \frac{\mu_0}{4\pi} \frac{(3(\vec{m}\cdot\hat{r})\hat{r} - \vec{m})}{r^3} & \text{outside} \quad r > R \end{cases}$$

where $\vec{m} = \frac{4}{3}\pi R^3 \vec{M}$ = total magnetic moment



$\vec{B}$