UNM: Physics 405
Electricity and Magnetism

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Lecture 1: Intro and Review

Fundamental Forces in Nature:

Decreasing strength

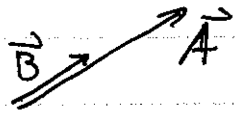
- Strong
- * Electromagnetic *
- Weak
- Gravitational

Almost every force we experience in daily life is electromagnetic in nature. Everything we perceive to be "mechanical" is really electromagnetic. When you pound your fist on the desk, when the wheels on your car accelerate you as they grip the pavement, when you push open a door, the underlying force is electromagnetic. Under ~~m~~ all are atoms whose electron clouds repell each other. It is this repulsion that keeps your hand from passing through the desk when you pound your first. It is electromagnetic forces that are at the heart of chemistry and thus life itself. Electricity and magnetism drive our technology. We like in an electromagnetic world.

Review of Mathematical Foundations

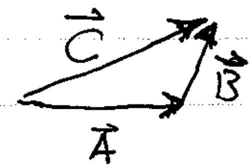
I. Vector Algebra: Scalars and Vectors

- **Scalar multiplication**: $\vec{A} = c\vec{B}$ c: real number



Here $c > 1$

- **Linearity** $\vec{C} = \vec{A} + \vec{B}$



- **Basis**: "Complete set": Any vector can be decomposed in a linear combo of basis vectors (minimum # = dimension)

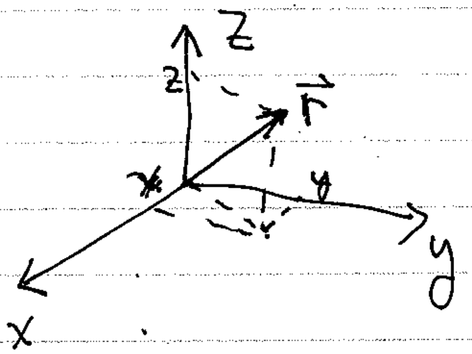
$$\vec{A} = A_1\vec{e}_1 + A_2\vec{e}_2 + A_3\vec{e}_3 + A_4\vec{e}_4$$

basis vectors (Here 4D)

Set (A_1, A_2, A_3, A_4) = "Coordinates"

- **Real space** $\mathbb{R}^3$

The notion of vector space is an abstraction of 3D physical space



position vector

$$\vec{r} = x\,\hat{x} + y\,\hat{y} + z\,\hat{z}$$

basis vectors

Other structures on vector space

- **Inner (scalar) product = Dot Product**

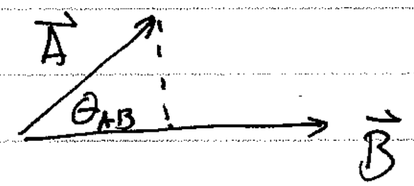
Given $\vec{A} = (A_x, A_y, A_z)$

$\vec{B} = (B_x, B_y, B_z)$

relative to basis $(\hat{x}, \hat{y}, \hat{z})$

Define: $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

Geometrically:



Can show: $\vec{A} \cdot \vec{B} = |\vec{A}|\,|\vec{B}| \cos(\theta_{AB})$ (Use law of cosine)

Dot product defines length: $\vec{A} \cdot \vec{A} = |\vec{A}|^2$

$|\vec{A}|^2 = A_x^2 + A_y^2 + A_z^2$ (Pythagarus theorem!)

Think of dot-product as a "projection" onto a given direction:

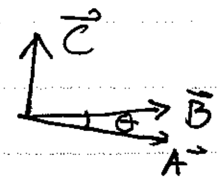
Basis vector are unit vector $\hat{x} \cdot \hat{x} \equiv 1$

$\Rightarrow \quad \hat{x} \cdot \vec{A} = A_x \quad \hat{y} \cdot \vec{A} = A_y$ etc.

- Vector Product = Cross Product

Defines "handedness" $\quad \hat{x} \times \hat{y} = \hat{z}$

Right hand rule



$$\vec{C} \equiv \vec{A} \times \vec{B}$$

$$\vec{C} \perp \vec{A} \text{ and } \vec{C} \perp \vec{B}$$

$\Rightarrow \vec{C} \perp$ to plane $(\vec{A}, \vec{B})$ and right hand

$$|\vec{C}| = |\vec{A}|\,|\vec{B}| \sin\theta$$

In term of coordinates: $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

$$\Rightarrow \vec{A} \times \vec{B} = \hat{x}(A_y B_z - B_y A_z) + \hat{y}(A_z B_x - B_z A_x) + \hat{z}(A_x B_y - B_x A_y)$$

Not Commutative: $\quad \vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

- Many identities

e.g. "Scalar triple product"

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = (\vec{A} \times \vec{B}) \cdot \vec{C}$$

(can "switch" dot and cross)

• e.g. <u>Vector triple product</u>

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$$

Note: $(\vec{A} \times \vec{B}) \times \vec{C} = -\vec{C} \times (\vec{A} \times \vec{B})$

$$= -\vec{A}(\vec{C} \cdot \vec{B}) + \vec{B}(\vec{C} \cdot \vec{A})$$

$\Rightarrow$ Cross Product is <u>not</u> associative

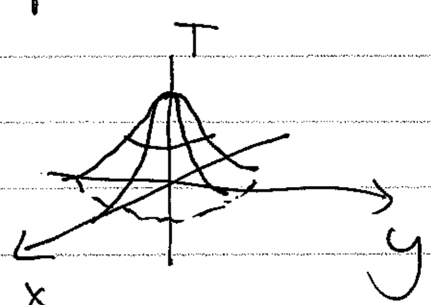
II. Fields – Functions of spatial position

(i) Scalar field $F(\vec{r}) = F(x, y, z)$

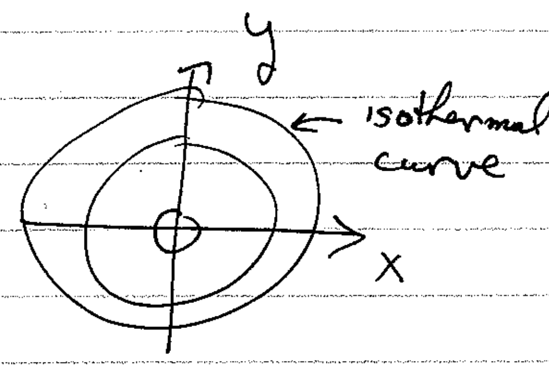
examples:
- Temperature as function of posit.
- Height above sea level
- Mass density in inhomogeneous sample

Example: Temperature in plane $T(x,y) = T_0 e^{-(x^2+y^2)}$

Representations:



Surface plots



Contour Plot

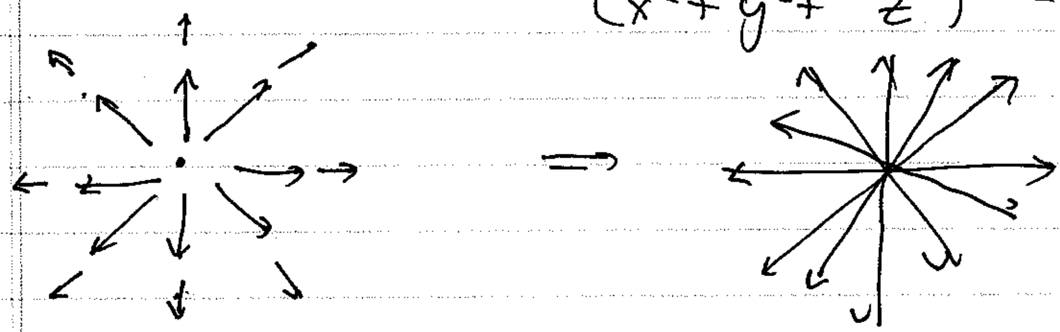
(ii) <u>Vector Field</u> : Vector-valued function of $\vec{r}$

- Velocity in a fluid
- Gravitational field
- Electric and Magnetic fields

- Example: $\vec{E}$ field of a point charge q

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{q}{|\vec{r}|^2} \frac{\vec{r}}{|\vec{r}|}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q\,(x\hat{x} + y\hat{y} + z\hat{z})}{(x^2+y^2+z^2)^{3/2}}$$



(field lines)

- <u>Example</u>: $\vec{B}$ field around a steady current I_0 for an infinite wire along z

$$\vec{B}(x,y,z) = \frac{\mu_0}{4\pi} \frac{2I_0}{(x^2+y^2)} (-y\hat{x} + x\hat{y})$$ Independent of z