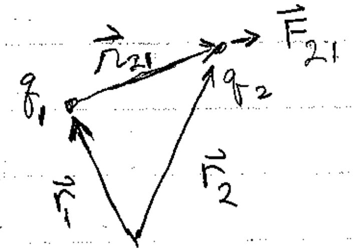
UNM Physics 405: Lecture 5

Coulomb's Law & Electric Fields

- Electrostatics: All Charges are Stationary

Coulomb's Law: Inverse square law between points, with Force directed on line between them



Origin (arbitrary)

Point charges, q_1 and q_2 located at $\vec{r}_1$ and $\vec{r}_2$

$\vec{r}_{21} \equiv \vec{r}_2 - \vec{r}_1$ vector from 1 to 2

$\Rightarrow$ Force on 2 due to 1 $\vec{F}_{21} = k \dfrac{q_1 q_2}{(r_{21})^2}\, \hat{r}_{21} = -\vec{F}_{12}$

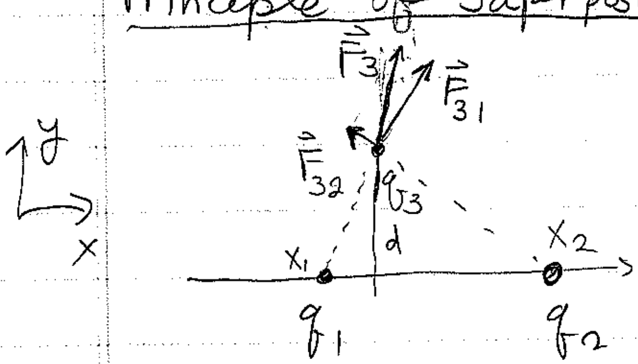
$$\Rightarrow \boxed{\vec{F}_{21} = k\, \frac{q_1 q_2\, (\vec{r}_1 - \vec{r}_2)}{|\vec{r}_2 - \vec{r}_1|^3}}$$

Units? Mechanical units Length, Mass, time
New quanitity Charge
k determines proportionality

SI Units: meter, kilogram, second (MKS)
Charge: Coulomb

$k \equiv \dfrac{1}{4\pi\epsilon_0}$, $\epsilon_0 = 8.85\times 10^{12}\ \dfrac{C}{Nm^2}$ = "permittivity of free space"

Principle of superposition: Forces add as vectors



$$\vec{r}_{31} = -x_1\hat{x} + d\hat{y}$$

$$\vec{r}_{32} = -x_2\hat{x} + d\hat{y}$$

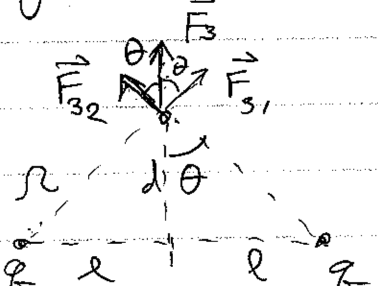
$$\Rightarrow \vec{F}_3 = \vec{F}_{31} + \vec{F}_{32}$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{q_1 q_3}{(x_1^2+d^2)^{3/2}}(-x_1\hat{x}+d\hat{y}) + \frac{q_2 q_3}{(x_2^2+d^2)^{3/2}}(-x_2\hat{x}+d\hat{y})\right]$$

Note: If $q_1 = q_2 = q$ and $x_2 = -x_1 = l$

$$\Rightarrow \vec{F}_3 = \frac{1}{4\pi\epsilon_0}\left(q q_3 \frac{2d}{(l^2+d^2)^{3/2}}\right)\hat{y}$$

Only y-component by symmetry



$$\vec{F}_3 = |\vec{F}_3|\hat{y}$$

$$(F_{31})_y = (F_{32})_y = \frac{1}{4\pi\epsilon_0}\frac{q q_3}{r^2}\cos\theta$$

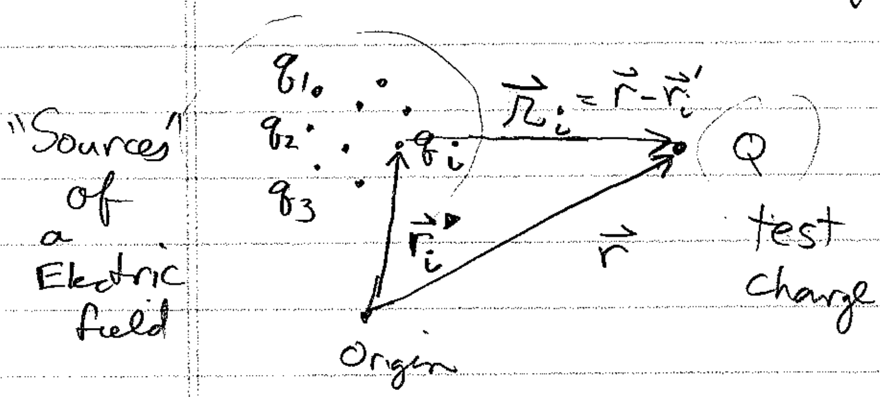
$$= \frac{1}{4\pi\epsilon_0}\frac{q q_3}{r^2}\left(\frac{d}{r}\right) = \frac{q q_3}{4\pi\epsilon_0}\frac{d}{(d^2+l^2)^{3/2}}$$

$$\Rightarrow |\vec{F}_3| = (F_{31})_y + (F_{32})_y = \frac{q_1 q_3}{4\pi\epsilon_0}\frac{2d}{(d^2+l^2)^{3/2}}$$

as before

Electric field

Coulomb's law, as stated, gives a notion of "action at a distance". A different perspective is that the charges generate an electric field, and the local field at a position determines the force on a "test charge" at the position



Notation

- "primed" coordinates for sources
- Script $\vec{\mathscr{r}}$ = vector from source to position of interest

By principle of superposition:

Force on Q due to all the "sources"

$$\vec{F} = \sum_i \frac{Q q_i}{4\pi\epsilon_0 \mathscr{r}_i^2}\hat{\mathscr{r}}_i = Q \underbrace{\sum_i \frac{1}{4\pi\epsilon_0}\frac{q_i}{\mathscr{r}_i^2}\hat{\mathscr{r}}_i}_{\vec{E}(\vec{r})}$$

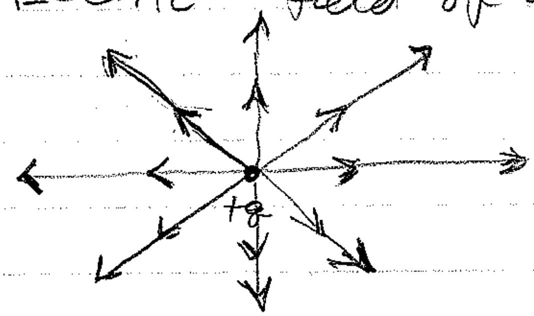
$$\Rightarrow \boxed{\vec{F}_{\text{on } Q}(\vec{r}) = Q\vec{E}(\vec{r})}$$

$$\boxed{\vec{E}(\vec{r}) = \sum_i \frac{1}{4\pi\epsilon_0}\frac{q_i}{\mathscr{r}_i^2}\hat{\mathscr{r}}_i = \sum_i \frac{1}{4\pi\epsilon_0}\frac{q_i(\vec{r}-\vec{r}_i')}{|\vec{r}-\vec{r}_i'|^3}}$$

Physical reality of $\vec{E}$? Right now, only a convenience. Next semester essential when we consider electromagnetic radiation

Electric field = lines of force on a "test charge"

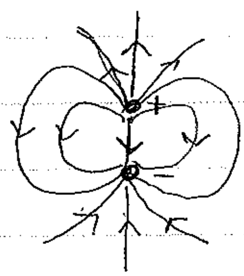
e.g. Electric field of a positive point charge



$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

radially outward from charge

e.g. Electric dipole: One positive, One negative charge (equal and opposite)



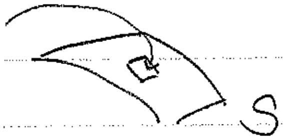
- Field lines go from positive to negative charges
- Field line terminate at charges

Continuous Charge distribution

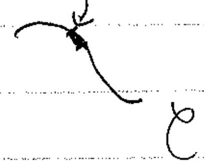
For macroscopic materials, we "coarse grain" over discrete point charges to yield a continuous charge density

Volume distribution: $\rho(\vec{r})$ $dq = \rho\, d^3r$

Surface distribution: $\sigma(\vec{r})$ $dq = \sigma\, da$



Line distribution: $\lambda(\vec{r})$ $dq = \lambda\, dl$



Point: q_i