

## Physics 405 Spring 2009

### Problem Set #1: DUE Fri. 1/30/07

#### Read Griffiths, "Advertisement" and Chap. 1

#### (1) Gradients (Griffiths 1.12)

The height of a certain hill (in feet) is given by

$$h(x, y) = 10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12)$$

where  $y$  is the distance (in miles) north,  $x$  is the distance east of South Hadley.

- (a) Where is the top of the hill located?
- (b) How high is the hill?
- (c) How steep is the slope (in feet per mile) at a point 1 mile north and one mile east of South Hadley? In what direction is the slope steepest at that point?

(d) Check your answer with *Mathematica* by:

$$h[x_, y_] := 10*(2 x y - 3 x^2 - 4 y^2 - 18 x + 28 y + 12)$$

```
Plot3D[h[x,y], {x,-5,0}, {y,0,5}]
```

```
contours = ContourPlot[h[x,y], {x,-4,0}, {y,0,6}]
```

```
Needs["VectorFieldPlots`"]
```

```
fields = GradientFieldPlot[h[x,y], {x,-4,0}, {y,0,6}, PlotPoints->10]
```

```
Show[contours, fields]
```

#### (2) Div and Curl

Calculate the divergence and curl of the following vector fields

(a)  $\mathbf{v} = x^2 \hat{\mathbf{x}} + 3xz^2 \hat{\mathbf{y}} - 2xz \hat{\mathbf{z}}$

(b)  $\mathbf{v} = \hat{\mathbf{x}} \cos(kz - \omega t)$  ,

#### (3) Practice with Gradient (fixed and variable points) (Griffiths 1.13)

#### (4) Notion of Flux: (Griffiths 1.29)