Physics 405 Spring 2009

Problem Set #3: DUE Fri. 02/13/2009

Read: Griffiths Chap. 2.1-2.2

(1) Electrostatic Fields of disk (10 Points)

Griffiths 2.6, page 64. (*Hint*: Use the results of part (a))

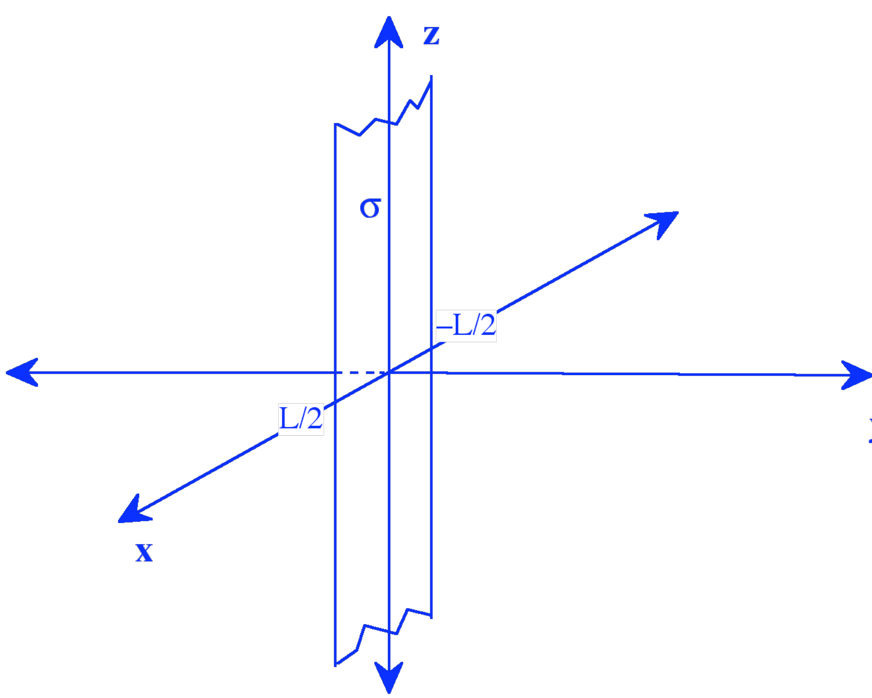
(2) Griffith 2.16, page 75 (10 Points)

(3) Griffith 2.17, page 75 (10 Points)

Check your answer in the limit d-->0 such that all the charge is concentrated on the surface (i.e. $2\rho d \to \sigma$: charge per unit area). What is the difference in the magnitude of the electric field and below and above the slab? In Prob. (3) you should have found that the electric field is discontinous at $r=b$. Determine a general rule for the change in $|\mathbf{E}|$ across a charge distribution.

(4) (15 points)

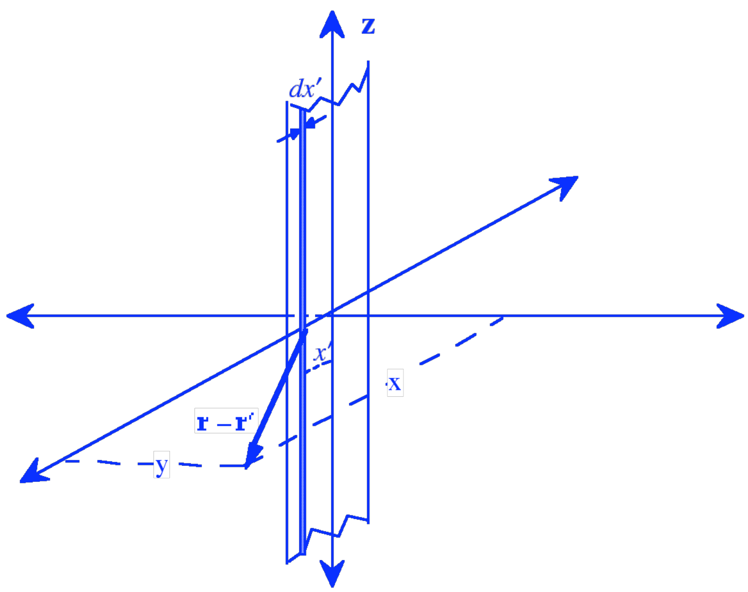
An infinitely long strip of width L (shown below) carries a charge per unit area σ



(a) Show that the electric field over all space is:

$$\mathbf{E}(\mathbf{r}) = \frac{\sigma}{2\pi\varepsilon_0}\left[\frac{1}{2}\ln\left(\frac{(x+L/2)^2+y^2}{(x-L/2)^2+y^2}\right)\hat{\mathbf{x}} + \left(\tan^{-1}\left(\frac{x+L/2}{y}\right) - \tan^{-1}\left(\frac{x-L/2}{y}\right)\right)\hat{\mathbf{y}}\right]$$

(Hint: Break up the strip into infinitesimal strips of width *dx* shown on the next page)



(b) Take the following limits, and explain why the result is what you expect.

(i) $\lim\limits_{x\to 0}\mathbf{E}(\mathbf{r})$ (ii) $\lim\limits_{L\to\infty}\mathbf{E}(\mathbf{r})$ (iii) $\lim\limits_{r\to\infty}\mathbf{E}(\mathbf{r})$

(c) Plot the electric field in the *x*-*y* plane, with **E** in units of $\sigma/2\pi\varepsilon_0$, using the following *Mathematica* code:

```
Needs["Graphics`PlotField`"]
Efield[x_,y_,L_] := { .5 Log[ ((x+L/2)^2+y^2) / ((x-L/2)^2+y^2) ] ,
                        ArcTan[(x+L/2)/y]–ArcTan[(x+L/2)/y] }
PlotVectorField[Efield[x,y,L], {x, xi, xf}, {y, yi, yf}]
```

Choose various values for *L* and plot regions {xi,xf}, {yi,yf}, and explain why the plots make sense.

(d) **Extra Credit**: Take the limit of **E**(**r**) with $L \to 0$, while $\sigma \to \infty$ such that the product of *L* and σ remains constant and goes to $L\sigma \to \lambda$, the charge per unit length. Explain your result.