Physics 405 Spring 2008

Problem Set #4: DUE Friday. 2/20

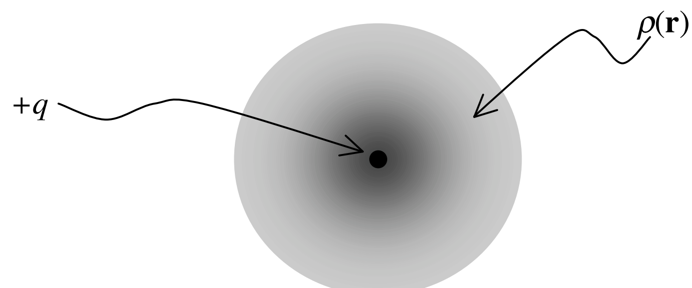
Read: Griffiths, Chap. 2.3-2.5

(1) The "Yukawa Potential" (15 Points)

The electric force dominates the interaction of particles because it is very long-range; i.e. the electrostatic potential falls off as an algebraic function of distance 1/r. Nuclear forces come into play only at short distances. The Yukawa form of the nuclear potential is

$$V(r) = G\ \frac{ae^{-r/a}}{r},$$

where G is some coupling constant, and a is a distance. This potential can be mimicked in electrostatics by a point charge q at the origin and "screened" by a smeared out negative charge distribution $\rho(r) = -\frac{q}{4\pi a^2}\frac{e^{-r/a}}{r}$. (Here a is a constant with dimensions of length)



(a) Sketch V(r) showing the asymptotic form for r<<a and r>>a. Explain the physical significance of the constant a.

(b) Show that the overall charge distribution is neutral. Does this make sense given the long range form of the potential?

(c) Show that this charge distribution gives a Yukawa potential
(*Hint* Use Gauss' law to find the electric field first)

(d) What is the potential energy stored in the charge distribution ρ(r)?

(2) Potentials and contours (25 points)

Consider again the strip of charge as given in Prob. Set #3, prob. (4)

(a) Show that the electrostatic potential in the *x*-*y* plane is

$$V(x,y)=\frac{\sigma}{4\pi\varepsilon_0}\left\{-2L+2y\left(\tan^{-1}\left(\frac{L/2-x}{y}\right)+\tan^{-1}\left(\frac{L/2+x}{y}\right)\right)\right.$$
$$\left.-(x-L/2)\ln\left[(x-L/2)^2+y^2\right]+(x+L/2)\ln\left[(x+L/2)^2+y^2\right]\right\}$$

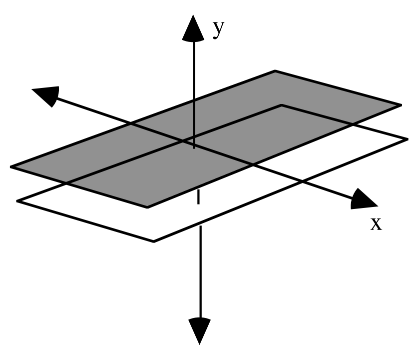
(The integral $\int dx\ln(x^2+y^2)=-2x+2y\tan^{-1}(x/y)+x\ln[x^2+y^2]$ may be useful).

(b) Use this potential to recover the electric field given in Problem Set #3, 4(a).

(c) Take the limit of $V(x,y)$ as in Problem Set #3, 4(b) and explain your results.

(d) Plot the equipotential contours using *Mathematica*, and plot the gradient field as in Problem Set #1. Do this 3 times with different ranges for the plots:
(i) x,y ~ L. (ii) x,y >>L. (iii) x,y<<L. Explain your Results

(e) Now suppose there are two strips, one with positive surface charge σ at y=0, and one with equal and opposite charge –σ, at y = –s as shown:



Use the *principle of superposition* to find V(x,y).

(f) Again plot the contours and Gradient field. Do this for two cases
(i) s<<L . (ii) s>>L Explain your results.