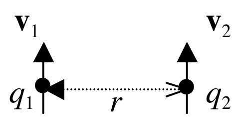
Physics 405

Problem Set #10: DUE 4/24/2009

Read Griffiths Chap. 5

Problem 1

(a) Consider two charges q_1 and q_2 separated by a distance r, moving with the same constant velocity $\mathbf{v}$, with v<<c (the speed of light).



Show that the repulsive force due to the electric (Coulomb repulsion) is much, much stronger than the attractive force due to the magnetic field.

(b) Show that in the presence of a time independent electric and magnetic field, the general equation of motion for a particle of mass m and charge q can be written as,

$$\frac{d^2\mathbf{v}}{dt^2} = -\omega_c^2\mathbf{v} + \frac{q^2}{m^2}\left[(\mathbf{E}\times\mathbf{B}) + (\mathbf{B}\cdot\mathbf{v})\mathbf{B}\right],$$

where $\omega_c = \dfrac{qB}{m}$ is the cyclotron frequency.

(Note that the equation of motion depends only on the charge-to-mass ratio)

(c) Now let $\mathbf{E} = E_0\hat{y}$, $\mathbf{B} = B_0\hat{z}$. Assuming an initial condition such that the particle has no velocity along $\mathbf{B}$ at t=0, show that,

$$\mathbf{v}(t) = v_0\cos(\omega_c t + \phi)\hat{\mathbf{x}} + v_0\sin(\omega_c t + \phi)\hat{\mathbf{y}} + \frac{\mathbf{E}\times\mathbf{B}}{B^2},$$

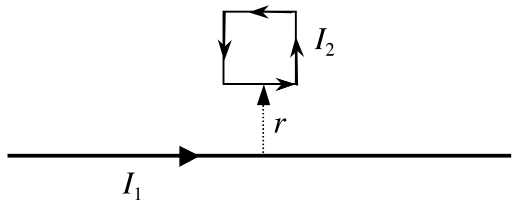
where v_0 and ϕ are constants that depend on the initial condition.

Sketch a trajectory in the x-y plane.

(d) If E_0=1 Volt/cm, B_0=1 Tesla, and $v_0=10^3$ cm/s, how many cyclotron radii will the particle drift in the x-direction in one cyclotron period, $T_c = 2\pi/\omega_c$.

Problem 2: **Griffiths, Problem 5.46**. Also plot the $B(z)$ in units of $\mu_0 I/R$ for the field found in part (b), in the region –R<z<R. Given coils of wire with radius 10 cm, what current is needed to cancel the Earth's magnetic field (~0.5 Gauss) at the midpoint of the Helmholtz coils?

Problem 3: A square loop (each side length s) carrying a steady current I_1 sits a distance r from an infinitely long wire, carrying current I_2



(a) Show that the force on the loop is

$$\mathbf{F} = -\frac{\mu_0 I_1 I_2}{2\pi}\left(\frac{s}{r} - \frac{s}{s+r}\right)\hat{\mathbf{r}}\,.$$

(b) Show that in the limit s<<r,

$$\mathbf{F} = \nabla(\mathbf{m} \cdot \mathbf{B})$$

where $m = I_1 s^2 \hat{\phi}$ is the magnetic moment of the loop and **B** is the magnetic field due to the infinitely long wire at the center of the small loop.