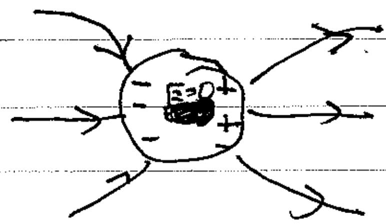
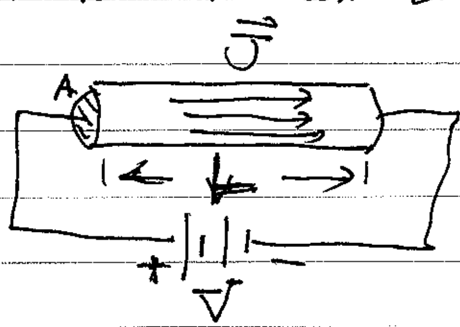
Physics 406: Lecture 2

Electric Currents and EMF

In electrostatics, electric field inside a ~~cond~~ (perfect) conductor is zero: charges move until they cancel field so there is no more force on them



Consider now a conductor connected to a battery



$$\boxed{\vec{J} = \sigma \vec{E}}$$

↑ conductivity

Ohm's "Law" (imperical)

Charges flowing inside conductor, not electro-~~magneto~~static.

More familiar form of Ohm's Law

$I \cong JA \qquad E \approx \frac{V}{L} \quad \Rightarrow$

$$I = JA = \sigma E A = \sigma A \frac{V}{L} = \frac{V}{R}$$

$$\Rightarrow \boxed{V = IR} \qquad \boxed{R = \frac{L}{\sigma A}} \text{ resistance}$$

Resistivity: $\frac{1}{\sigma}$ (Ohms/meter)

Conductors (Copper, Silver) : $\frac{1}{\sigma} \sim 10^{-8} - 10^{-7}\ \Omega/m$

Semiconductors (Silicon, Germanium) : $\frac{1}{\sigma} \sim 10^{-2} - 10^{-1}\ \Omega/m$

Insulators (Rubber, Wood, Glass) : $\frac{1}{\sigma} \sim 10^{5} - 10^{16}\ \Omega/m$

Ohm's Law is Surprising:

$$\vec{J} = \rho\vec{v} = \sigma\vec{E} \Rightarrow \vec{v} \propto \vec{E} = \frac{\vec{F}}{q}$$

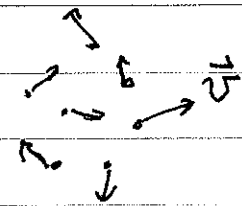
Newton's Laws ???

Answer: Motion of charges is not "ballistic" in typical material, but involves thermal motion in imperfect crystal. Electron acceleration is interrupted by constant collisions, never picking up much speed.

We will consider a simple classical model (Drude model). The real story needs quantum mechanics.

Drude model

- ~~Thermal electrons~~ in conductor



$$\langle \vec{v} \rangle = 0 \qquad \frac{1}{2} m_e \langle \vec{v}^2 \rangle = \frac{3}{2} k_B T$$ (Equipartition)

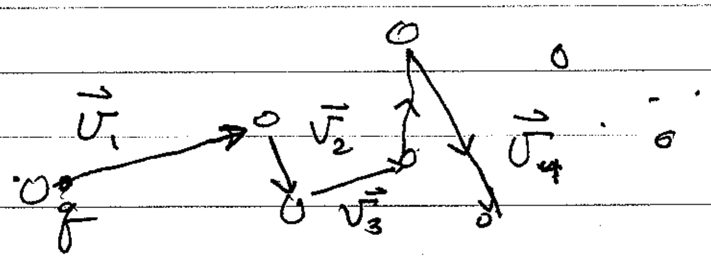
$$\langle \vec{v}^2 \rangle = 3 \langle v_x^2 \rangle$$ ← some direction

$$\Rightarrow \sqrt{\langle v_x^2 \rangle} = \sqrt{\frac{k_B T}{m_e}} \equiv v_{thermal}$$

Define: λ = mean free path for collisions

$\Rightarrow$ Mean time between collisions: $\tau \equiv \frac{\lambda}{v_{therm}}$

- Apply ~~elect~~ external electric field in x-direction



$$\Longrightarrow \vec{F} = q\vec{E}$$

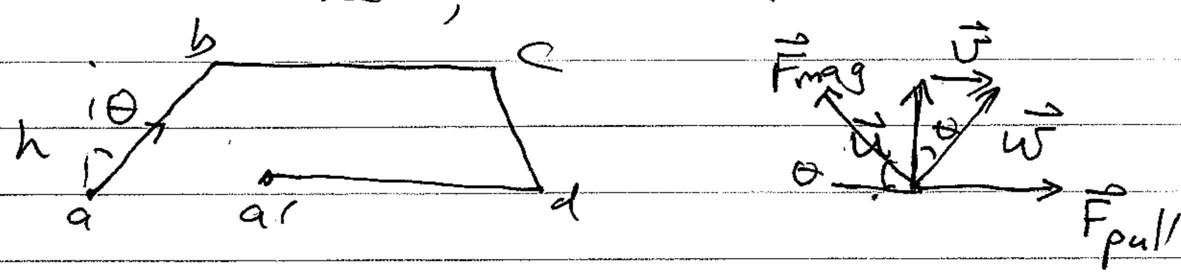
After each collision, velocity is randomized

After N_c collision

$$\langle \vec{v} \rangle = \frac{1}{N_c} \sum_{0} \left(\vec{v}_j^{\,c} + \frac{q\vec{E}}{m} t_j \right)$$

$$\Rightarrow \langle \vec{v} \rangle = \frac{qE}{m} \tau$$

In "Lab frame", after a time Δt



$$|\vec{F}_{mag}| = q w B$$

$\Rightarrow$ Must pull circuit to counter act "back EMF" of the flowing charges

No acceleration of loop

$$\Rightarrow \frac{|\vec{F}_{pull}|}{q} = \frac{|\vec{F}_{mag}|}{q}\cos\theta = w\cos\theta\, B$$

$$= uB$$

$$\oint \frac{\vec{F}_{pull}}{q}\cdot d\vec{l} = \sin\theta\,(uB)\frac{h}{\cos\theta} = w\sin\theta\, Bh$$

$$= vBh \quad \checkmark$$

$\Rightarrow$ Work done by $\vec{F}_{pull}$

density of chare carriers

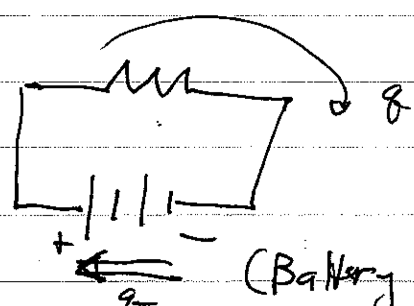
$$\vec{J} = qn\vec{v} = \boxed{\frac{nq^2\tau}{m}}\vec{E}$$

σ — General relation: Holds as long as $\vec{E}$ is small enough that it doesn't affect τ

$$\tau \propto \frac{1}{\sqrt{\text{Temp}}} \Rightarrow R \propto \sqrt{\text{Temp}}$$

Ohmic Heating

Because of resistance, the flowing charges lose some energy.



(Battery does work to move charge "up hill") $W = qV$

Power = Rate at which battery does work
= Rate at which energy is dissipated by resistor

$$\Rightarrow P = \frac{dq}{dt}V = IV = \boxed{I^2 R}$$

↑ familiar relation

General expression:

$$P = \vec{v}\cdot\vec{F} = \int d^3r\, \vec{v}\cdot\rho\vec{E} = \int d^3r\, \vec{J}\cdot\vec{E}$$

$\Rightarrow \vec{J}\cdot\vec{E} = \dfrac{\text{local rate of energy dissipation}}{\text{Volume}}$

"Ohmic heating"

EMF (Electromotive force)

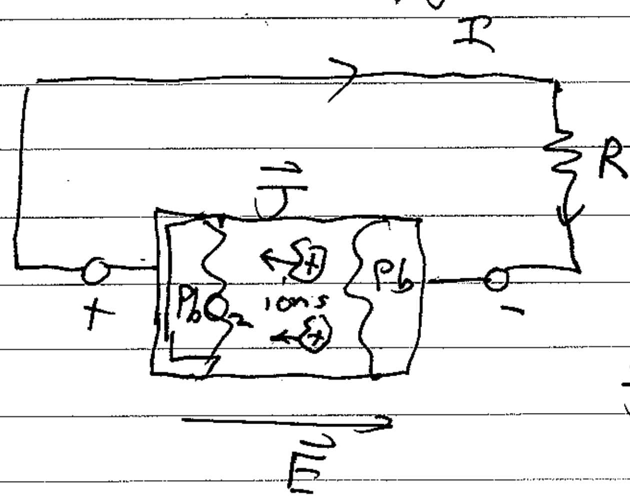
If energy is being dissipated, then there must be some force to maintain the steady flow

$\Rightarrow$ Battery supplies "electromotive force" (emf")

Define $\mathcal{E} = \frac{1}{q}\oint \vec{F}\cdot d\vec{l}$ $\left(\frac{\text{Force}\cdot\text{Distance}}{\text{charge}} = V\right)$

For Circuit $\mathcal{E} = \frac{1}{q}\oint\left(\vec{F}_{\text{Electrostatic}} + \vec{F}_{\text{battery}}\right)\cdot d\vec{l}$

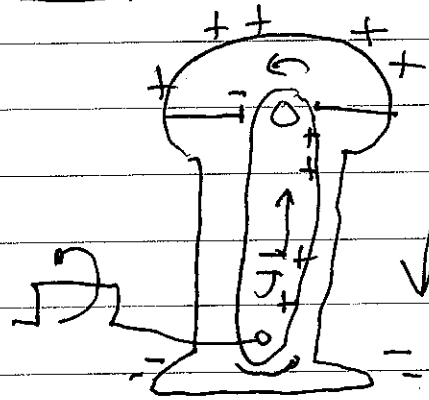
$\Rightarrow \mathcal{E} = \frac{1}{q}\oint \vec{F}_{\text{battery}}\cdot d\vec{l} = V$



$\vec{J}\cdot\vec{E} < 0$ in battery

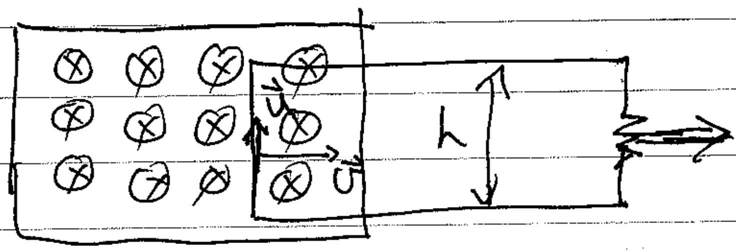
$\Rightarrow$ Work done

Example: Van de Graaff



$$\vec{J}\cdot\vec{E} < 0$$

Motional EMF in magnetic field



Magnetic field pushes current along wire

$$\mathcal{E} = \oint_{loop} \frac{\vec{F}}{q}\cdot d\vec{l} = \frac{(qvB)\,h}{q} = vBh$$

But, magnetic fields do no work?

- EMF arising from work done by the force pulling the circuit

- $\mathcal{E}$ calculated in instaneous frame of circuit

- Work calculated in lab frame