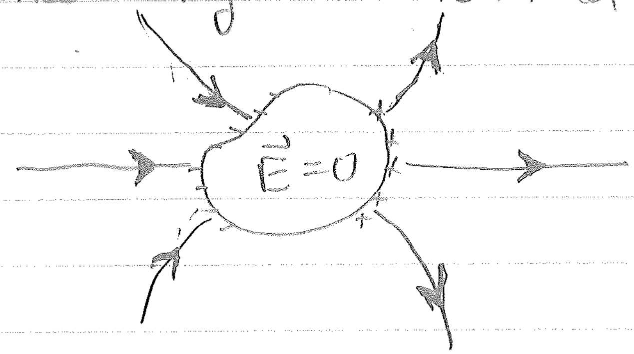
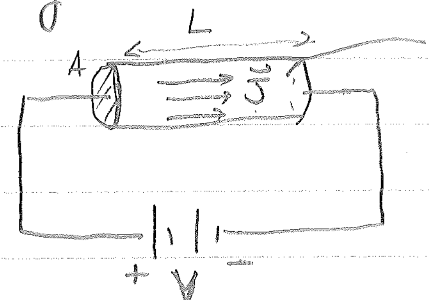
# Physics 406: Lecture 2

## Electric Currents & EMF

In electrostatics, the electric field inside a perfect conductor is zero: Charges move until they cancel any electric field in the bulk




Consider now a conducting bar connected to a battery




Conductivity: $\sigma$

Resistivity: $\rho_R = \frac{1}{\sigma}$

Empirical Law: $\boxed{\text{Ohm's Law} \quad \vec{J} = \sigma \vec{E}}$

$\Rightarrow$ Inside the conductor $\vec{E} \neq 0 \Rightarrow$ No electrostatics!

More familiar form of Ohm's Law:

$\vec{E}$ uniform $\Rightarrow \quad E = \frac{V}{L}, \quad J = \frac{I}{A}$

$V$ = Voltage drop , $I$ = current

$$J = \sigma E \Rightarrow E = \frac{J}{\sigma} \Rightarrow \frac{V}{L} = \frac{I}{\sigma A}$$

$$\Rightarrow \boxed{V = IR}, \quad \boxed{R = \frac{L}{\sigma A}}$$

Ohm's Law — resistance

Over 25 orders of magnitude between good and bad conductors

Excellent Conductors (e.g. Copper, Silver) $\frac{1}{\sigma} \sim 10^{-8} - 10^{-7}\ \Omega/m$ ← Ohms ← meter

Semi Conductors (e.g. Silicon, Germanium) $\frac{1}{\sigma} \sim 10^{-2} - 10^{-1}\ \Omega/m$

Insulators (e.g. Rubber, glass) $\frac{1}{\sigma} \sim 10^{5} - 10^{16}\ \Omega/m$

Ohm's Law is surprising

$$\vec{J} = \rho \vec{v} = \sigma \vec{E} \Rightarrow \vec{v} \propto \vec{E} = \frac{\vec{F}}{q}$$

$\rho$: local charge density; $\vec{v}$: local velocity

Huh?? What about Newton's Law!

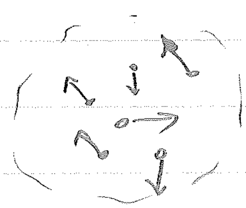
$$\vec{F} = m\vec{a}$$

<u>Answer:</u> the motion of the charges is not "ballistic" in a typical material, but instead involves thermal motion in imperfect crystals

$\Rightarrow$ Electron acceleration is interupded by very frequency collisions $\rightarrow$ drag force

$\rightarrow$ electron reach "terminal velocity"

<u>Drude Model</u> (Classical kinetic gas theory)



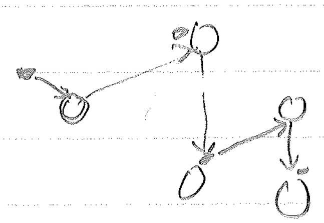
$\leftarrow$ Conduction electrons in thermal motion

$\langle \vec{v} \rangle = 0 \qquad \frac{1}{2} m_e \langle \vec{v}^2 \rangle = \frac{3}{2} k_B T$ (equipartition theorem)

In an direction $\frac{1}{2} m \langle v_x^2 \rangle = \frac{1}{2} k_B T$

$\Rightarrow$ RMS thermal velocity $\sqrt{\langle v_x^2 \rangle} = \sqrt{\frac{k_B T}{m}} = v_{therm}$

Thermal electrons undergo collisions w/ defects



$\lambda$ = mean-free path between collisions

$\tau \equiv \frac{\lambda}{v_{therm}}$ = average time between collisions

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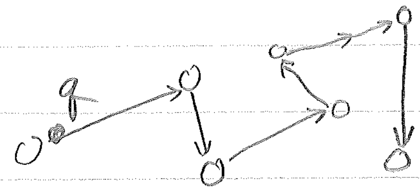
$\tau \propto \frac{1}{\sqrt{T}}$ $\leftarrow$ temperature

Now apply an electric field




$\rightarrow \langle \vec{v}_{drift} \rangle$ (drift velocity)

$\Longrightarrow \quad \vec{F} = q\vec{E}$

Because of thermal motion, there are many collisions with defects the essential randomize velocity

$$\vec{a} = \frac{d\langle \vec{v} \rangle_{drift}}{dt} = -\frac{1}{\tau}\langle \vec{v} \rangle_{drift} + \frac{q\vec{E}}{m}$$

Steady state $\vec{a} \Rightarrow 0 \qquad \langle \vec{v}_{drift} \rangle = \frac{q\tau}{m}\vec{E}$

Current density: $\vec{J} = q\underset{\uparrow}{n}\vec{v} = \frac{nq^2\tau}{m}\vec{E}$

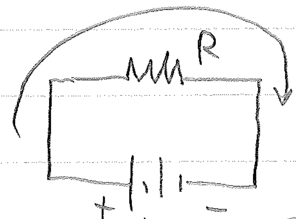
density of charge carriers

$$\Rightarrow \boxed{\begin{array}{l} \vec{J} = \sigma\vec{E} \\ \\ \sigma = \dfrac{nq^2\tau}{m} \end{array}}$$

Note: $\tau \propto \frac{1}{\sqrt{T}} \Rightarrow$ resistance $\propto \sqrt{T}$

## Ohmic Heating

Because of resistance, the flowing charges lose energy in inelastic collisions to heat




$I = \frac{dq}{dt}$

← Battery does work to move charge "up hill" $U = qV$. This potential energy is dissipated in resistor in steady-state.

Power = Rate at which battery does work

in steady state = Rate at which energy is dissipated in the resistor

$$P = \frac{d}{dt}(qV) = \frac{dq}{dt} V = IV = I^2 R$$

General expression:

$$P = \underbrace{\vec{v} \cdot \vec{F}_R}_{\text{electric force}} = \int d^3r \; \vec{v} \cdot \rho \vec{E} = \int d^3r \; \vec{J} \cdot \vec{E}$$

$\Rightarrow$ $\vec{J} \cdot \vec{E}$ = $\dfrac{\text{local rate field do work on charges}}{\text{Volume}}$

Ohmic heating if work is dissipated

# EMF (Electromotive Force)

If energy is being dissipated, then there must be some force to maintain the steady flow

$\Rightarrow$ Battery supplies "electromotive force" (EMF)

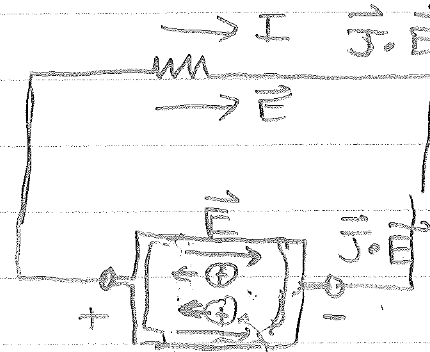
Define $\mathcal{E} = \frac{1}{q} \oint_{C} \vec{F} \cdot d\vec{l}$ : Force / charge around some closed loop at some instance

$\vec{\text{EMF}}$ = Voltage

For the circuit:

$$\mathcal{E} = \frac{1}{q} \oint \left( q\, \vec{E}_{\text{electro-static}} + \vec{F}_{\text{battery}} \right) \cdot d\vec{l}$$

Since $\oint \vec{E}_{\text{electro-static}} \cdot d\vec{l} = 0 \Rightarrow \boxed{\oint \frac{\vec{F}_{\text{battery}}}{q} \cdot d\vec{l} = V}$



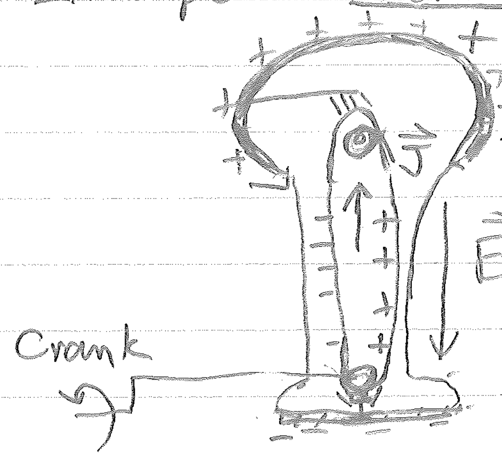

$\vec{J} \cdot \vec{E} > 0 \Rightarrow$ Ohmic heating

$\vec{J} \cdot \vec{E} < 0 \Rightarrow$ EMF

Ions in battery travel across potential barrier (up hill) because of electrochemical process

(quantum mechanics!)

• Example: Van de Graaff Generator

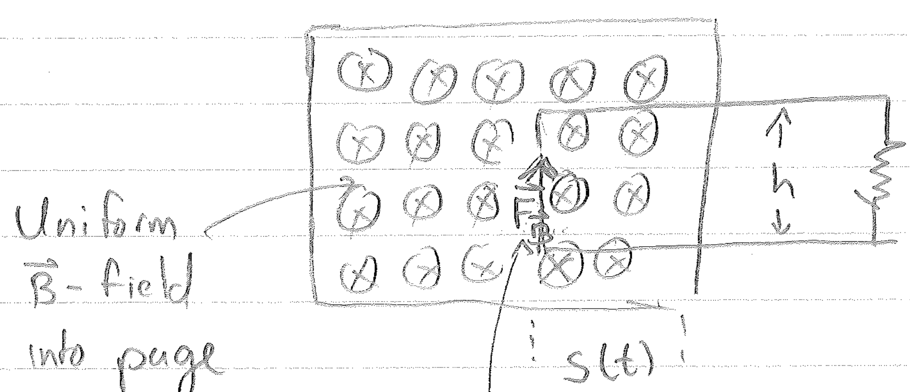



Brushes strip electrons making dome positive relative to base

$\vec{J}\cdot\vec{E} < 0 \Rightarrow$ Work being done mechanically to to move charge against potential

$\Rightarrow$ EMF

• "Motional EMF" (Lorentz force)




Pulling Loop with force $\vec{F}_{pull}$ @ constant $\vec{v}$

$\vec{F}_B = q\,\vec{v} \times \vec{B}$

$$\Rightarrow \boxed{\mathcal{E} = \oint \frac{\vec{F}_B}{q} \cdot d\vec{l} = vBh}$$

Moving loop through region of different $\vec{B}$-fields $\Rightarrow$ EMF

But, magnetic fields do no work!

Force magnetic $q\vec{v}\times\vec{B} = \vec{F}_{mag}$

Rate at which work is done $= \vec{F}_{mag}\cdot\vec{v} = 0$ !

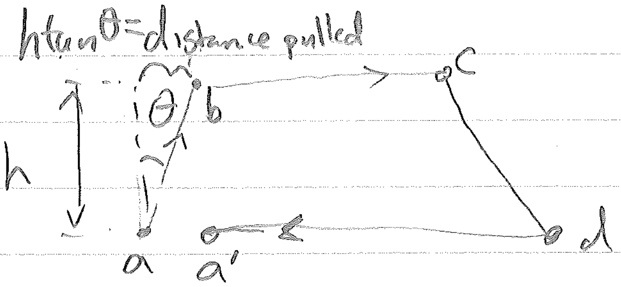
So who is doing the work? Answer – whoever is pulling the loop.

Note: EMF is defined $\mathcal{E} = \oint_{\mathcal{C}(t)} \frac{\vec{F}}{q}\cdot d\vec{l}$

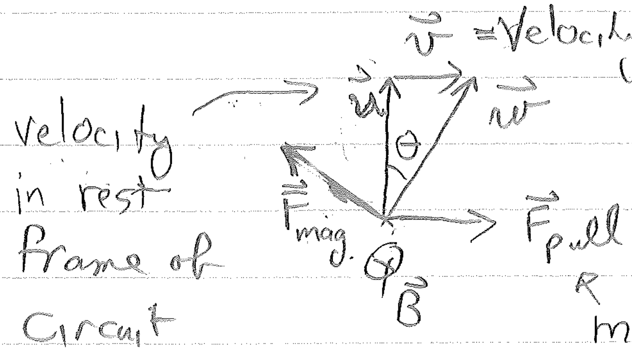
Where $\mathcal{C}(t)$ = contour @ any instant.

But Work $W = \int \vec{F}\cdot d\vec{l}(t)$

path of particle in a give frame




Path of a charge $q$ in the lab frame




$|\vec{F}_{mag}| = q w B$

must pull to mainstain constant $\vec{v}$

$|\vec{F}_{pull}| = |\vec{F}_{mag}|\cos\theta = q\, w\cos\theta\, B = quB$

Work done in pulling a distance $h\tan\theta$

$$\oint \vec{F}_{pull} \cdot d\vec{l} = quBh\tan\theta = qvBh$$

$$\Rightarrow \mathcal{E} = \frac{\oint \vec{F}_{pull} \cdot d\vec{l}}{q} = vBh \checkmark$$

$\Rightarrow \mathcal{E}$ from mechanical force