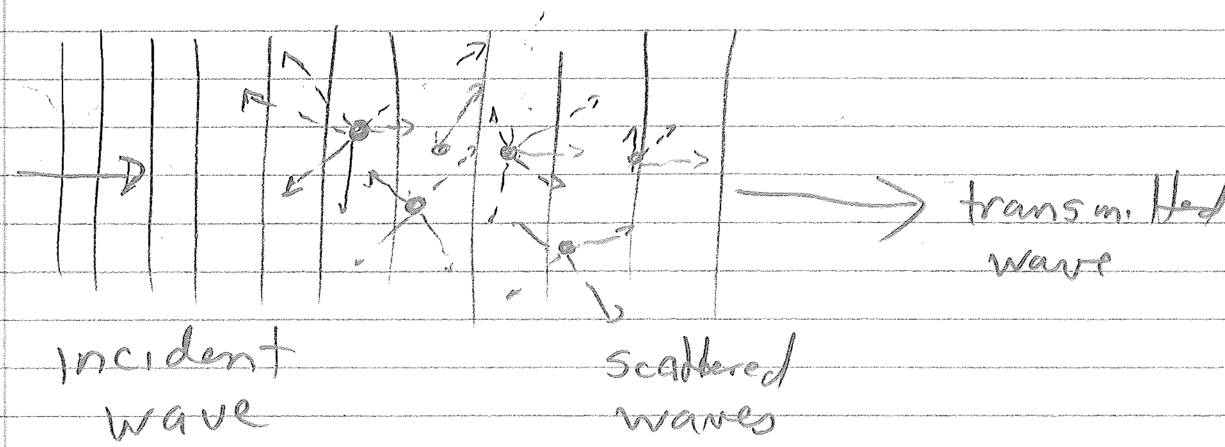
Physics 406: E&M II

Lecture 21: Scattering of Electromagnetic Waves (I)

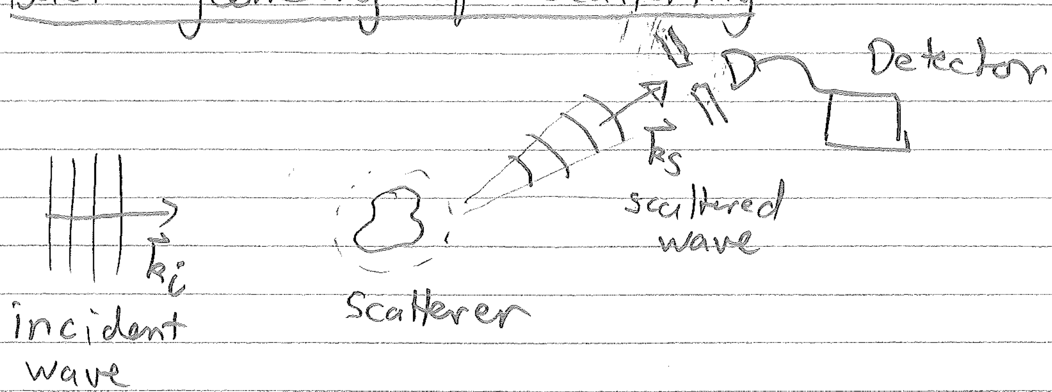
When a beam of electromagnetic radiation passes through matter, the fields cause charges to oscillate and thus radiate. For example, when light passes through a gas, the electric field will cause the electrons to oscillate and radiate in random directions



This process is known as scattering. The transmitted wave will be attenuated as some of the energy is radiated into other directions. Scattering also underlies the index of refraction, as a interference process between the incident wave as the forward scattered wave.

Scattering is a fundamental process in physics, explaining basic phenomena such as why the sky is blue, and also an important experimental tool for probing systems. By send in a beam and seeing how it scatters, we learn a lot about how systems interact and what they are made up. The scattering could beam a beam of waves, or a beam of particles. Or, in the case of quantum mechanics, it could be scattering of probability amplitudes which ultimately are detected as particles.

Basic geometry of scattering



Definition: Cross-Section

Scattering cross-section $\sigma = \dfrac{\text{Total stuff scattered / time}}{\text{Incident flux density of stuff}}$

For E/M wave, stuff = electromagnetic energy

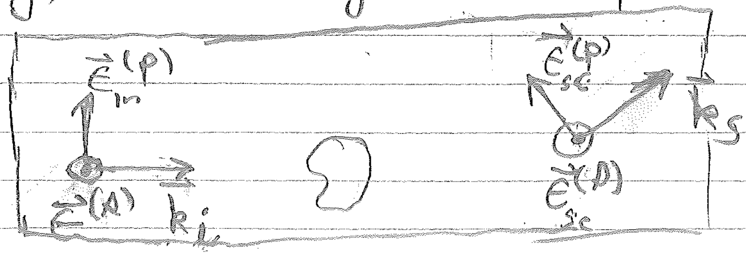
$$\sigma = \frac{\text{Total Power Scattered}}{\text{Incident Intensity}}$$

$$\Rightarrow \boxed{I_{inc}\,\sigma = P_{scat}}$$

Differential scattering cross-section

$$\frac{d\sigma}{d\Omega}(\hat{k}_i, \hat{k}_s) = \frac{\dfrac{dP_{scat}}{d\Omega}(\hat{k}_i, \hat{k}_s)}{I_{inc}(\hat{k}_i)}$$

Generally, the scattering will be polarization dependent

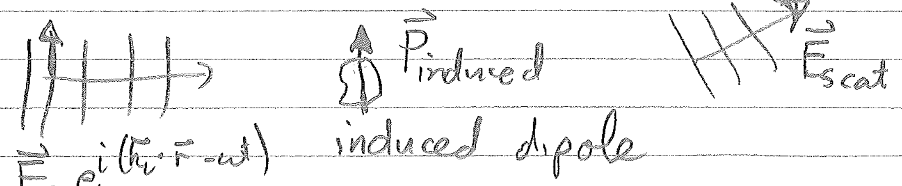


- p-polarization – plane of $\vec{k}_i$ & $\vec{k}_s$
- s-polarization $\perp$ to plane

⇒ Define polarization-dependent scattering cross-section

$$\frac{d\sigma}{d\Omega}(\hat{k}_i, \vec{\epsilon}_{in}\,;\hat{k}_s, \vec{\epsilon}_{sc}) = \frac{\frac{dP_{scat}}{d\Omega}(\hat{k}_s, \vec{\epsilon}_{sc})}{I_{inc}(\hat{k}_i, \vec{\epsilon}_{in})}$$

Long-wavelength scattering: $\lambda >>$ size of scatterer



The radiation will be dominated by electric dipole

$$\Rightarrow \vec{E}_{scat} = \frac{k^2}{4\pi\epsilon_0}\vec{P}_\perp^{\,induced}\frac{e^{ikr}}{r} \qquad \left(k = \frac{\omega}{c}\right)$$

$$\frac{dP_{scat}}{d\Omega}(\hat{k}_s, \vec{\epsilon}_{sc}) = \underbrace{\left\langle \vec{S}_{scat}(\hat{k}_s, \vec{\epsilon}_{sc})\right\rangle}_{\frac{c\epsilon_0}{2}|\vec{\epsilon}_{sc}\cdot\vec{E}_{scat}|^2}\cdot\hat{r}\,r^2$$

$$= \frac{c\,k^4\,|\vec{\epsilon}_{sc}\cdot\vec{P}_\perp^{\,induced}|^2}{32\pi^2\epsilon_0}$$

$$I_{inc}(\hat{k}_{in}\vec{\epsilon}_{in}) = \left\langle \vec{S}_{inc}(\hat{k}_{in}, \vec{\epsilon}_{in})\right\rangle\cdot\hat{k}_{in}$$

$$= \frac{c\epsilon_0}{2}|\vec{\epsilon}_{in}\cdot\vec{E}_{inc}|^2$$

$$\Rightarrow \boxed{\frac{d\sigma}{d\Omega}(\hat{k}_{in}, \vec{\epsilon}_{in}\,;\hat{k}_{sc}, \vec{\epsilon}_{sc}) = \left(\frac{1}{4\pi\epsilon_0}\right)^2 k^4\frac{|\vec{\epsilon}_{sc}\cdot\vec{P}_\perp^{\,ind}|^2}{|\vec{\epsilon}_{in}\cdot\vec{E}_{inc}|^2}}$$

Linear Response

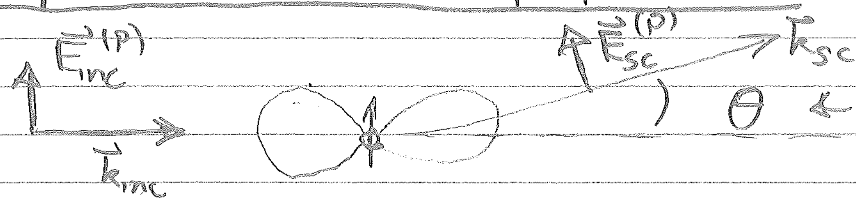
Typically, the induced dipole moment will be linearly proportionally to the field.

$$\vec{P}_{induced} = \tilde{\alpha}\, \vec{E}_{incident}$$

"polarizability"

$$\Rightarrow \frac{d\sigma}{d\Omega}(\hat{k}_{in}, \vec{\epsilon}_{in} ; \hat{k}_{sc}, \vec{\epsilon}_{sc}) = \frac{k^4 |\tilde{\alpha}|^2}{(4\pi\epsilon_0)^2} \frac{|\vec{\epsilon}_{sc} \cdot \vec{E}^{\perp}_{inc}|^2}{|\vec{\epsilon}_{in} \cdot \vec{E}_{inc}|^2}$$

p-polarization $\Rightarrow$ p-polarization

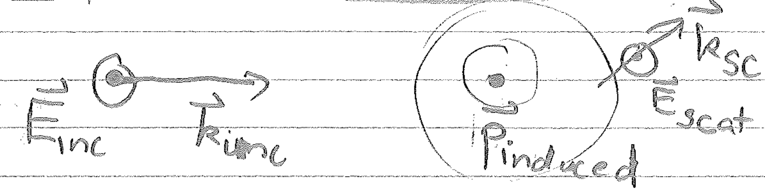


$\leftarrow$ In scattering θ - taken w.r.t. incident $\vec{k}_{inc}$

$$\Rightarrow |\vec{E}^{\perp}_{inc}| = E_{inc} \cos\theta$$

$$\Rightarrow \boxed{\frac{d\sigma}{d\Omega}(\hat{k}_{in}, \vec{\epsilon}^{(p)}_{in} ; \hat{k}_{sc}, \vec{\epsilon}^{(p)}_{sc}) = \frac{k^4 |\tilde{\alpha}|^2 \cos^2\theta}{(4\pi\epsilon_0)^2}}$$

s-polarization $\Rightarrow$ s-polarization

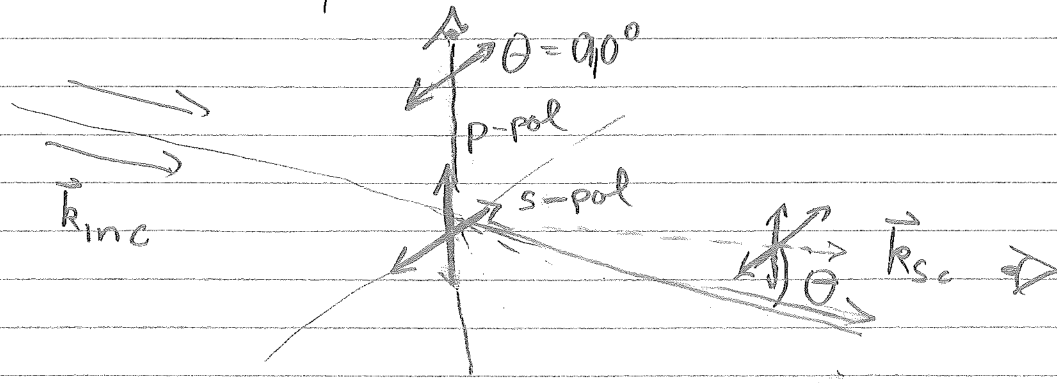


$$|\vec{E}^{\perp}_{inc}| = E_{inc} \quad \forall \theta$$

$$\Rightarrow \boxed{\frac{d\sigma}{d\Omega}(\hat{k}_{in}, \vec{\epsilon}^{(s)}_{in} ; \vec{k}_{sc}, \vec{e}^{(s)}_{in}) = \frac{k^4 |\tilde{\alpha}|^2}{(4\pi\epsilon_0)^2}}$$

Scattering of unpolarized light

Light from natural sources (like stars) has a random polarization. That is, the two possible directions oscillate with a random phase between them.



As a function of θ, the light is partially polarized. At $\theta_{scat} = 0°$ we see both polarizations. At $\theta_{scat} = 90°$ we see only s-polarization $\Rightarrow$ Only $\frac{1}{2}$ of the intensity is scattered into $90°$ and 100% into $0°$ $\Rightarrow$ forward scattering is stronger.

$\Rightarrow$ Total differential scattering cross-section of unpolarized light into all directions

$$\boxed{\frac{d\sigma}{d\Omega}(\theta) = \frac{k^4 |\tilde{\alpha}|^2}{(4\pi\epsilon_0)^2} \left(\frac{1+\cos^2\theta}{2} \right)}$$