Physics 406

Problem Set #1: DUE Friday. Aug. 31 2012

Read: Griffiths Chap. 7.1

(1) Mathematical Review (10 points)

(a) Show that for any vector field $\mathbf{F}(\mathbf{r})$, $\nabla\times\nabla\times\mathbf{F} = \nabla(\nabla\cdot\mathbf{F}) - \nabla^2\mathbf{F}$.

(b) Consider a physical quantity Q (this could be charge, energy, momentum, etc.). If $\rho(\mathbf{r},t)$ is the local density of Q per unit volume, $\mathbf{J}(\mathbf{r},t)$ is the local flux density of Q (i.e. rate at which Q flows through a unit area), and $R(\mathbf{r},t)$ is the local rate at which Q is produced or destroyed per unit volume, show that the general "continuity equation"

$$\nabla\cdot\mathbf{J} + \frac{\partial\rho}{\partial t} = R ,$$

is a local conservation law for Q. That is, show that the total amount of "Q" inside of a closed volume changes because Q flows through the volume, or because it is created/destroyed inside the volume.

(c) Express each of the following complex numbers in both real/imag and polar forms

(i) $(-1-i)^2$ (ii) $\dfrac{1}{\omega^2-\omega_0^2-i\omega\Gamma}$

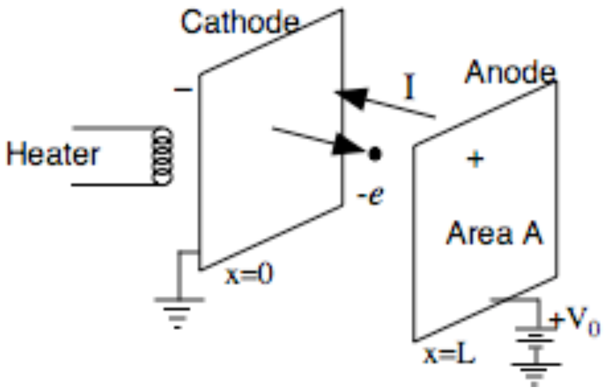
(iii) $\sqrt{-i}$ (iv) $\dfrac{e^{-i\omega t}}{\omega^2-\omega_0^2-i\omega\Gamma}$

Here ω, ω_0, Γ are real numbers.

(2) Space Charge Limited Diode (15 points)

We showed in class that if the factor limiting the velocity of the charge carriers is collisions, then the current-voltage relationship satisfies Ohms law, I=V/R. Here is an example where the "law" is violated.

Consider a vacuum diode, consisting of a parallel plate capacitor (with area A) in which electrons are "boiled" off the negative plate (cathode) which are accelerated through a potential difference V0 to the positive plate (anode) (next page).



Parallel plate capacitor,
Area A, distance L between plates

(assume $L^2 << A$, not drawn to scale)

Eventually, enough electrons will boil off the plates, that the factor limiting the current will be their mutual Coulomb repulsion ("space charge").
Our goal is to show that

$$I = K\,(V_0)^{3/2} \quad \text{The } \textbf{Child-Langmuir law}.$$

(a) Write down the Poisson differential equation relating the potential as a function of *x* to the charge density as a function of *x*.

(b) Write the relationship between the charge density of electrons as a function of *x*, the velocity of the electrons as a function of *x*, the area of the plates, and the current *I*. (In steady state the current *I* is constant across the diode)

(c) Find the relationship between the velocity of an electron as a function of *x* and the potential as a function of *x*. (Hint, using energy considerations).

(d) Put (a-c) together to obtain the differential equation for the potential,

$$\frac{d^2V}{dx^2} = C\,V^{-1/2} \quad \text{where,} \quad C = \frac{I}{\varepsilon_0 A}\sqrt{\frac{m}{2e}}\,,$$

with *m* the electron mass, *e* its charge.

(e) Show that $V(x) = \left(\frac{9C}{4}\right)^{2/3} x^{4/3}$ is a solution to (d), and use the boundary conditions to obtain the Child-Langmuir law.