

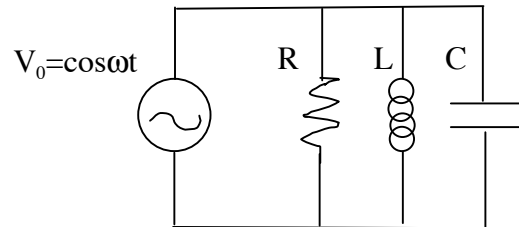
Physics 406

Problem Set #3: DUE Fri. Sept. 14, 2012

Read: Griffiths Chap. 7.3

Problem 1 (10 points)

Consider a parallel R-L-C circuit driven by an AC source



(a) Find the steady state current $I(t)$ drawn from the source (for a general ω).

(b) What is the current when the drive frequency is on resonance, $\omega = \omega_0 = 1/\sqrt{LC}$
(Explain your answer)

(2) Quality Factor of an Oscillator (15 points)

(a) Consider the two sinusoidal functions of time

$$A(t) = A_0 \cos(\omega t - \phi_A) = \text{Re}(\tilde{A}e^{-i\omega t}), \quad B(t) = B_0 \cos(\omega t - \phi_B) = \text{Re}(\tilde{B}e^{-i\omega t}),$$

where $\tilde{A} = A_0 e^{i\phi_A}$ and $\tilde{B} = B_0 e^{i\phi_B}$ are the complex amplitudes. Show that the time average of $A(t)$ and $B(t)$ over one period can be expressed in terms of the complex amplitudes as:

$$\langle A(t)B(t) \rangle \equiv \frac{1}{T} \int_0^T A(t)B(t) dt = \frac{1}{2} \text{Re}(\tilde{A}\tilde{B}^*) = \frac{1}{2} \text{Re}(\tilde{A}^* \tilde{B}),$$

where $T=2\pi/\omega$ is the period of oscillation and $*$ is the complex conjugate symbol.

(b) Consider the series RLC circuit from class.

Show that the average power dissipated by the load impedance in the near resonance approximation is

$$\langle P(t) \rangle = \langle I(t)V(t) \rangle \equiv \frac{V_0^2}{2R} \frac{\Gamma^2/4}{(\omega - \omega_0)^2 + \Gamma^2/4}, \text{ where } \omega_0 = 1/\sqrt{LC} \text{ and } \Gamma = R/L$$

Sketch this as a function of ω . What is the FWHM?

(c) The quality factor of an oscillator (used called the “Q”) is defined as

$$Q \equiv \omega_0 \frac{\text{average energy stored/cycle}}{\text{average power dissipated/cycle}}$$

$$\text{Show that } Q = \frac{\omega_0}{\Gamma}.$$

Hints: Since the Q doesn't depend on the driving frequency, pick a convenient one to calculate the average energy stored. Remember that the capacitor and inductor store equal amounts of energy (half the cycle the capacitor has more energy and half the inductor does).

(3) Griffiths, Problems 7.16 (10 points).