Physics 406

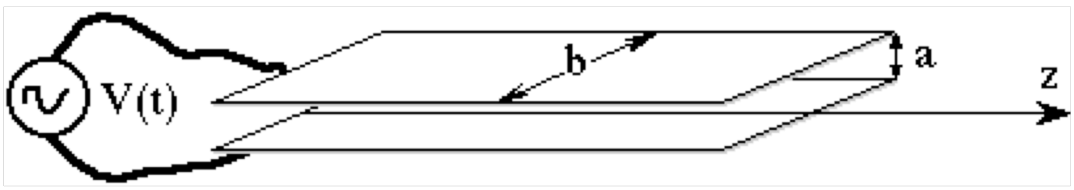
Problem Set #4: DUE Thurs. Sept. 21 2012

Read: Griffiths Chaps. 7.3 and 8.1

Problem 1: Griffiths 7.33

Problem 2: Transmission line (15 points)

Consider a long transmission line consisting of two parallel conducting strips of width b, separated by a distance a. A voltage signal $V(t)$ is to be transmitted down the line.



For small circuit elements, we know that the capacitors and inductors satisfy the voltage/current relations Capacitor: $I = -C\dfrac{dV}{dt}$ Inductor: $V = -L\dfrac{dI}{dt}$.

Because the signal does not propagate instantaneously, the voltage and current along the transmission line now vary as a function of distance.

(a) Use Maxwell's Equations to show that

$$\frac{\partial I}{\partial z} = -C\frac{\partial V}{\partial t}, \qquad \frac{\partial V}{\partial z} = -\mathcal{L}\frac{\partial I}{\partial t}$$

where C and $\mathcal{L}$ are the capacitance and inductance (respectively) *per unit length* along z.

(b) Show that the voltage and current signals obey the wave equation of the form

$$\left(\frac{\partial^2}{\partial z^2} - \mathcal{L}C\frac{\partial^2}{\partial t^2}\right)V(z,t) = 0 \qquad \left(\frac{\partial^2}{\partial z^2} - \mathcal{L}C\frac{\partial^2}{\partial t^2}\right)I(z,t) = 0$$

What is the speed of propagation of these wave compared with the speed of light in vacuum?

(c) Show that the wave equation admits monochromatic waves solutions of the form

$$V(z,t) = V_0 \cos(kz - \omega t), \quad I(z,t) = I_0 \cos(kz - \omega t), \text{ and show } V_0 = Z_c I_0$$

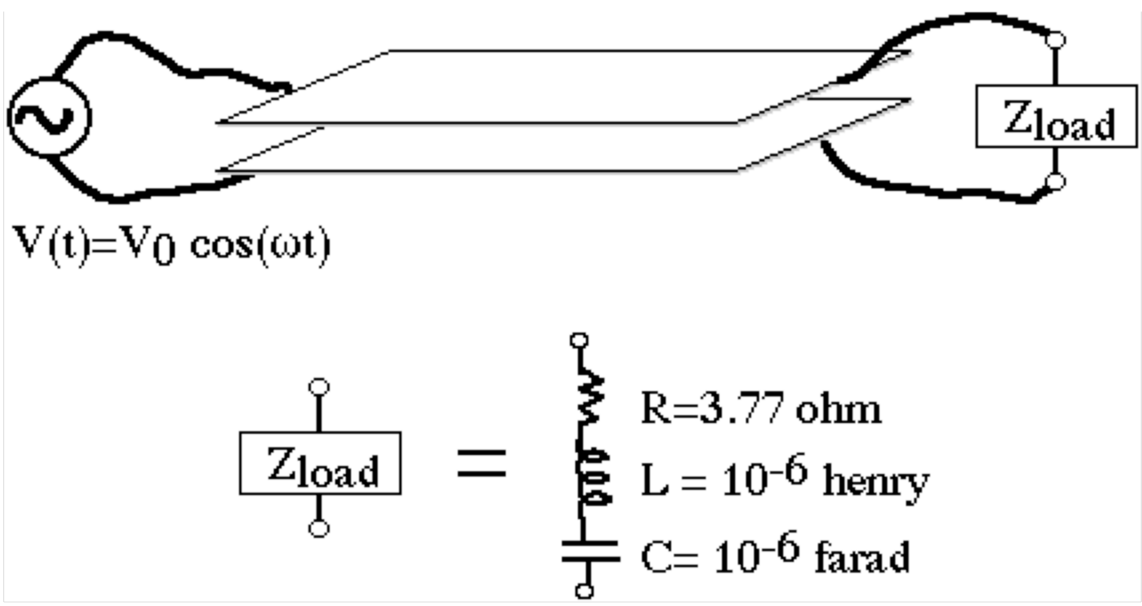
$$\text{where } Z_c = \sqrt{\frac{\mathcal{L}}{C}} = Z_0 \frac{a}{b}, \quad Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377 \text{ ohms.}$$

Z_c is known as the "characteristic impedance" of the transmission line.

(d) If a "load" circuit terminates the line, maximum power will be delivered if the impedance of the load *equals* the characteristic impedance of the line. If there is an impedance mismatch some of the signal will be reflected, with a reflection coefficient

$$r = \frac{V_{ref}}{V_{inc}} = \frac{Z_c - Z_{load}}{Z_c + Z_{load}}$$

Consider an *RLC* circuit terminating the line with the following parameters (let $b = 100a$):



Given a sinusoidal signal incident on the load, $V_{inc} \cos(\omega t)$. **At what frequency ω will there be zero reflection?**

Problem 3: Characteristics of Plane Waves (10 Points)

Consider a plane wave whose electric field is

$$\mathbf{E}(\mathbf{r},t) = \left(\hat{\mathbf{x}} + 2\hat{\mathbf{y}} + E_z\hat{\mathbf{z}}\right)\cos\left(-3x + y + z - \omega t\right),$$

in MKS-SI units. Determine:

(a) The direction of propagation
(b) The direction of polarization
(c) The wavelength
(d) The frequency (in Hz)
(e) The magnetic field