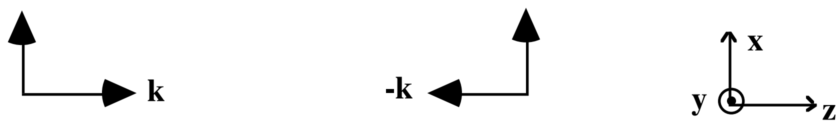
Physics 406 Electricity and Magentism
Problem Set #5: DUE Thursday Oct. 5 2012
Read: Griffiths Chap. 8, 9.1-9.2

Problem 1: Standing Wave (10 points).

Consider the superposition of two counterpropagating and plane waves with the same frequency, whose electric fields have the same polarization



$\mathbf{E}_1 = \hat{\mathbf{x}}\, E_0 \cos(kz - \omega t)$ $\qquad$ $\mathbf{E}_2 = \hat{\mathbf{x}}\, E_0 \cos(kz + \omega t)$

(a) What is the total electric field $\mathbf{E}_3 = \mathbf{E}_1 + \mathbf{E}_2$?

(b) What is the total magnetic field, $\mathbf{B}_3$?

(c) Sketch $|\mathbf{E}_3(z,t)|$ and $|\mathbf{B}_3(z,t)|$ as a function of z over one wavelength for $t = 0, T/4, T/2, 3T/4, T$, where T is the period of oscillation.

(d) What are the electric and magnetic energy densities as a function of z and t.

(e) What is the time average energy flux (intensity) - Explain you answer.

Problem 2: Spherical Waves (10 Points)

Consider the wave equation in three dimensions for a *scalar* field $\psi(\mathbf{r},t)$.

$$\nabla^2\psi - \frac{1}{\mathrm{v}^2}\frac{\partial^2\psi}{\partial t^2} = 0.$$

We seek solutions for monochromatic wave in spherical coordinates, independent of θ and ϕ - this corresponds, e.g., to waves generated by a point source.

(a) Let $\psi(\mathbf{r},t) = \tilde{\psi}(r)e^{-i\omega t}$. **Show** that $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\tilde{\psi}}{\partial r}\right) + k^2\tilde{\psi} = 0$, where $k = \omega/\mathrm{v}$.

(b) Let $\tilde{\psi}(r) = \frac{u(r)}{r}$. **Show** that $\frac{d^2u}{dr^2} + k^2 u = 0$.

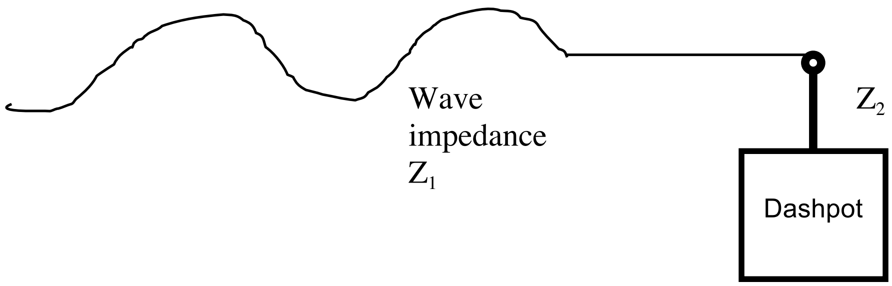
(c) **Show** that the most general solution can be written as superpositions of

$$\psi_1(\mathbf{r},t) = u_0\frac{\cos(kr - \omega t - \phi)}{r} \quad \text{and} \quad \psi_2(\mathbf{r},t) = u_0\frac{\cos(kr + \omega t - \phi)}{r}.$$

What is the physical difference between ψ_1 and ψ_2?

Problem 3: Reflection of Traveling Waves due to Impedance Mismatch (10 points)

Suppose we have a string with characteristic impedance Z_1 extending from $z = -\infty$ to z=0. At z=0 it is attached to a load consisting of the input terminal of a dashpot having impedance Z_2. At $z = -\infty$ there is a transmitter emitting a traveling wave in the +z direction.



Some fraction of the wave energy will be absorbed by the dashpot, but because of the impedance mismatch, some fraction will be reflected. The goal of this problem is to find the amplitude of the reflected wave compared to the incident wave.

Given Facts

- We showed in class that the local "drag" force on the string for a wave traveling in the +z direction is $F_+(z,t) = -Z_1 \dfrac{\partial f_+(z,t)}{\partial t}$, where Z_1 is the characteristic impedance.
- For a wave traveling in the –z direction is $F_-(z,t) = +Z_1 \dfrac{\partial f_-(z,t)}{\partial t}$.
- The wave amplitude at the dashpot is the superposition of incident and reflected waves.
- The *net force* acting on the string at z=0 is provided by the drag of the dashpot,

$$F_{total}(z,t) = -Z_2 \frac{\partial f_{total}(z,t)}{\partial t}.$$

That is the dashpot does not accelerate (it is taken to have negligible mass).

Use these facts to show that the ratio of the amplitude of the reflected and transmitted waves is

$$r \equiv \frac{f_{reflected}}{f_{incident}} = \frac{Z_1 - Z_2}{Z_1 + Z_2}$$