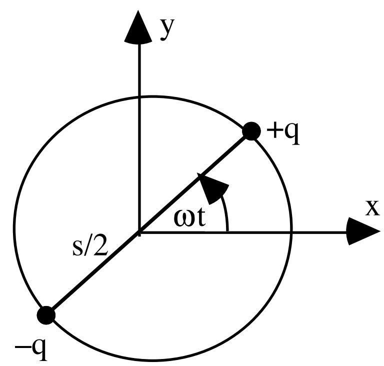
Physics 406: Electricity and Magnetism II

Problem Set #9: Due Fri. Nov 16, 2012

Problem 1. Radiation by a rotating electric dipole (15 points)

Consider an electric dipole rotating in the x-y plane



The motion of this dipole can be model by the superposition of two linearly oscillating dipoles that are 90° out of phase with one another: $\mathbf{p}(t) = p_0 \cos(\omega t)\hat{\mathbf{x}} + p_0 \sin(\omega t)\hat{\mathbf{y}}, \quad p_0 \equiv qs$.

(a) An observer is located at position (r, θ, ϕ), with respect to the origin at the center of the ring. If we write the electric field associated with the dipole radiation as $\mathbf{E}(\mathbf{r},t) = \mathrm{Re}\left[\tilde{\mathbf{E}}(\mathbf{r})e^{-i\omega t}\right]$, show that the complex amplitude in the far field is,

$$\tilde{\mathbf{E}}(r,\theta,\phi) = \frac{1}{4\pi\varepsilon_0} k^2 p_0 \left(\cos\theta\,\hat{\theta} + i\hat{\phi}\right) \frac{e^{i(kr+\phi)}}{r}$$

What is the polarization of the wave at an arbitrary point?

(b) As a check show that on the x, y, and z axes the real part of the electric fields are:

On x-axis (i.e. θ=π/2, ϕ=0) $\mathbf{E}(\mathbf{r},t) = \frac{1}{4\pi\varepsilon_0} k^2 p_0 \frac{\sin(\omega t - kx)}{x}\,\hat{\mathbf{y}}$

On y-axis (i.e. θ=π/2, ϕ=π/2) $\mathbf{E}(\mathbf{r},t) = \frac{1}{4\pi\varepsilon_0} k^2 p_0 \frac{\cos(\omega t - ky)}{y}\,\hat{\mathbf{x}}$

On z-axis (i.e. θ=0, ϕ=0) $\mathbf{E}(\mathbf{r},t) = \frac{1}{4\pi\varepsilon_0} k^2 p_0 \left(\frac{\cos(\omega t - kz)\hat{\mathbf{x}} + \sin(\omega t - kz)\hat{\mathbf{y}}}{z} \right)$

(Remember that the values of the spherical unit vectors depend on position, e. g., along the x-axis (i.e. θ=π/2, ϕ=0) $\hat{\theta} = \hat{\mathbf{z}}$, $\hat{\phi} = \hat{\mathbf{y}}$) Please explain why these are what you expect.

(c) Find the time-averaged rate at which electromagnetic energy is radiated per solid angle. Sketch the angular distribution of both the radiated power.

(d) Find the total time-averaged rate at with energy radiated to infinity. Comment on your result.

Problem 2. Death of a Classical Atom (10 points)

In the early part of the 20^{th} century, after Rutherford's discovery of the atomic nucleus, a classical model of the atom was proposed. This consisted of an electron orbiting the nucleus, much in the same way as the planets orbit the sun, only now the binding force is the Coulomb attraction rather than the gravitational force. There is a problem here. The electron-nucleus system is a rotating dipole and thus the orbiting electron should radiate electromagnetic waves that carry away energy. As the electron loses energy the radius of the orbit will decrease and eventually it will spiral into the nucleus!

Calculate the time it would take for a hydrogen atom, modeled as an electron in a circular orbit around a proton, to lose all of its energy to electric dipole radiation. Take the initial energy of the electron to be ½ the Rydberg energy (13.5 eV) and the initial radius to be the Bohr radius (0.5 Å). This paradox led Bohr to make his giant leap of faith into the quantum theory by *postulating* that the orbits are by definition "stationary states" that do not radiate.