Physics 406: Electricity and Magnetism II
Extra Credit -- Problem Set #11: Due Fri. Dec. 7, 2012

Problem 2: Radiation reaction (10 points)

Consider an electron on a spring. Even without friction, the oscillator will experience a damping force as since it is loosing energy to electromagnetic radiation ("radiation resistance").

(a) Given some initial displace x_0 and velocity v_0, derive an expression for the rate at which the oscillating electron radiates electromagnetic energy. Give the rate averaged over a period of oscillation (assume the amplitude decays very little over a period)

(b) Show that the electron can be expected to radiate its energy in at a rate

$$\Gamma_{rad} = \frac{e^2\omega_0^2}{6\pi\varepsilon_0 mc^3}.$$

This is the classical picture of "spontaneous emission" for a quantum mechanical oscillator. Estimate numerically the "radiative lifetime" found for the case of an electron oscillating a typical optical frequency (i.e. a frequency of visible light).

(c) The back reaction force on a point charge due to the radiation it emits leads to unphysical **noncausal** solutions to the equation of motion. In particular, show that in the absence of any binding force, the reaction force on an accelerating the equation of motion for a charge undergoing acceleration is,

$$m\ddot{\mathbf{x}} = \tau m\dddot{\mathbf{x}} + \mathbf{F}_{external}\text{, where } \tau = \frac{1}{6\pi\varepsilon_0}\frac{e^2}{mc^3},$$

with run away solution even when $\mathbf{F}_{external} = 0$:

$$\mathbf{x}(t) = \mathbf{x}(0) + (\dot{\mathbf{x}}(0) - \tau\ddot{\mathbf{x}}(0))t - \tau^2\ddot{\mathbf{x}}(0)(1 - \exp(t/\tau)).$$

This is known as the Abraham-Lorentz equation (see Griffiths).

As stated in "Classical Field Theory", by F. E. Low,
"The contradiction between energy conservation and causality is a genuine difficulty of classical electromagnetic theory describing the interaction of electromagnetic fields with *point* particles. This problem (the unwanted pole) does not appear in relativistic quantum electrodynamics; however other problems do [i.e. divergence of Feynman diagrams]"

Problem 2: Rayleigh scattering from a dielectric sphere (10 points)
The original analysis of Lord Rayleigh regarding scattering of waves in the atmosphere assumed the particles to be small dielectric sphere (Rayleigh precedes Lorentz and his oscillator model). Give a dielectric sphere of radius $a \ll \lambda$, and index of refraction *n*, show that its scattering cross section that it presents to unpolarized light is given by

$$\frac{d\sigma}{d\Omega} = k^4 a^6 \left(\frac{n^2 - 1}{n^2 + 2} \right) \left(\frac{1 + \cos^2 \theta}{2} \right).$$

Note, when the radius of the sphere starts to become large, this formula no longer holds. In that case, we need to keep many more multipole moments. The general solution is known as *Mie scattering*, which has properties quite different from Rayleigh scattering. Mie scattering explain why clouds (with large droplets of water) look white, while the sky is blue.