

Physics 491: Quantum Mechanics I

Problem Set #9

Due: Wed, Dec. 10, 2003

Problem 1: Liboff 10.37 (Page 460)

Problem 2: Liboff 10.38 (Page 460-61)

Problem 3: s-wave scattering from a repulsive shell

Consider a “paper lantern” potential $V(r) = U_0\delta(r - a)$, i.e. a shell of radius a , and strength U_0 . This is a crude model for a nucleus with binds protons and neutrons.

(a) Calculate the s-wave scattering phase shift (Hint: Look back at Exam II)

$$\text{Now let } \gamma_0 = \frac{2m}{\hbar^2}U_0. \quad \text{Assume } \gamma_0 \gg \frac{1}{a}, k$$

(b) Show that if $\tan(ka)$ is *not* close to zero, the phase shift resembles that of a hard sphere.

(c) Show that if $\tan(ka)$ is close (but not equal to) zero, resonance is possible (i.e., the cross section reaches its maximum value $\sigma_0 = 4\pi\lambda^2$). Compare the resonance energy with that of a bound state of the spherical shell with an infinitely impenetrable wall.

This is an example of a “scattering resonance”, when the incident energy matches a quasi bound-state.