Lecture 5a: Supplement

The Uncertainty Principle and Complimentarity

The incompatibility of x and p fundamentally changes our notions about physics. Our new ways of thinking about the quantum mechanics are due in great part to Niels Bohr of Copenhagen. His notions are sometimes hard to pin down and semi-philosophical in style. Nonetheless they have great impact and should be examined.

Einstein was famously unhappy with the new ideas. In his view, the randomness observed in quantum experiments must be due to unknown information ("hidden variables") whose value actually determine the paths of particles. We assign probabilities because we don't have access to these variables. In a series of epoch-making debates, Einstein and Bohr argued about this through "though experiments" known by its ~~germ~~ German name, "gedanken" experiments. In these debates, the relationship between the uncertainty principle and wave-particle duality were brought ~~to~~ the fore. The larger principle was dubbed by Bohr complimentarity.

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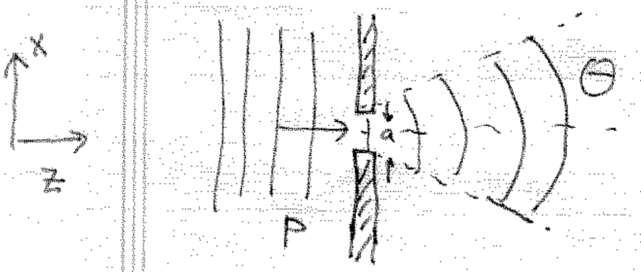
Preliminary: Diffraction and the Uncertainty Principle

Consider a plane wave incident on a slit, width a



The wave will diffract at an angle

$$\theta \sim \frac{\lambda}{a}$$

(basic wave theory)

This is consistent with the uncertainty principle. Immediately after passing through the slit the particle is localized to a width $\Delta x \sim a$. According to the uncertainty principle, the particle has an uncertainty in moment $\Delta p_x \sim \frac{\hbar}{a}$. The resulting wave packet will thus spread in the x-direction. The angular spread $\Theta \sim \frac{\Delta p_x}{p} \approx \frac{\hbar/a}{p} = \frac{\lambda}{a}$

The uncertainty principle is thus compatible with diffraction

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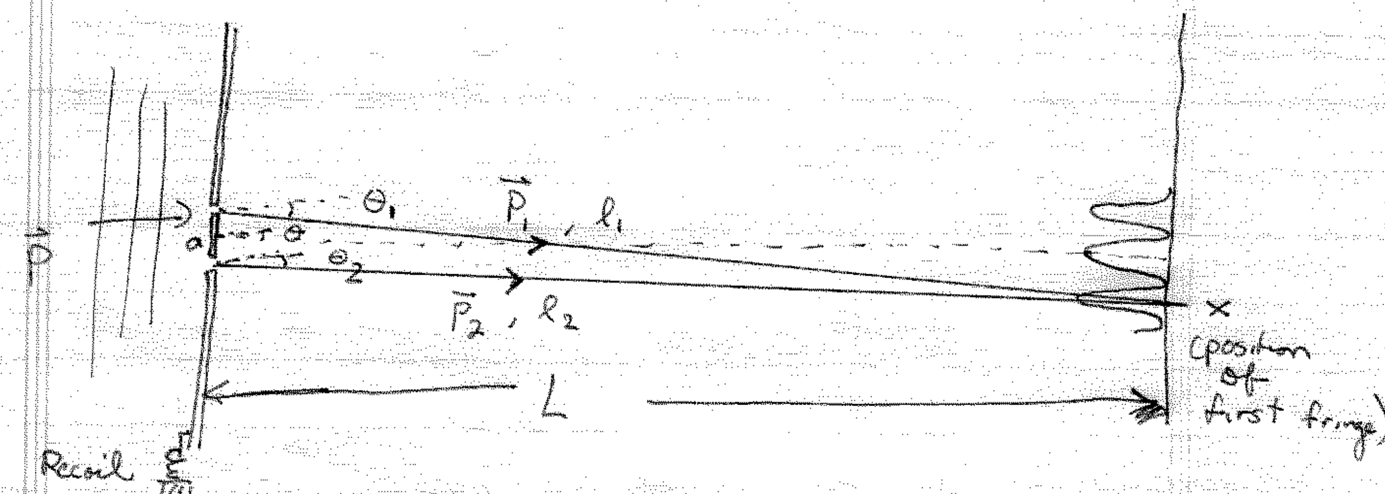
Einstein's Gedanken Experiment

Consider again the double slit experiment.



In the "far field" $L >> a$ $\quad \theta_1 \approx \theta_2 \approx \theta = \frac{x}{L}$

Constructive interference for first fringe

$$l_1 - l_2 = \lambda \Rightarrow a \sin\theta \approx a\theta = \lambda$$

$$\Rightarrow \theta = \frac{\lambda}{a} = \frac{x}{L} \Rightarrow x = L\frac{\lambda}{a}$$

Now, according to Einstein we should be able to measure through which slit the particle travelled to get to x since the momenta $\vec{p}_1$ and $\vec{p}_2$ differ. Since momentum is conserved the difference between $\vec{p}_1$ and $\vec{p}_2$ should be measurable through the recoil of the diffraction gratting. Thus, in principle one should be able to measure both the position x and which slit.

Bohr's response

In Bohr's view one must look at the *whole apparatus*. If we measure the deflection of the particle through its recoil, we can only do so with a finite precision.

The momentum difference is in the x direction

$$p_{1x} - p_{2x} = p \sin\theta_1 - p\sin\theta_2 \cong p(\theta_1 - \theta_2)$$

$$\theta_1 \approx \frac{x + a/2}{L} \qquad \theta_2 \approx \frac{x - a/2}{L}$$

$$\Rightarrow p_{1x} - p_{2x} \approx p\,\frac{a}{L} = \frac{h a}{\lambda L} = \frac{h}{x}$$

Thus the uncertainty of the measurement of p_x must be much smaller than this value

$$\Delta p_x \ll \frac{ha}{\lambda L}$$

In addition we must measure the position of the particle with a resolution better than a fringe:

$$\Delta x \ll \frac{x}{2} = \frac{\lambda L}{a 2}$$

$$\Rightarrow \Delta p_x \, \Delta x \ll \frac{h}{2} \Rightarrow$$

Violates the uncertainty principle $\Delta p_x \Delta x > \frac{h}{4\pi}$

The measuring apparatus itself must be included in thinking about the physics.

Bohr's argument highlights an important fact. If we try to determine which path the particle took we wash out the interference fringes. Bohr dubbed this "complementarity" — the wave and particle nature are <u>complementary</u> — no experiment can reveal both. Measurements are no longer <u>passive observations</u> but must be included in a complete description of the system.

The role of measurement in quantum mechanics is central and the most confusing aspect of the theory. What constitutes a measurement? How is it different from other interactions? these are questions we will continue to grapple with over the ~~course~~ course of the year.