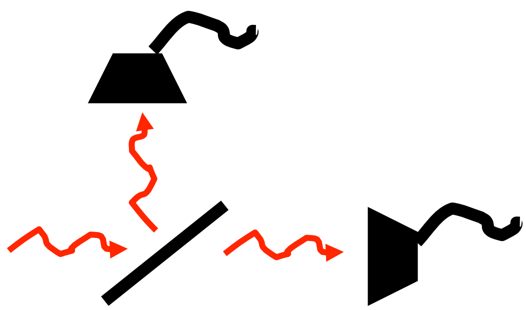
Physics 491: Quantum Mechanics I

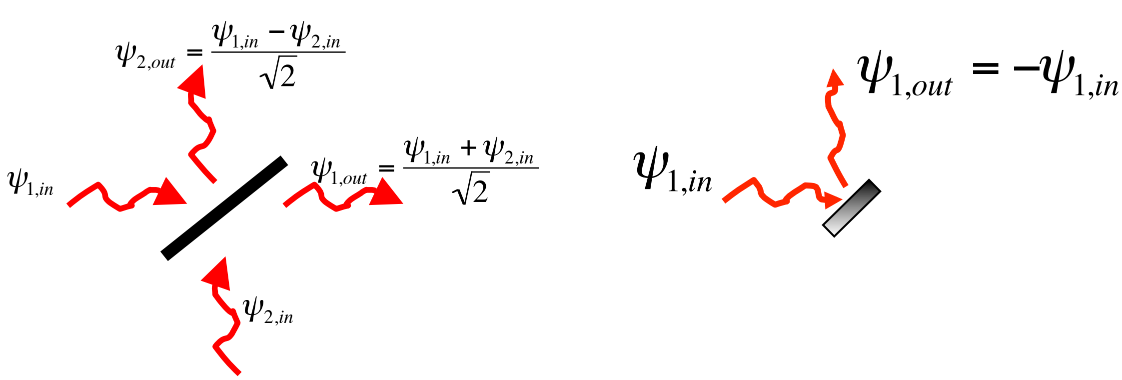
Problem Set #1
Due: Wednesday, August 29, 2018

Problem 1: The Mach-Zender Interferometer and Quantum Interference (15 points)

Background information:
Consider a single photon incident on a 50-50 beam-splitter (i.e., a partially transmitting, partially reflecting mirror, with equal coefficients). Whereas classical electromagnetic energy divides equally, the photon is *indivisible*. That is, if a photon-counting detector is placed at each of the output ports, only **one** of them fires. Which ones fires is completely random (i.e. we have no better guess for one over the other).



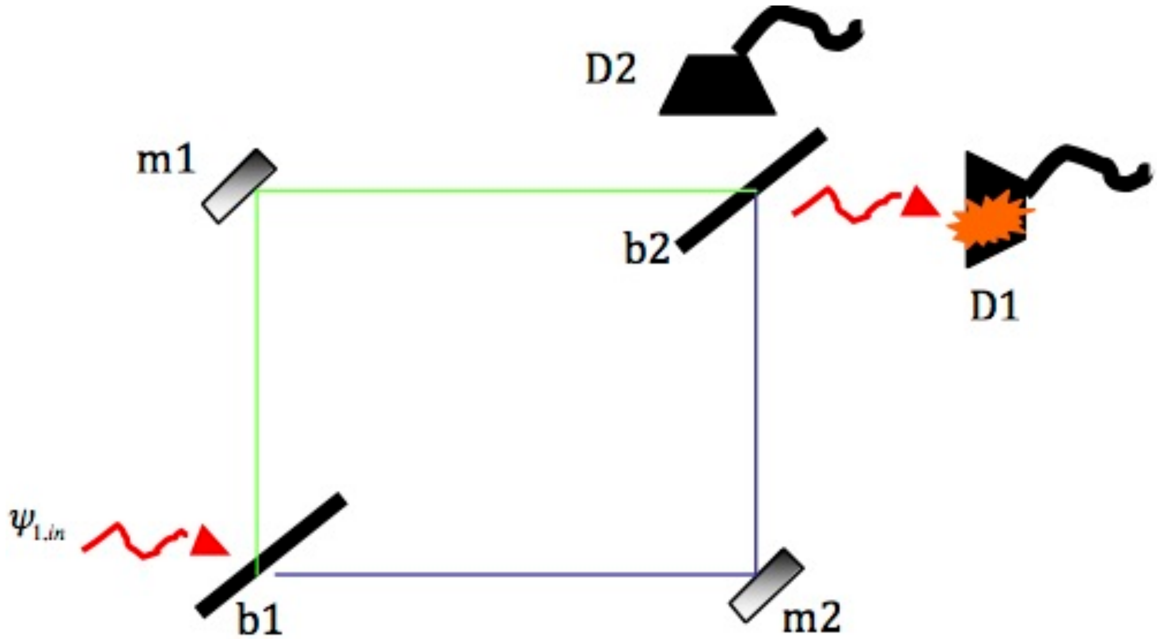
The input-output transformation of the waves incident on 50-50 beam-splitters and perfectly reflecting mirrors are shown below.



(a) Show that with these rules, there is a 50-50 chance of either of the detectors sketched above to fire.

(b) Next Page

(b) Now let us set up an interferometer known as the Mach-Zender:



The wave is split at beam-splitter b1, where it travels either the green path or blue path. Mirrors are then used to recombine the beams on a second beam-splitter, b2. Detectors D1 and D2 are placed at the two output ports.

Assuming the paths are perfectly balanced (i.e. equal length) show that the probability for detector D1 to fire is 100% - **no randomness!**

(c) Classical logical reasoning would predict a probability for D1 to fire,

$$P_{D1} = P(\text{transmission at b2} \mid \text{green path})\, P(\text{green path}) + P(\text{reflection at b2} \mid \text{blue path})\, P(\text{blue path})$$

Calculate this and compare to the quantum result. *Explain*.

(d) **Extra credit:** How would you set up the interferometer so that detector D2 fired with 100% probability? How about making them fire completely at random? Leave the *basic geometry the same*, i.e. do not change the direction of beam splitters or the direction of the incident light. (5 points)

Problem 2: Bayes rules with Gaussians. (15 points)

Let's consider a *classical* problem (no quantum uncertainty). Suppose we're trying to measure the position of a particle and we assign a prior probability distribution, $p(x) = \frac{1}{\sqrt{2\pi\sigma_0^2}} \exp\left[-(x - x_0)^2 / 2\sigma_0^2\right]$. Our measuring device is not perfect. Due to noise is can only measure with a resolution Δ. That is, when I measure the position, I must put error bars on this. Thus, if my detector registers the position as y, I assign likelihood that the position was x to a Gaussian, $p(y \mid x) = \frac{1}{\sqrt{2\pi\Delta^2}} \exp\left[-(y - x)^2 / 2\Delta^2\right]$.

Use Bayes theorem to show that, given the new data, I must now update my probability assignment of the position to a new Gaussian,

$$p(x \mid y) = \frac{1}{\sqrt{2\pi\sigma'^2}} \exp\left[-(x - x')^2 / 2\sigma'^2\right],$$

Where $x' = x_0 + K_1(y - x_0)$, $\sigma'^2 = K_2\,\sigma_0^2$, with $K_1 = \frac{\sigma_0^2}{\sigma_0^2 + \Delta^2}$ $\quad K_2 = \frac{\Delta^2}{\sigma_0^2 + \Delta^2}$.

Comment on the behavior as the measurement resolution improves.