Physics 491: Quantum Mechanics I
Problem Set #3
Due: Wednesday, September 19, 2018

Problem 1: Square wave packet (20 points)

Consider a free particle. initially with a well defined momentum p_0, whose wave function is well approximated by a plane wave. At t=0, the particle is the *localized* in a region $-\frac{a}{2} \le x \le \frac{a}{2}$, so that is wave function is

$$\psi(x) = \begin{cases} Ae^{ip_0 x/\hbar}, & -\frac{a}{2} \le x \le \frac{a}{2} \\ 0, & \text{otherwise} \end{cases}.$$

(a) Find the normalization constant A and sketch $\mathrm{Re}(\psi(x))$, $\mathrm{Im}(\psi(x))$, and $|\psi(x)|^2$.
(b) Find the momentum space wave function $\tilde{\phi}(p)$ and show that it too is normalized.

$$\text{Hint: Use } \int_0^\infty \frac{\sin p}{p} dp = \frac{\pi}{2}.$$

(c) *Estimate* the uncertainties Δx and Δp at this time. How close is this to a minimum uncertainty wave packet? If you calculate the standard deviations exactly you will find $\Delta p \to \infty$. Why do you think this is the case?
(d) Estimate the spreading of the wave packet's rms as a function of time, $\Delta x(t)$.

Problem 2: More examples in momentum space (30 points)

Consider the following wave function $\psi(x) = Axe^{-x^2/4\sigma^2}$.
(a) Find the normalization constant A. Sketch $\psi(x)$ and $|\psi(x)|^2$
(b) Find the momentum space wave function and sketch $|\tilde{\phi}(p)|^2$
(c) Find Δx and Δp. Does this satisfy the uncertainty principle. Is it a minimum uncertainty packet?

Now consider the following wave function $\psi(x) = A\left(e^{-(x-a/2)^2/4\sigma^2} + e^{-(x+a/2)^2/4\sigma^2}\right)$.
(d) Repeat parts (a)-(c). To do the sketches consider two cases: $a = \sigma$ and $a = 4\sigma$.