Physics 491: Quantum Mechanics I
Problem Set #7
Due: Wed, Nov. 7, 2018

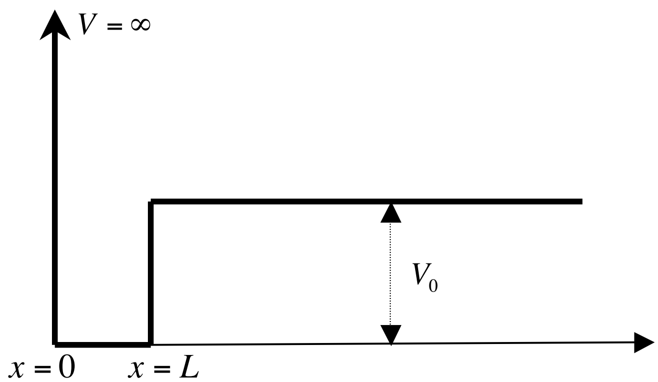
Problem 1: Finite Square Well Encore (15 points)

Consider the symmetric finite square well of depth V_0 and width a as discussed in class.

(a) Let $k_0 = \sqrt{\frac{2mV_0}{\hbar^2}}$. Sketch the bound states for the following choice of $k_0 a/2$.

$$\text{(i)}\ \frac{k_0 a}{2} = 1,\ \text{(ii)}\ \frac{k_0 a}{2} = 1.6,\ \text{(iii)}\ \frac{k_0 a}{2} = 5$$

(b) Consider now the half-infinite, half-finite potential well



Without doing any calculation, show that there are no bound states unless $k_0 L > \pi/2$.

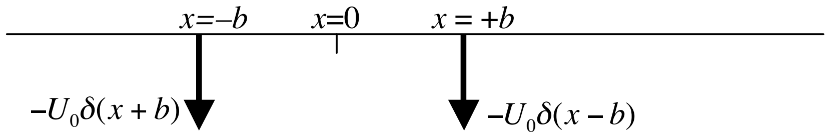
(Hint, think about erecting an infinite wall down the center of a symmetric finite well of width a=2L. Also, think about parity)

(c) Show that in general, the binding energy eigenvalues satisfy the eigenvalue equation

$$\kappa = -k\cot(kL),\ \text{where}\ \kappa = \sqrt{\frac{2mE_b}{\hbar^2}}\ \text{and}\ k^2 + \kappa^2 = k_0^2.$$

Problem 2: Double Delta Function Well (30 Points)

Consider a potential corresponding to two delta function wells



Because the potential is invariant under a parity reflection, the energy eigenfunctions are eigenfunctions of parity. Thus, we can look for even/odd parity solutions to the T.I.S.E.

(a) For even parity, we can make the Ansatz (or "assumed form of solution"),

$$u_{even}(x) = \begin{cases} Be^{+\kappa x} & x < -b \\ A\left(e^{\kappa x} + e^{-\kappa x}\right) & -b < x < b \\ Be^{-\kappa x} & x > b \end{cases}, \text{ where } \kappa = \sqrt{\frac{2mE_b}{\hbar^2}} \text{ with } E_b = -E$$

Justify this.

(b) Use the boundary conditions to that binding energy follows from the transcendental equation,

$$\text{Even parity: } \frac{\kappa}{\kappa_0} = 1 + \exp(-2\kappa b), \text{ where } \kappa_0 = \frac{mU_0}{\hbar^2}.$$

(c) For odd parity, justify the Ansatz,

$$u_{odd}(x) = \begin{cases} -Be^{+\kappa x} & x < -b \\ A\left(e^{\kappa x} - e^{-\kappa x}\right) & -b < x < b \\ Be^{-\kappa x} & x > b \end{cases}, \text{ where } \kappa = \sqrt{\frac{2mE_b}{\hbar^2}} \text{ with } E_b = -E.$$

(d) Now show that the binding energy follows from the equation

$$\text{Odd parity: } \frac{\kappa}{\kappa_0} = 1 - \exp(-2\kappa b), \text{ where } \kappa_0 = \frac{mU_0}{\hbar^2}.$$

(e) As $b \to 0$, the solution goes to the expected case of a single delta function potential. Explain the limit $b \to \infty$.

(f) In general show a graph which represents the numerical solution to these equations by plotting both the left hand side and right hand side as a function of κ/κ_0. Sketch the possible eigenfunctions.

(g) Show that there is only one solution if $2\kappa_0 b < 1$. Is it even or odd parity? Explain.