Physics 491: Quantum Mechanics I

Problem Set #9
Due: Wed, Nov. 21, 2018

Problem 1: Physical examples of harmonic oscillators. (15 Points)
No potential is harmonic for arbitrarily large displacements from the origin. Eventually nonlinearities set in. But, as long as the displacement is small, so that the lowest order quadratic term dominates, we can treat the potential as harmonic. However, there is zero point motion of the quantum state. If the extent of the ground state is large compared to the range over which the potential is quadratic, the spectrum in no way look like that of an SHO.

Consider the following potentials. Find the characteristic oscillation frequency near an equilibrium. Determine the extent of the ground state for the corresponding SHO. By comparing this to the characteristic scale over which the potential is harmonic, estimate the number of levels for which the energy spectrum looks harmonic (i.e. equally space). Sketch the energy level diagram on top of the potential for some choice of the parameters.

(a) A periodic potential: $V(x) = V_0 \sin^2(kx)$.

(b) A Leonard-Jones potential binding a diatomic molecule: $V(r) = \dfrac{C_{12}}{r^{12}} - \dfrac{C_6}{r^6}$.

(c) The effective potential for radial motion of the electron in hydrogen in a *p*-state:

$$V(r) = \frac{\hbar^2}{mr^2} - \frac{e^2}{r}.$$

Problem 2: 15 points

7.16 A large dielectric cube with edge length L is uniformly charged throughout its volume so that it carries a total charge Q. It fills the space between condenser plates, which have a potential difference Φ_0 across them. An electron is free to move in a small canal drilled in the dielectric normal to the plates (Fig. 7.12).

The Hamiltonian for the electron is

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{\mathrm{K}x^2}{2} + \frac{e\Phi_0}{L}x$$

(a) What is the spring constant K in terms of the total charge Q?

(b) What are the eigenenergies and eigenfunctions of $\hat{H}$? [*Hint:* Rewrite the potential energy of the electron as

$$V = \frac{\mathrm{K}}{2}(x^2 + 2\gamma x) = \frac{\mathrm{K}}{2}[(x+\gamma)^2 - \gamma^2]$$

$$\gamma \equiv \frac{e\Phi_0}{L\mathrm{K}}$$

then change variables to $z \equiv x + \gamma$. To evaluate K, use Gauss's law (neglecting "edge effects").]

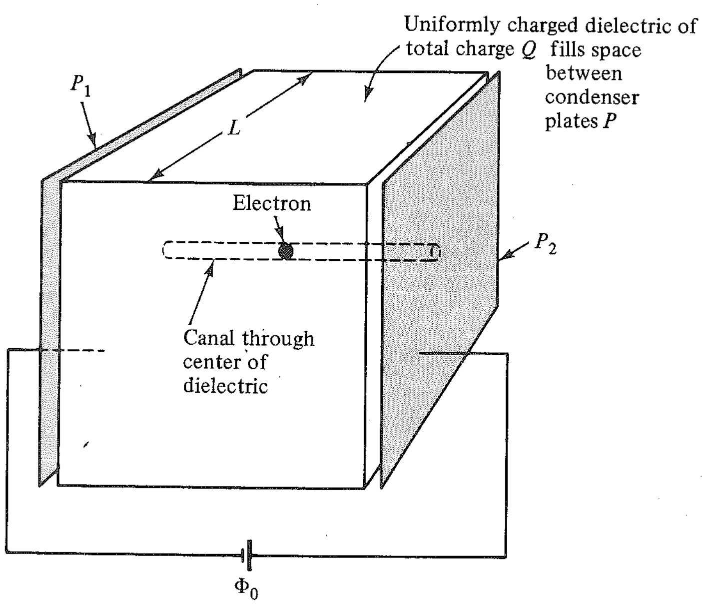

FIGURE 7.12 **Configuration described in Problem 7.16.**

Problem 3: More exercises with the SHO (10 points)

(a) The n^{th} excited state the simiple harmonic oscillator is defined in terms of the ground state by $|n\rangle = \frac{1}{\sqrt{n!}}\left(\hat{a}^{\dagger}\right)^{n}|0\rangle$, If $|0\rangle$ is normalized, use operator algebra to show that $|n\rangle$ is normalized.