Physics 492, Spring 2004
Quantum Mechanics II

March 10, 2004

Midterm Exam

Name...

UNM I.D. #.................................

The exam consists of two problem each worth 50 points. Partial credit will be given, so please show all work.

The exam is OPEN NOTES (including problem sets and solutions) but CLOSED BOOK.

Problem 1: A wave packet of the harmonic oscillator.

Consider a one dimensional simple harmonic oscillator with mass m and frequency ω. At t=0 the system in prepared in a superposition of the ground and second excited states.

$$|\psi(0)\rangle = \sqrt{\frac{2}{3}}\,|0\rangle + \frac{i}{\sqrt{3}}\,|2\rangle,$$

where $|n\rangle$ is a "number state" of the harmonic oscillator.

(a) What is the average number of excitation in the system, and the uncertainty in n?
(b) How do these values change with time?
(c) What is the average position and momentum of the particle as a function of time?
(d) What are the uncertainties in position and momentum as a function of time? Is this a minimum uncertainty wave packet?
(e) Sketch a cartoon of the position probability density as a function of time.

Problem 2: A spin-3/2 particle.

Consider a particle with spin-angular momentum j=3/2. There are four sublevels with this value of j but different eigenvalues of $\hat{j}_z$, $|m=3/2\rangle, |m=1/2\rangle, |m=-1/2\rangle, |m=-3/2\rangle$. The "raising operator" in this 4-dimensional subspace is,

$$\hat{j}_+ = \sqrt{3}\left|\frac{3}{2}\right\rangle\left\langle\frac{1}{2}\right| + 2\left|\frac{1}{2}\right\rangle\left\langle-\frac{1}{2}\right| + \sqrt{3}\left|-\frac{1}{2}\right\rangle\left\langle-\frac{3}{2}\right|,$$

where the states are labeled by the m quantum number.

(a) What is the lowering operator $\hat{j}_-$?

(b) Find a matrix representation of $\hat{j}_x, \hat{j}_y, \hat{j}_z$ in the standard basis.

(c) Check that that the state $|\psi\rangle = \frac{1}{2\sqrt{2}}\left(\left|\frac{3}{2}\right\rangle + \sqrt{3}\left|\frac{1}{2}\right\rangle + \sqrt{3}\left|-\frac{1}{2}\right\rangle + \left|-\frac{3}{2}\right\rangle\right)$ is a normalized eigenstate of $\hat{j}_x$ with eigenvalue 3/2.

(d) Suppose this particle describes the nucleus of an atom, which has a magnetic moment described by the operator $\hat{\vec{\mu}} = g_N \mu_N \hat{\vec{j}}$, where g_N is the "g-factor" and μ_N is the so-called "nuclear magneton". At time t=0, the system is prepared in the state given in (c). A magnetic field, pointing in the y-direction of magnitude B, is suddenly turned on.

What is the evolution of $\langle \hat{j}_z \rangle$ as a function of time?