Problem 1: (From P.S.#6) (15 Points)

Problem 2: Addition of spin and orbital angular momentum (15 Points)

Consider an electron with orbital angular momentum quantum number $l = 1$ and spin quantum number $s = 1/2$. The total angular momentum operator, is $\hat{\vec{j}} = \hat{\vec{l}} + \hat{\vec{s}}$.

(a) Find the simultaneous eigenvectors of $\hat{\mathbf{j}}^2, \hat{j}_z, \hat{\mathbf{s}}^2, \hat{\mathbf{l}}^2$ (i.e. direct diagonalization of $\hat{\mathbf{j}}^2$).
Hint: Order your basis so that your matrices are block diagonal.

(b) Find the matrix elements of $\hat{\mathbf{l}} \cdot \hat{\mathbf{s}}$ in the coupled basis.

Hint: Consider $\hat{\vec{j}}^2 = \left| \hat{\vec{l}} + \hat{\vec{s}} \right|^2$.