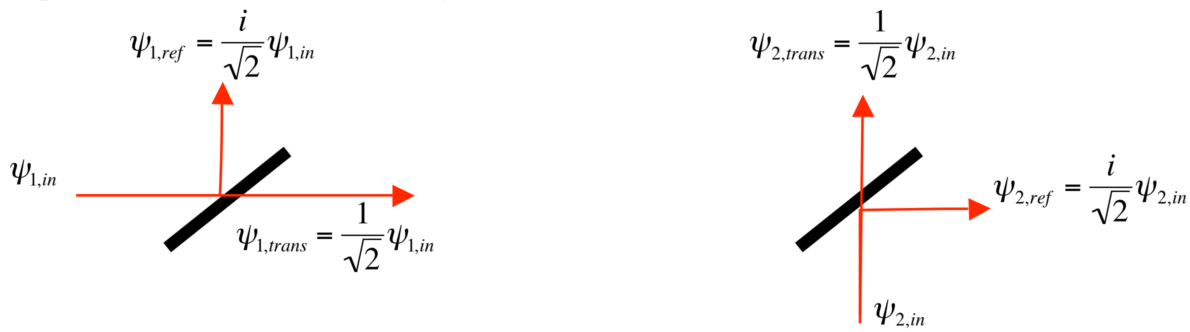
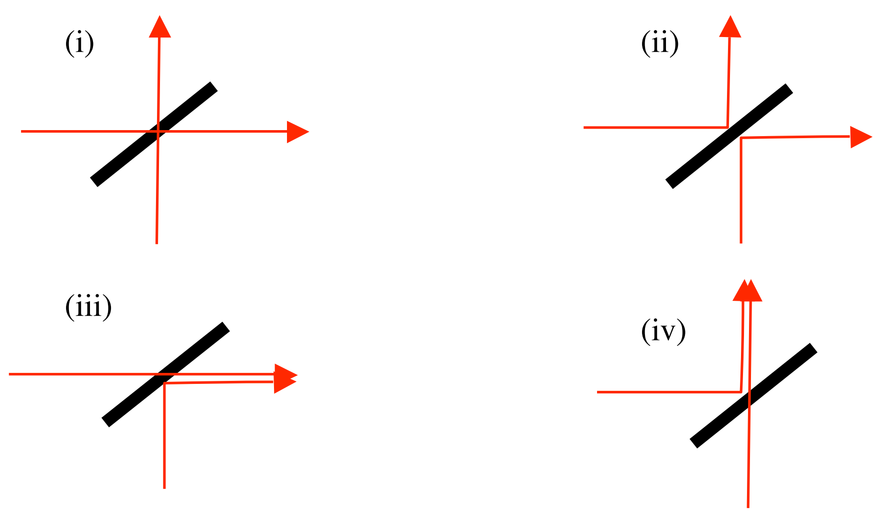
Physics 492: Quantum Mechanics II
Problem Set #8
Due: Thursday, April 15, 2004

Problem 1: Scattering of Identical Particles (15 points)
Collisions between *identical particles* act differently than collisions between distinguishable particles due to interference between indistinguishable processes.

(a) In 491 P.S #1 last semester, we considered single particle interference with 50-50 beam-splitters. Drawn here is a more symmetric version:

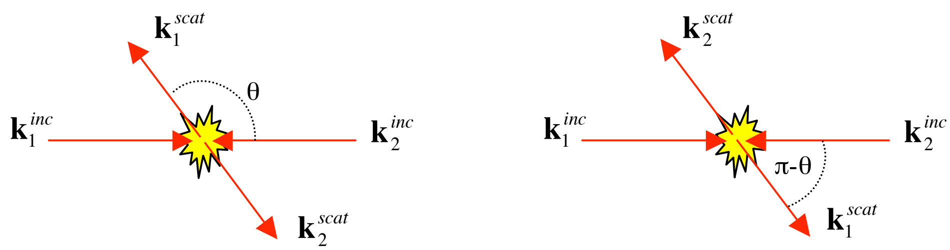


Consider two noninteracting *identical* particles (e.g. photons or neutrinos), one incident in each port of the beam-splitter. There are four possible processes: (i) both transmitted, (ii) both reflected, (iii) port-1 transmitted port-2 reflected, (iv) port-1 reflected port-2 transmitted.



If the two particles are *identical*, then processes (i) and (ii) are *indistinguishable*.
Show: If the particles are bosons (and in the same spin state) these processes *destructively interfere* and thus the particles never go off to different ports but instead go off to the same port. Show the reverse is true for fermions in the same spin state– they *always* go off to separate ports, never together into the same.

Consider now two *identical* particles which interact and scatter directly from one another. For any scattering event, there are two indistinguishable processes (show here in the center of mass frame).



Let $\mathbf{k}$ be the relative momentum $\mathbf{k}=\mathbf{k}_1-\mathbf{k}_2$ wave vector and θ be the angle between the incident and scattered relative $\mathbf{k}$. The differential scattering cross-section is thus the interference of the two scattering amplitudes $f(\mathbf{k})$,

$$\frac{d\sigma}{d\Omega} = \left| f(\mathbf{k}^{inc} \to \mathbf{k}^{scat}) \pm f(\mathbf{k}^{inc} \to -\mathbf{k}^{scat}) \right|^2$$

depending upon whether the spatial wave function is symmetric or antisymmetric under exchange of the particles.

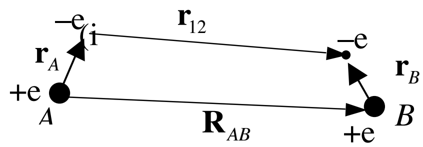
(b) For the case of spherical symmetry $f(\mathbf{k}) = f(k,\theta)$. Show

$$\frac{d\sigma}{d\Omega} = \left| f(k,\theta) \right|^2 + \left| f(k,\pi-\theta) \right|^2 \pm 2\,\mathrm{Re}\Big(f(k,\theta) f(k,\pi-\theta)^* \Big).$$

(c) Apply this to the case of Coulomb scattering of two electrons where the Rutherford scattering amplitude is $f(k,\theta) = \dfrac{me^2}{4(\hbar k)^2}\dfrac{1}{\sin^2(\theta/2)}$. Find the scattering differential cross section if the two electrons' spin state is triplet and when it is singlet. Sketch this as function of 0<θ <180 and compare with the classical Rutherford solution. Comment on the result at θ=90.

Problem 2: Covalent bonds – Diatomic Hydrogen (15 points)

Consider the simplest neutral molecule, diatomic hydrogen H_2, consisting of two electrons and two protons.



(a) Classically, where would you put the electrons so that the nuclei are *attracted* to one another in a bonding configuration? What configuration maximally repels the nuclei (anti-bonding)?

(b) Consider the two-electron state of this molecule. When the nuclei are far enough apart, we can construct this state out of "atomic orbitals" and spins. Write the two possible states as product of oribital and spin states. Which is the "bonding configuration"? Which is the "anti-bonding"?

(c) Sketch the potential energy seen by the nuclei as a function of the internuclear separation R for the two different electron configurations. Your "bonding configuration" should allow for bound-states of the nuclei to one another. This is a covalent bond.