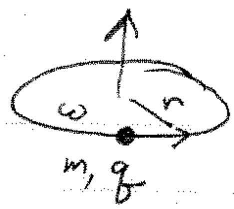
Physics 492 - Quantum II

Lecture 11: Magnetic Coupling and the Zeeman Effect

Magnetic moment of electrons

Because electrons move the produce currents. For example, consider the simplest classical circular orbit

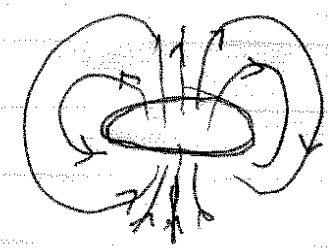


Current $I = \frac{q}{T} = \frac{\omega}{2\pi} q$

The angular momentum $L = m\omega r^2$

$$\Rightarrow \quad I = \frac{q}{2\pi r^2 m} L$$

Associated with this current loop is a magnetic field



This is ~~the~~ field is of the dipole form

A magnetic dipole moment for a loop of area A is $\vec{\mu} = \underbrace{\left[\frac{\mu_0}{4\pi}\right]}_{=1 \text{ in cgs}} \frac{I}{c} A \, \vec{n}$ ← normal

$$\begin{pmatrix} \text{For the charge on trajectory} \\ \text{in a loop} \quad A = \pi r^2 \end{pmatrix} \Rightarrow \vec{\mu} = \frac{q}{2mc} L \, \vec{n}$$

$$\Rightarrow \boxed{\vec{\mu} = \frac{q}{2mc}\vec{L} = \gamma \vec{L}}$$

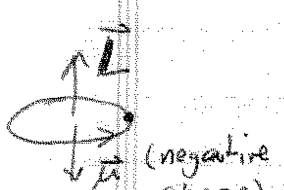
(cgs units)

γ = "gyromagnetic ratio"

General expression

Quantum description for electron

$$\hat{\vec{\mu}} = -\frac{e}{2m}\hat{\vec{L}} = -\frac{e\hbar}{2mc}\hat{\vec{l}}$$



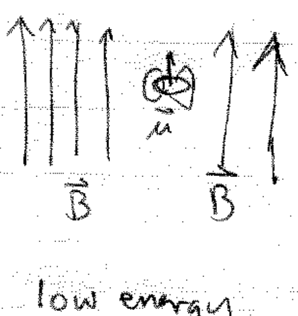
$$\hat{\vec{\mu}} = -\mu_B \hat{\vec{l}} \quad , \quad \mu_B = \frac{e\hbar}{2mc}$$

$$\mu_B = 0.93 \times 10^{-20} \ \frac{\text{ergs}}{\text{Gauss}} = \text{Bohr magneton}$$

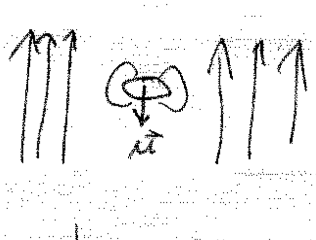
Interaction with external $\vec{B}$-field

Because electrons with orbital angular momentum produce currents, they will interact with externally applied magnetic fields. The interaction energy is

$$H_{int} = -\vec{\mu} \cdot \vec{B} \qquad \left(\begin{array}{c}\text{Try to align} \\ \vec{\mu} \text{ with } \vec{B}\end{array}\right)$$



low energy



high energy

Zeeman effect: Splitting of spectral lines in an external magnetic field

$$\hat{H} = \hat{H}_0 + \underbrace{\hat{H}_{int}}_{\text{"perturbation"}}$$

Hydrogen $\quad \hat{H}_0 = \frac{\hat{P}_r^2}{2m} + \frac{\hat{L}^2}{2mr^2} - \frac{e^2}{r} = \sum_{n,l,m}' E_n \, |n l m_l\rangle\langle n l m_l|$

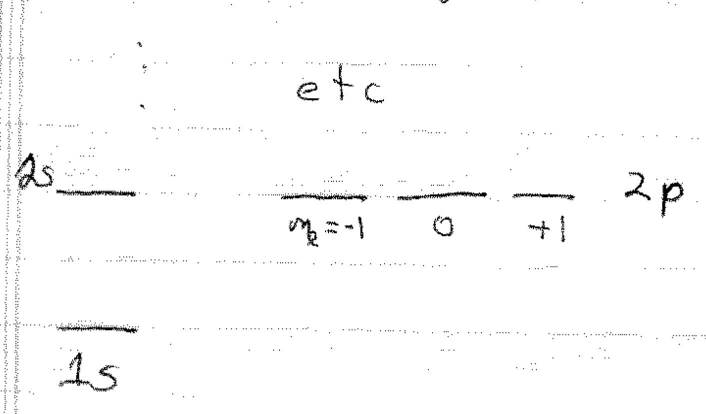
(for bound part of spectrum $\quad E_n = \frac{-R}{n^2}$)

$$\hat{H}_{int} = +\frac{\mu_B}{\hbar}\,\hat{\vec{L}}\cdot\vec{B}$$

Take $\vec{B} = B\,\vec{e}_z$ (choose z-axis along $\vec{B}$)

$$\Rightarrow \hat{H}_{int} = +\left(\frac{\mu_B B_z}{\hbar}\right)\hat{L}_z$$

Without $\vec{B}$-field



(Degeneracy for $2l+1$ sub-states for any l)
essential; spherical symmetry

(Recall: For hydrogen, also degeneracy for different l with same principle quantum number ('accidental'))

Now, adding $\vec{B}$ - field, the system is no longer spherical symmetric ($\vec{B}$ field along <u>one</u> axis).

~~However~~, the eigenvectors of $\hat{H}_0$, $|n, \ell, m_\ell\rangle$ are still eigenvectors of $\hat{L}_z$

$$\hat{L}_z |n, \ell, m_\ell\rangle = \hbar m_\ell |n, \ell, m_\ell\rangle$$

$\Rightarrow$ Magnetic interaction

$$\hat{H}_{int} |n, \ell, m_\ell\rangle = + m_\ell \underbrace{(\mu_B B_z)}_{\text{energy}} \quad \ll \frac{e^2}{a_0} = 13.6\,eV$$

example: Earth's magnetic field $B \sim 0.5$ Gauss

$$\Rightarrow \mu_B B_{earth} \approx 0.5 \times 10^{-20}\,\text{erg} \cong 3 \times 10^{-9}\,eV$$

<u>Tiny</u>

New energy levels: $\hat{H} |n \ell m_\ell\rangle = \left[-\frac{R}{n^2} + m_\ell (\mu_B B_z) \right] |n, \ell, m_\ell\rangle$

2s ——

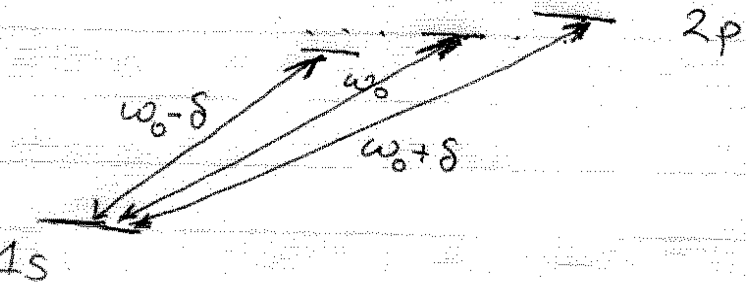
m_ℓ: -1, 0, $+1$; $\mu_B B_z$ 2p (degeneracy <u>broken</u> by perturbation)

1s ——

"Zeeman shift"

$m_\ell \equiv$ "magnetic quantum number"

Spectral lines



(Allowed transition)

Single spectral line without $\vec{B}$ $\Rightarrow$ ~~One~~ Three spectral lines with $\vec{B}$

"Normal Zeeman effect"

Historically, the Zeeman effect was observed in the early 20^{th} century, soon after the discovery of the electron, but before quantum theory. Lorentz explained it <u>classically</u>, one of the first applications of the electron theory, for which he and Peter Zeeman shared the nobel prize. However, it was already clear that this wasn't the whole story. For alkali atoms (e.g. sodium) the spectrum was split into <u>more</u> than three lines, something unexplainable by the classical theory. This was known as the "anomalous Zeeman effect". Landé fit this measured spectrum with a "fudge factor" $\mu = g \frac{\mu_B}{\hbar}$ (g = Landé "g factor")

and half-integer m values in some cases.

Pauli, introduced a additional "degree of freedom" in order to account for the periodic table via his exclusion principle.

In 1924 Uhlenbeck and Goudsmit explained this and the Zeeman effect (as well as fine structure, to be discussed later in the semester) through the introduction of spin angular momentum.

The electron spin has associated with it an intrinsic magnetic moment.

$$\hat{\vec{\mu}}_{\substack{spin \\ electron}} = -g_s \frac{\mu_B}{\hbar} \hat{\vec{S}}$$

Where $\hat{\vec{S}}$ = spin angular momentum operator (with $s = 1/2$ for electron)

Empirically $g_s = 2$ (plus QED correction)

The derivation of the g-factor for electrons waited for the Dirac equation ~~the~~ which gave a relativistic description of the electron.