Physics 492: Quantum Mechanics II

Problem Set #3
Due: Tuesday, Feb. 12, 2019 (5:00pm in TA mailbox)

Problem 1: Finish up Problem 3 from P.S. #2 (25 points)

Problem 1: 1D, 2D vs. 3D geometries (20 points)

(a) Consider a particle subject to a two-dimensional infinite square well potential

$$V(x,y)=\begin{cases}0, & 0<x<a_x, 0<y<a_y \\ \infty, & \text{otherwise}\end{cases}$$

and free in the *z*-direction. Show that the energy spectrum is

$$E_{n_x,n_y}(k_z)=\frac{\hbar^2}{2m}\left(k_{n_x}^2+k_{n_y}^2+k_z^2\right),\ \text{where}\ k_{n_x}=n_x\frac{\pi}{a_x},\ k_{n_y}=n_y\frac{\pi}{a_y},\ n_x,n_y=1,2,...,\ k_z\ \text{arbitrary.}$$

What are the eigenfunctions? What is their degeneracy?

(b) Now consider a 3D well with $a_z << a_x, a_y$. Let us choose $a_x = a_y \equiv a$ and $a = 10a_z$. This is a kind of "slab geometry". What is the energy spectrum? What is their degeneracy. Make a sketch of the energy levels. Please comment.

(c) Now consider a 3D well with $a_z >> a_x, a_y$. Let us choose $a_x = a_y \equiv a$ and $a = a_z/10$. This is a kind of "wire geometry". What is the energy spectrum? What is their degeneracy. Make a sketch of the energy levels. Please comment.

(d) Suppose you wanted to "engineer" a situation to study a gas in 2D or a gas in 1D. Explain how you might do this. Note, the temperature will play an important role.