Quantum information processing in ion traps II

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Part I, Rainer Blatt

Lecture 1: Nuts and bolts

- Ion trapology
- Qubits based on ground-state hyperfine levels
- Two-photon stimulated-Raman transitions
    * Rabi rates, Stark shifts, spontaneous emission

Lecture 2: Quantum computation (QC) and quantum-limited measurement

- Trapped-ion QC and DiVincenzo's criteria
- Gates
- Scaling
- Entanglement-enhanced quantum measurement

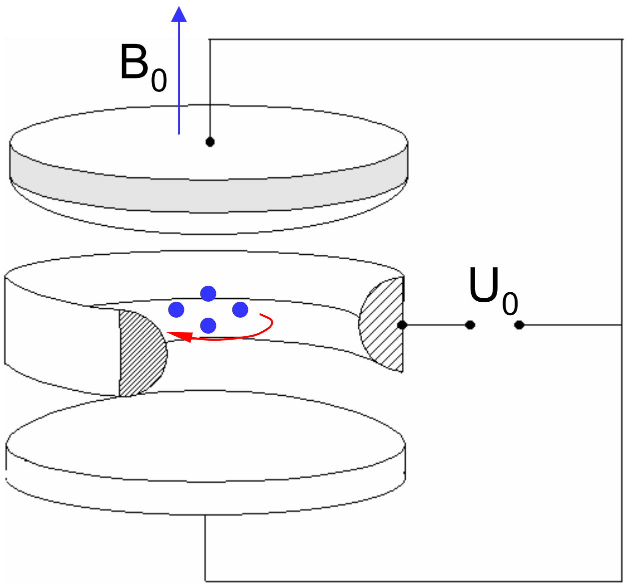
Lecture 3: Decoherence

- Memory decoherence
- Decoherence during operations
    * technical fluctuations
    * spontaneous emission
    * scaling
- Decoherence and the measurement problem

Ion trapping 101

"Earnshaw's theorem": ≅ In a charge free region, cannot confine a charged particle with static electric fields.
Proof: For confinement, must have $(\partial^2(q\Phi)/\partial^2 x_i)_{\text{trap location}} < 0$ ($x_i \in \{x,y,z\}$)
But from Laplace's equation: $\nabla^2\Phi = 0$, cannot satisfy confinement condition for all x_i.

Solution 1: Penning trap:



$$q\Phi \propto U_0\,[2z^2 - x^2 - y^2]$$

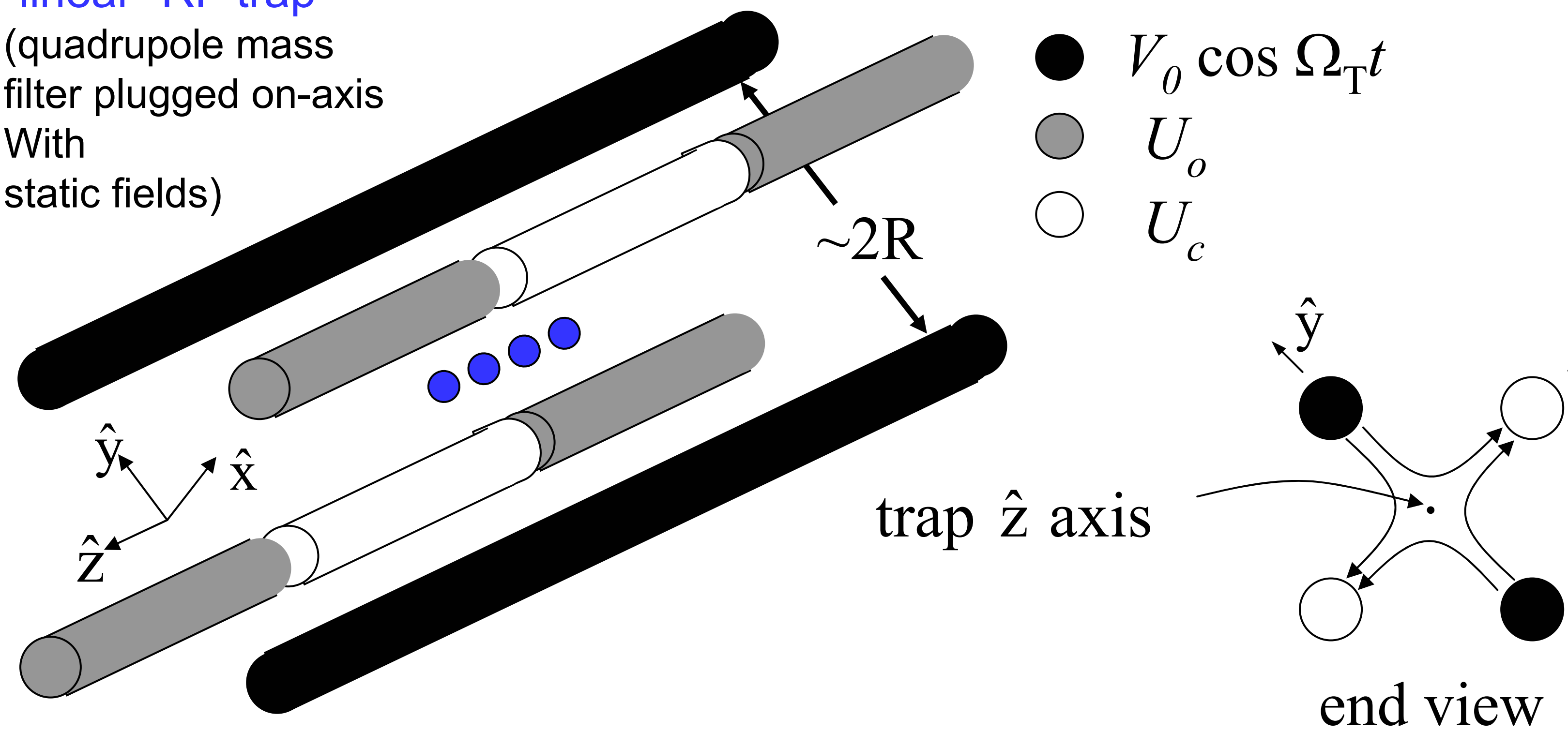
Difficult to accomplish individual ion addressing.
(However, see: Ciaramicoli, Marzoli, Tombesi, PRL **91**, 017901 (2003))

Solution 2: RF-Paul trap:

$$\Phi = (\alpha x^2 + \beta y^2 + \gamma z^2)V_0\cos\Omega t + U_0(\alpha' x^2 + \beta' y^2 + \gamma' z^2)$$

$$[\alpha + \beta + \gamma = \alpha' + \beta' + \gamma' = 0] \quad \text{(Laplace)}$$

Special case:
"linear" RF trap
(quadrupole mass filter plugged on-axis With static fields)



$$\Phi = \frac{(x^2 - y^2)}{2R^2} V_0 \cos \Omega_T t + \frac{(U_x x^2 + U_y y^2 + U_z z^2)}{2R^2}, \quad \boxed{\sum_{i=x,y,z} U_i = 0}$$

$$U_x = \alpha_{xo} U_o + \alpha_{xc} U_c, \quad U_y = \alpha_{yo} U_o + \alpha_{yc} U_c, \quad U_z = \kappa (U_o - U_c)$$

(Get α's and κ numerically)

$$\Phi = \frac{(x^2 - y^2)}{2R^2} V_0 \cos \Omega_T t + \frac{(U_x x^2 + U_y y^2 + U_z z^2)}{2R^2}$$

$$\boxed{\sum_{i=x,y,z} U_i = 0}$$

<u>Equations of motion</u>

(classical treatment adequate) $\vec{F} = m\vec{a}$

Mathieu equation

$$\frac{d^2 x_i}{d\xi^2} + \left[a_i - 2q_i \cos 2\xi\right] x_i = 0 \qquad (i \in \{x, y, z\}) \qquad \text{(2)}$$

$$a_i \equiv 4qU_i/m\Omega_T^2 R^2, \quad \xi \equiv \Omega_T t/2$$

$$q_x = -q_y \equiv -2qV_0/m\Omega_T^2 R^2, \quad q_z = 0$$

z-motion, q_z = 0 (static harmonic well)

$$\omega_z = (qU_z/mR^2)^{1/2} = \sqrt{a_z}\,\Omega_T/2$$

x,y motion, Mathieu equation:

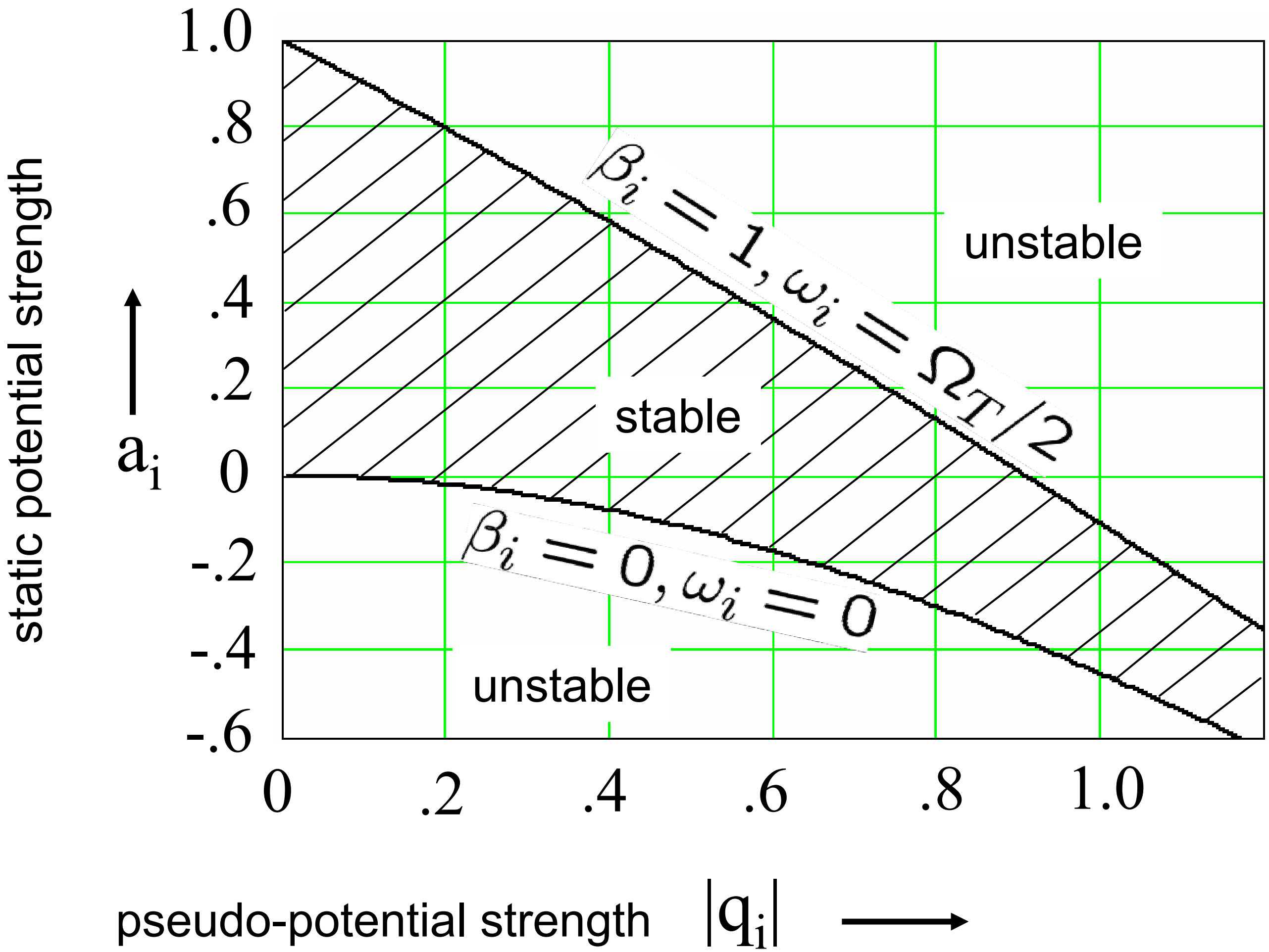
$$x_i(\xi) = Ae^{i\beta_i \xi} \sum_{n=-\infty}^{\infty} C_{2n} e^{i2n\xi} + Be^{-i\beta_i \xi} \sum_{n=-\infty}^{\infty} C_{2n} e^{-i2n\xi} \qquad (i \in \{x, y\})$$

plug into (2), find (recursion relation for) C_{2n}

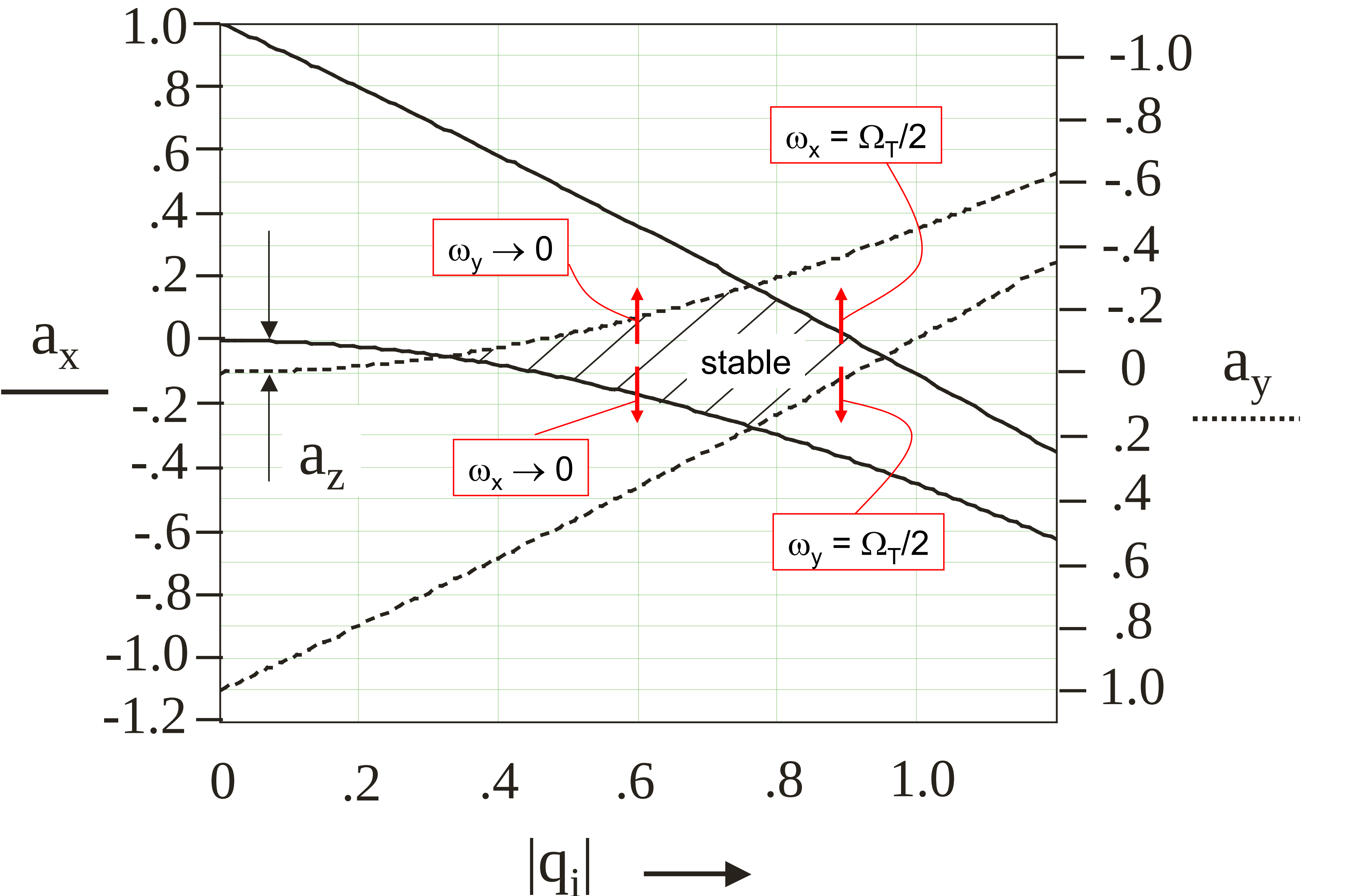
Solution in ith direction ($i \in \{x,y\}$):

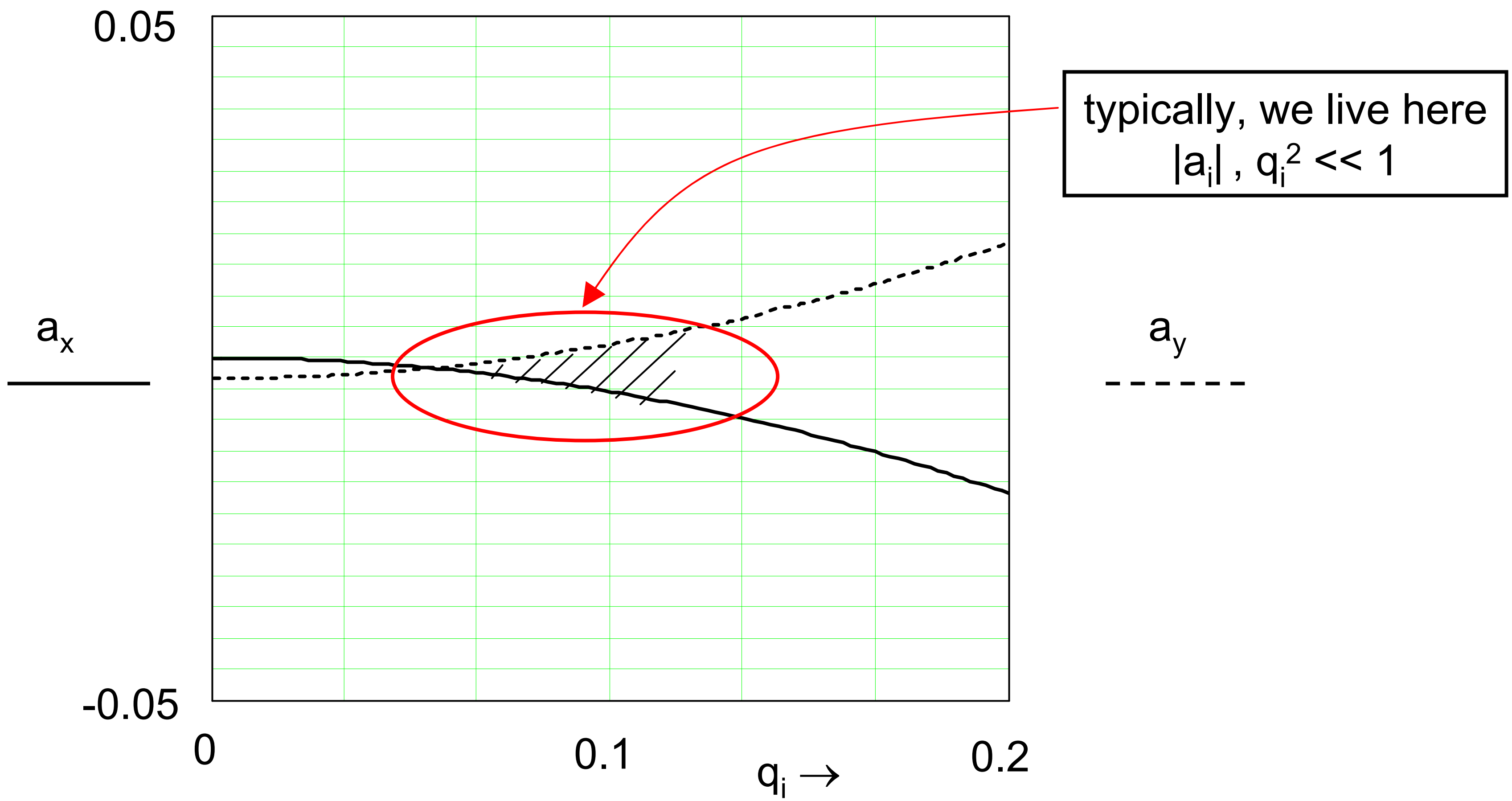
$$a_i \equiv 4qU_i/m\Omega_T^2 R^2$$

$$q_x = -q_y \equiv -2qV_0/m\Omega_T^2 R^2$$



Simultaneous solution for x, y, z: $a_i \propto U_i$, $a_x + a_y + a_z = 0$





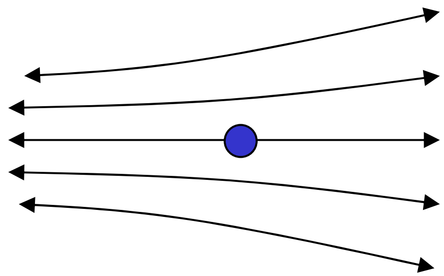
$$x_i(\xi) = Ae^{i\beta_i \xi} \sum_{n=-\infty}^{\infty} C_{2n} e^{i2n\xi} + Be^{-i\beta_i \xi} \sum_{n=-\infty}^{\infty} C_{2n} e^{-i2n\xi} \qquad (i \in \{x, y\})$$

$$\Rightarrow \quad x_i(t) \simeq \underbrace{X_{i0} \cos \omega_i t}_{\text{“secular” motion}} + \underbrace{\left[X_{i0} \cos \omega_i t\right] \frac{q_i}{2} \sin \Omega_T t}_{\text{“micromotion”}}$$

$$\omega_i = \frac{\Omega_T q_i}{2\sqrt{2}} = \frac{qV_o}{\sqrt{2} m \Omega_T R^2} \ll \Omega_T$$

Typically, $\omega_i < \Omega_T/5$

Heuristic approach: “pseudo-potential” approximation



$$\vec{E}(\vec{x},t) = \vec{E}(\vec{x})\cos\Omega_T t$$

- <u>assume</u> mean ion position changes negligibly in duration $2\pi/\Omega_T$

$$m\frac{\partial^2 \vec{x}_\mu}{\partial t^2} = q\vec{E}(\vec{x})\cos\Omega_T t \quad \Rightarrow \quad \vec{x}_\mu = -\frac{q\vec{E}(\vec{x})}{m\Omega_T^2}\cos\Omega_T t$$

- pseudo-potential from micromotion kinetic energy:

$$U_{pseudo} = \langle KE\rangle = \frac{1}{2}m\langle v_\mu^2\rangle = \frac{q^2E^2(\vec{x})}{4m\Omega_T^2}$$

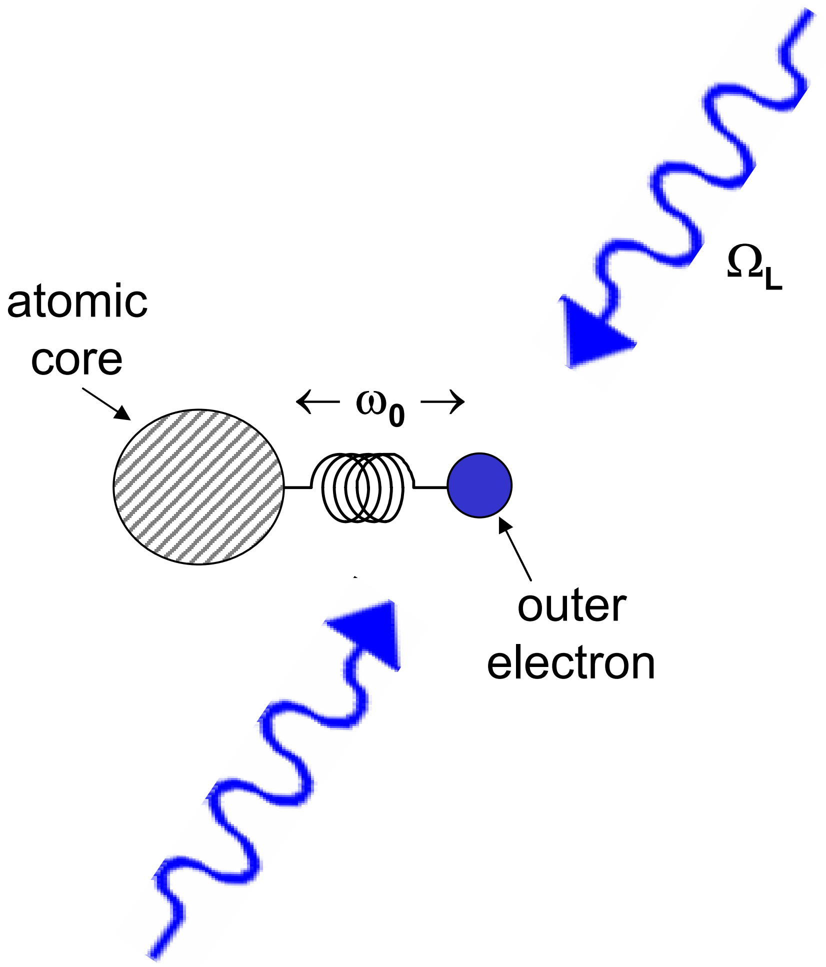
agrees with Mathieu equation limit $|a_i|$, $q_i^2 << 1$

- for linear RF trap,

$$U_{pseudo} = \frac{q^2V_0^2}{2m^2\Omega_T^2R^4}(x^2+y^2) = \frac{1}{2}m\omega_{x,y}^2(x^2+y^2) \qquad \omega_{x,y} = \frac{qV_0}{\sqrt{2}m\Omega_T R^2}$$

Or, calculate avg. force $\langle\vec{F}\rangle$ over one RF cycle. Then $U_{pseudo} = -\int\langle\vec{F}\rangle\cdot d\vec{x}$

Digression: Optical dipole traps:

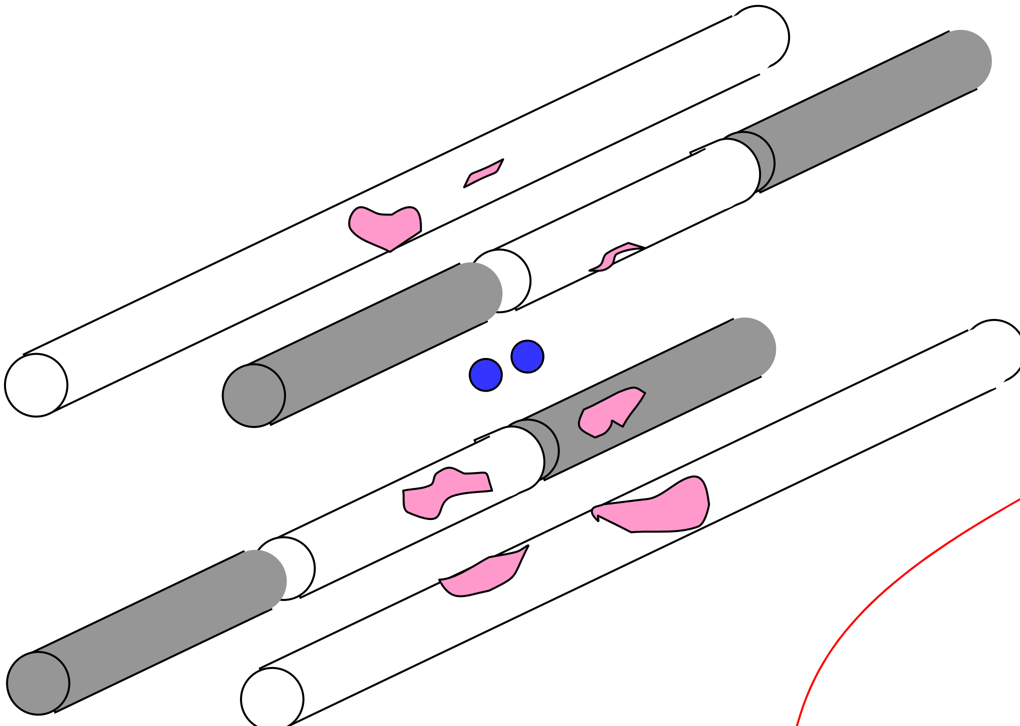


- Response of atomic core to laser field is negligible because of heavy mass.

- For Ω_L **>>** ω_0 (blue detuning) electron response out of phase with electric force. Electron trapped in ponderomotive (pseudo-potential) laser potential (just like RF trap). Trapping in field minima.
Core is attached to electron

- <u>Dispersion</u>:
For Ω_L **<<** ω_0 (red detuning) electron response in phase with electric force. Trapping in field maxima.

(some) ion-trap realities

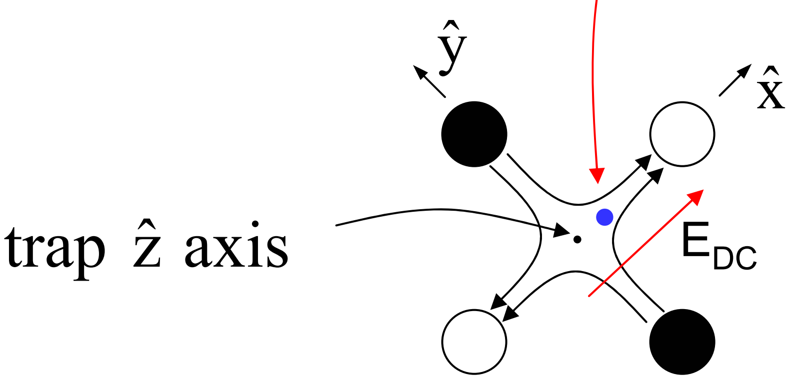


Patch potentials:

Static potentials: pushes ions away from trap axis $\Rightarrow$ micromotion $x_\mu \sin\Omega_T t$ can cause X-tal heating

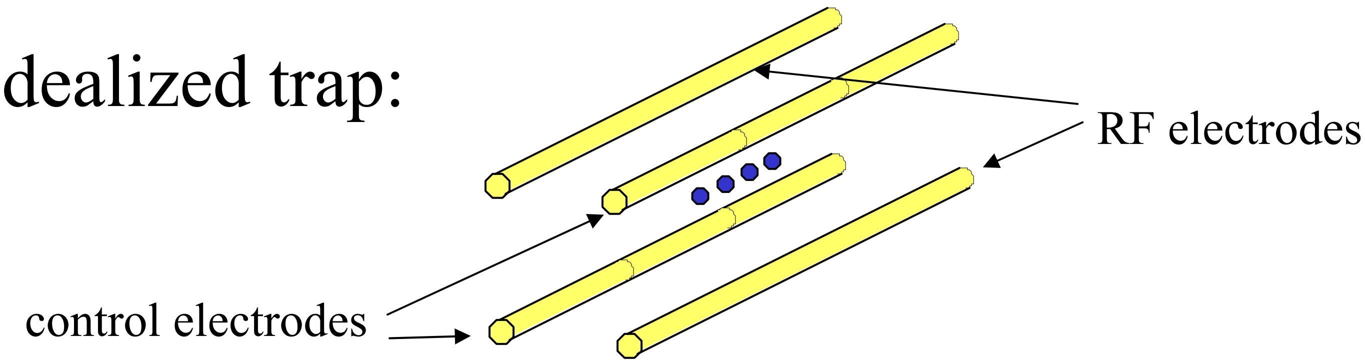
Fluctuating patch fields: causes heating; COM primarily affected

Source: unknown! (mobile electrons on oxide layers,….. ??)

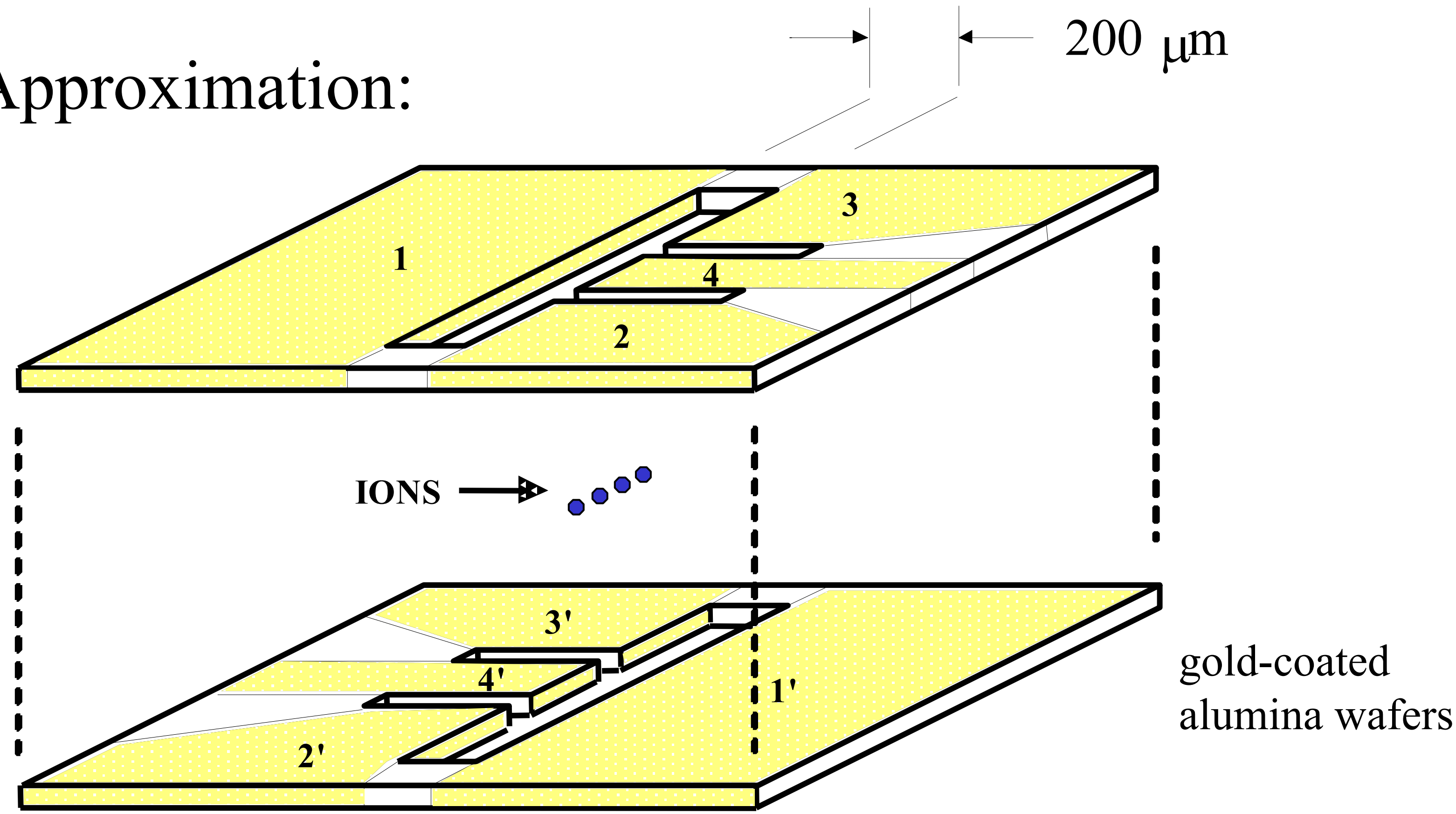


$$x_\mu = X_{displacement} \frac{q_i}{2} \sin \Omega_T t$$

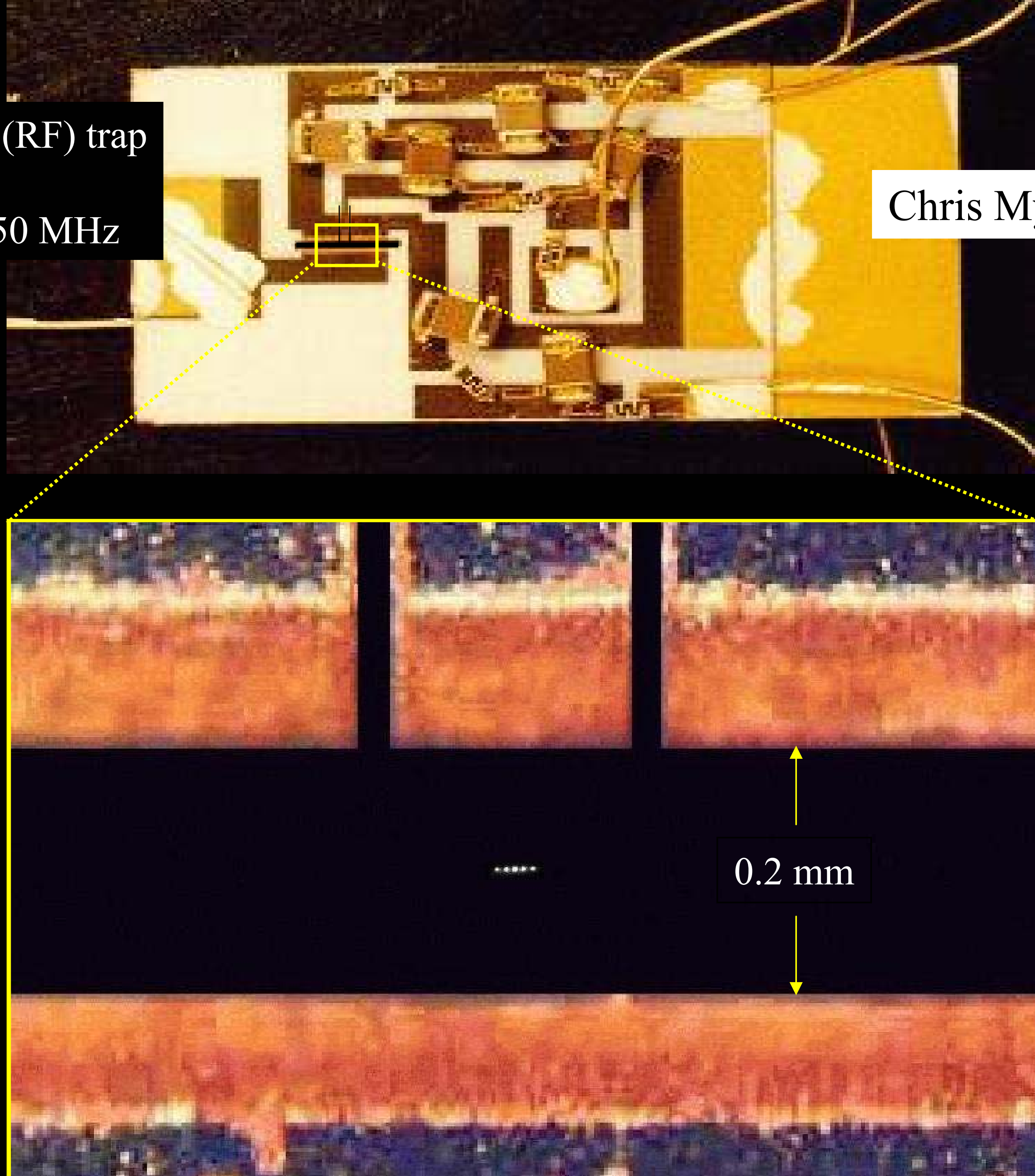
Idealized trap:



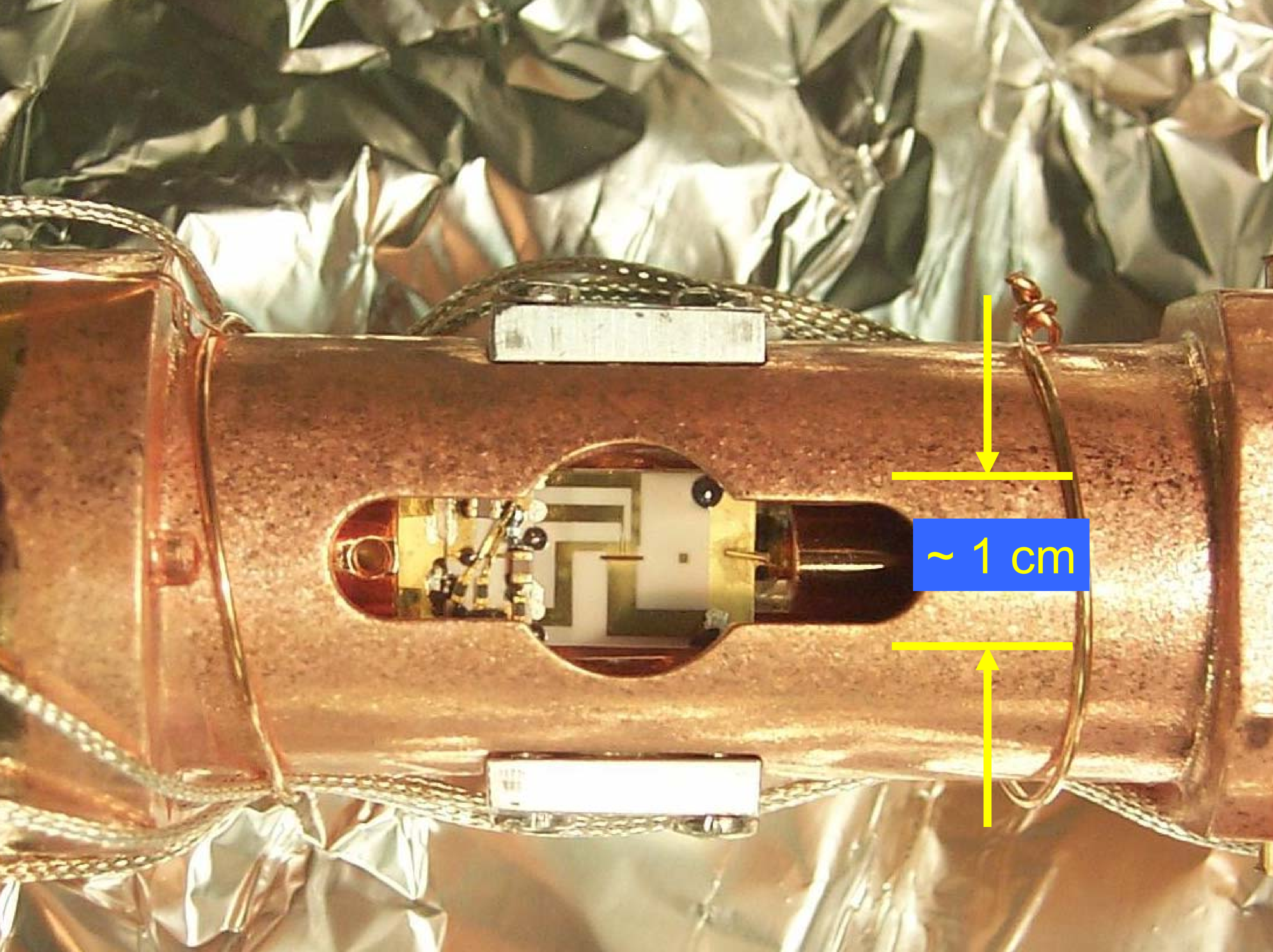
Approximation:

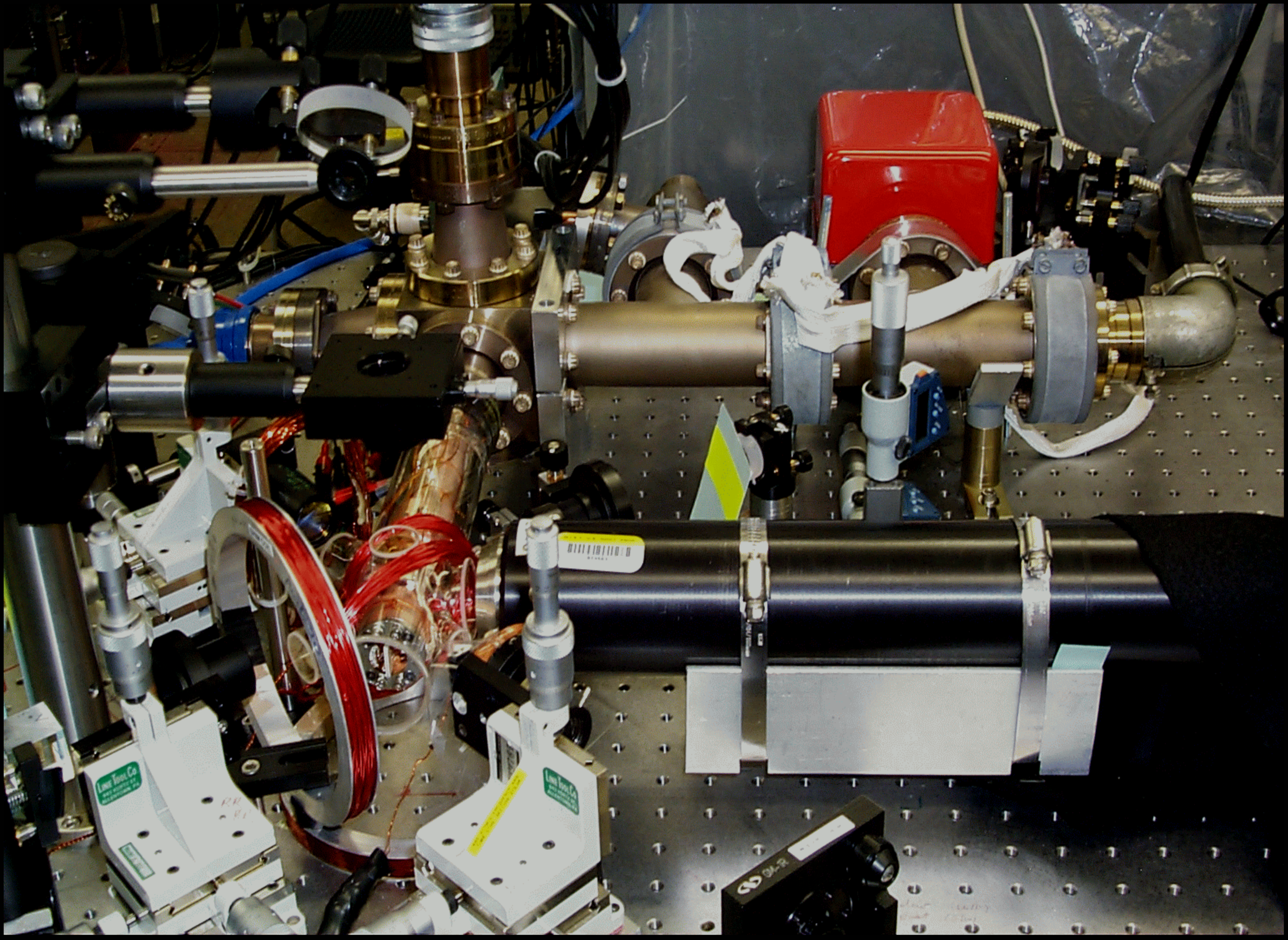


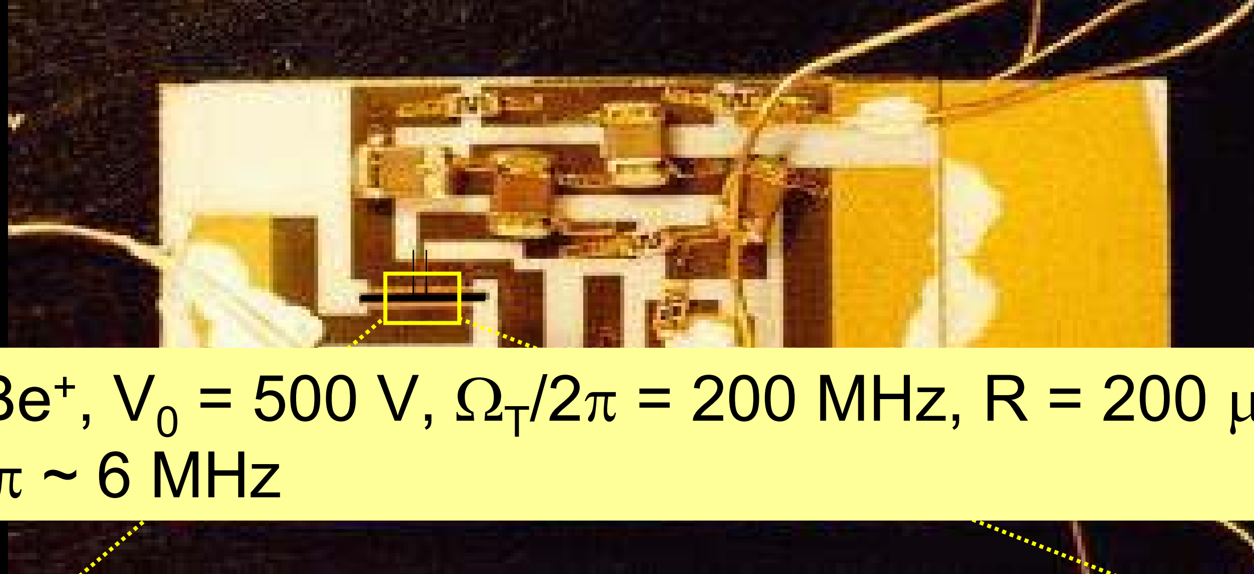
"linear" Paul (RF) trap
$V_{RF} \sim 500$ V
$\Omega_{RF} \sim 50 - 250$ MHz



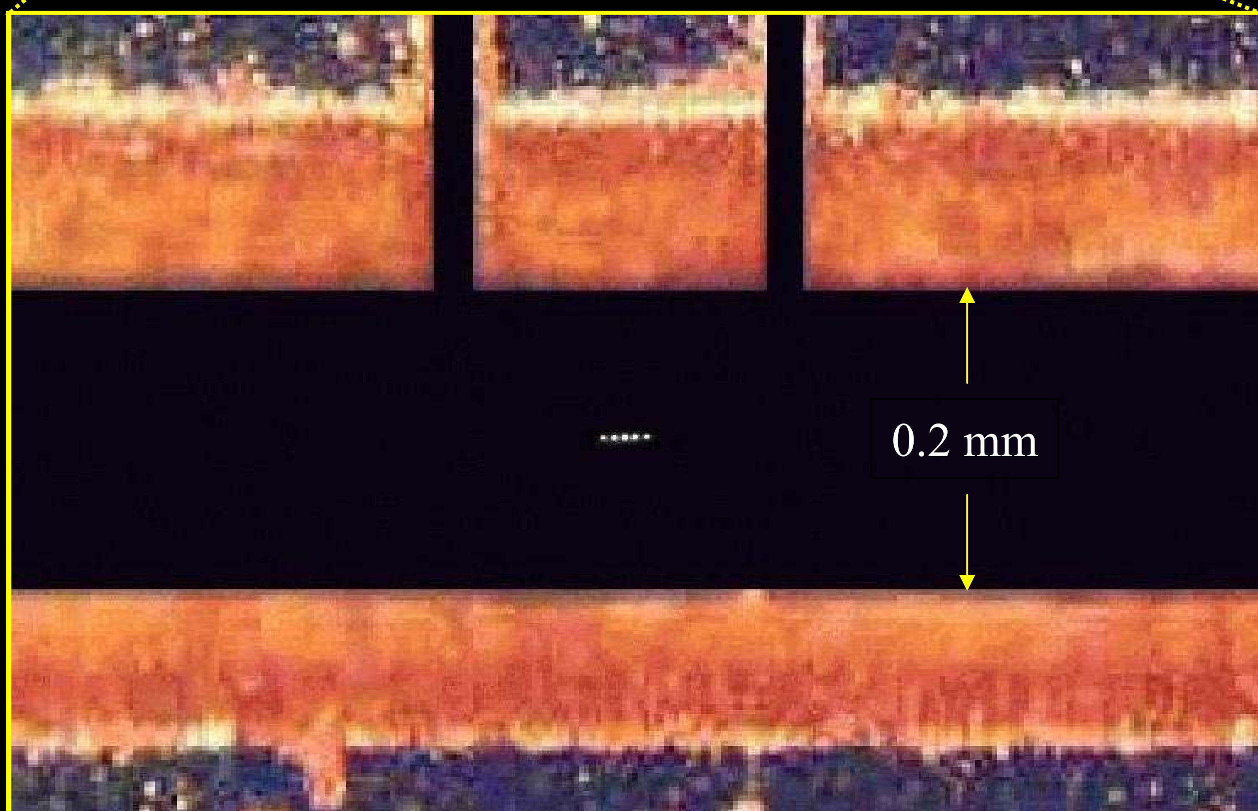
Chris Myatt *et al.*


~ 1 cm

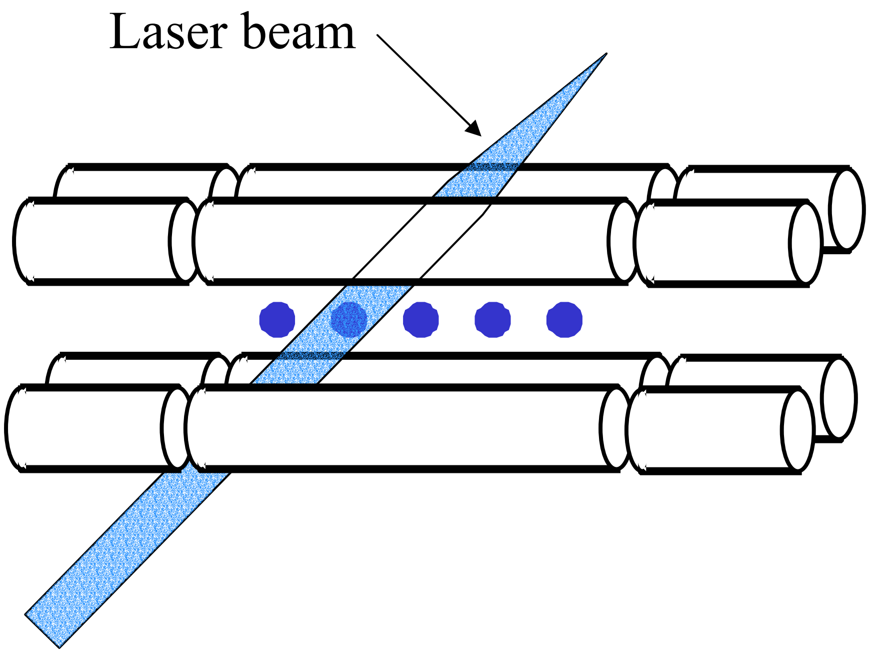




For $^{9}Be^{+}$, V_0 = 500 V, $\Omega_T/2\pi$ = 200 MHz, R = 200 μm
$\omega_{x,y}/2\pi \sim 6$ MHz

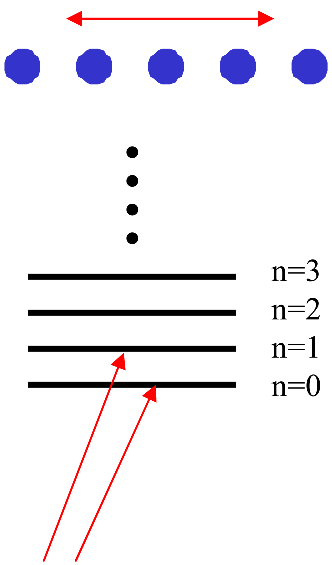


Ion Trap QC: Proposal: J. I. Cirac and P. Zoller, PRL **74**, 4091 (1995)



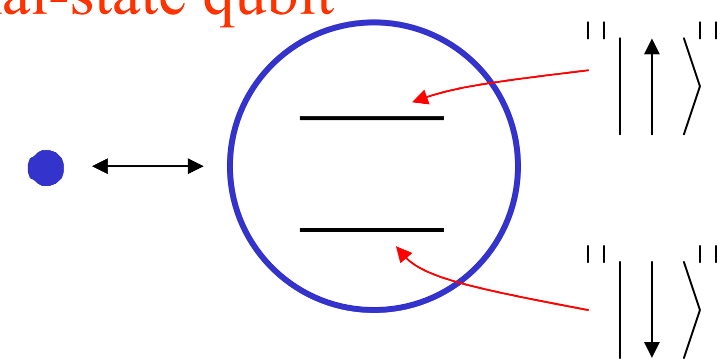
Motion "data bus"

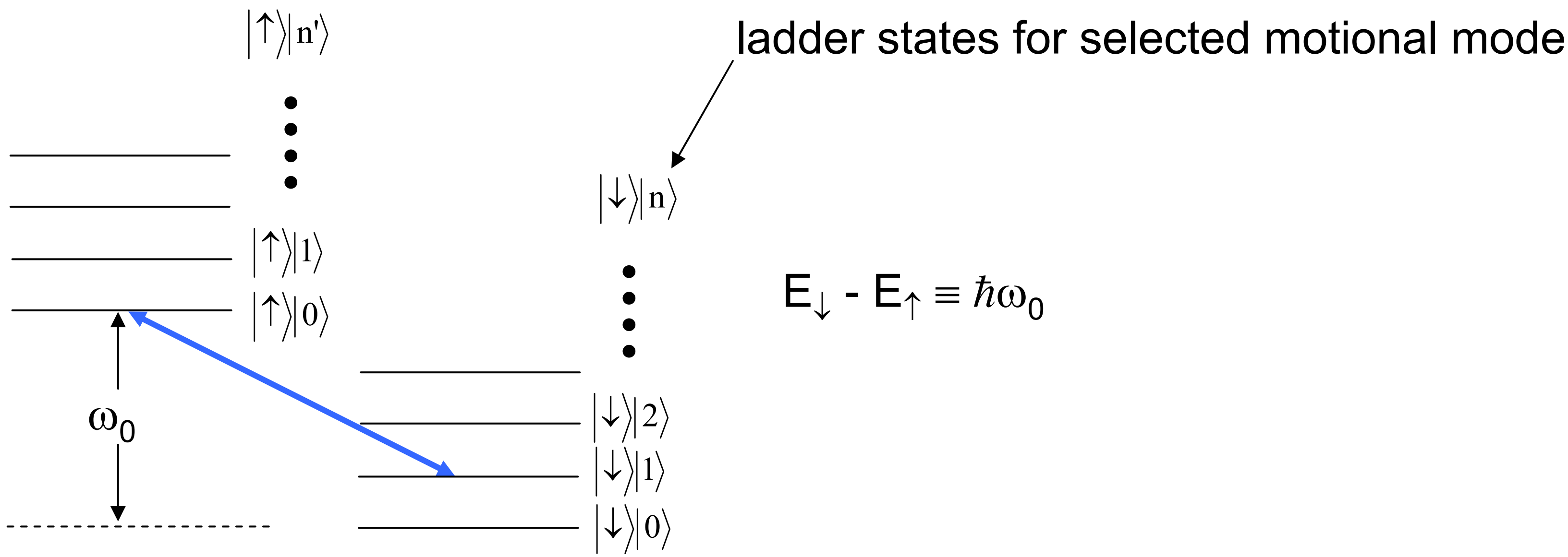
(e.g., center-of-mass mode)



Stay in two lowest motional states (motion qubit)

Internal-state qubit





$E_\downarrow - E_\uparrow \equiv \hbar\omega_0$

ω_0 = optical frequency:

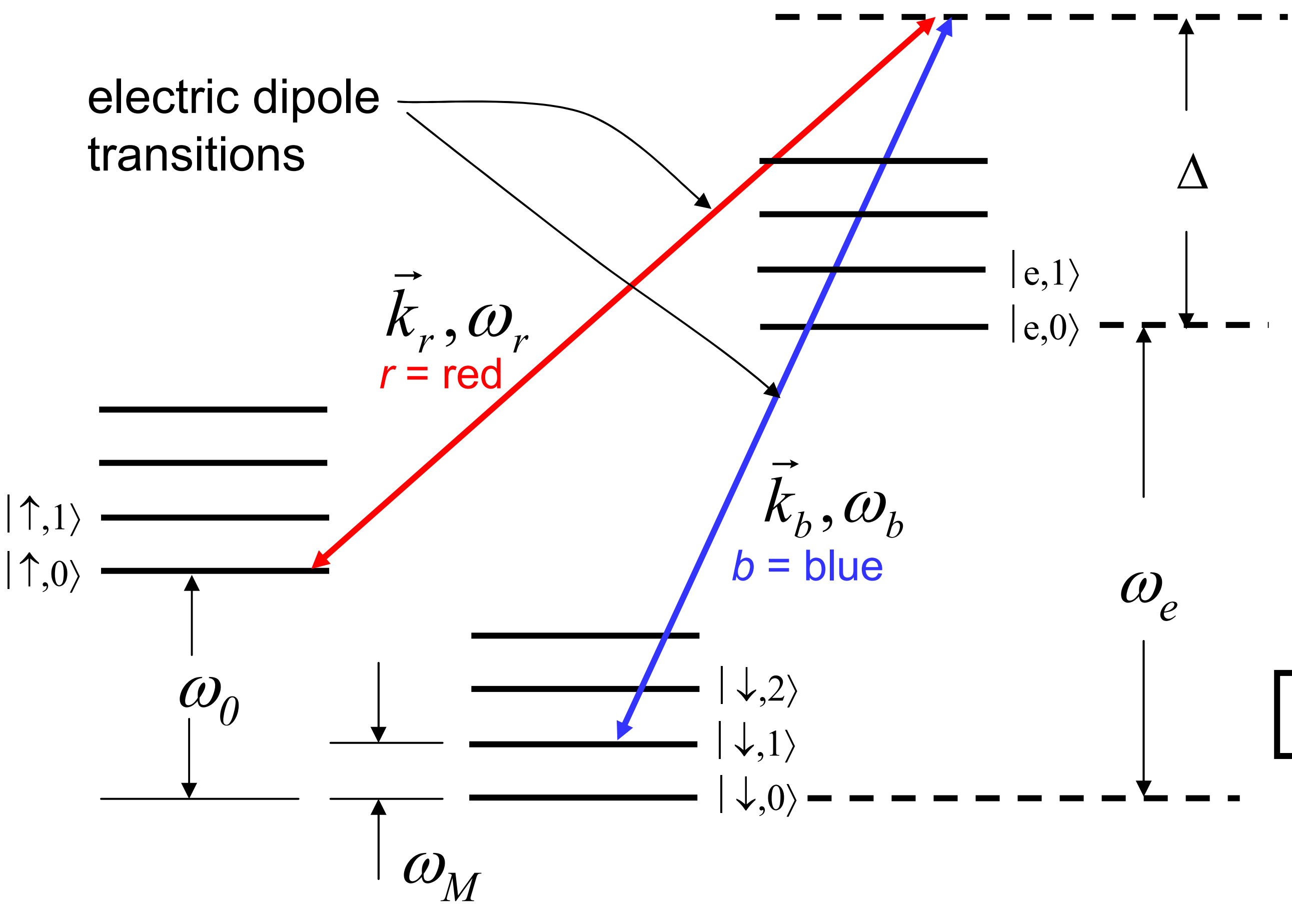
☺ single photon
☹ need good laser frequency stability
☹ memory and gate coherence limited by upper state lifetime (~ seconds)

ω_0 = RF/microwave frequency

☺ memory coherence limited by upper-state lifetime (>> days)
☹ sideband transitions weak at RF $\Rightarrow$ 2-photon optical stimulated-Raman transitions
- ☺ frequency stability = RF modulator stability
- ☺ vary sideband coupling (Lamb-Dicke parameter) with $\mathbf{k}_2 - \mathbf{k}_1$

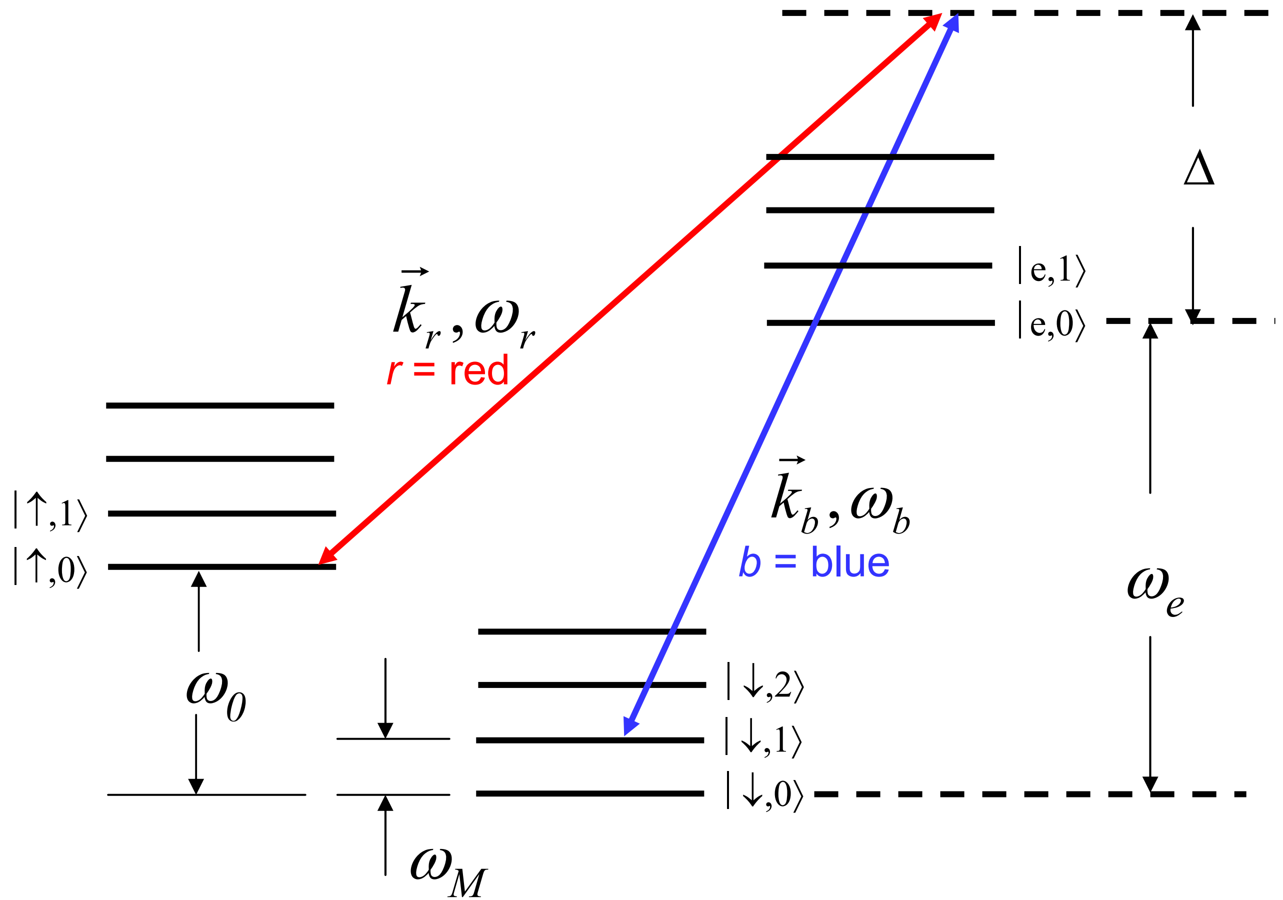
☹ gate decoherence: spontaneous-Raman scattering (<u>fundamental limit</u>)

Stimulated-Raman transitions:
Simple case: Motion: 1-D harmonic well (frequency ω_M),
Internal states: 3-level Λ system



$\omega_e >> \Delta >> \omega_0 >> \omega_M$

In Be^+, $\omega_e/2\pi \simeq 10^{15}$ Hz, $\Delta/2\pi \simeq 10^{11}$ Hz,
$\omega_0/2\pi \simeq 10^9$ Hz, $\omega_M/2\pi \simeq 10^7$ Hz



$$H = H_0 + H_I$$

$$H_0 = \hbar\omega_0|\uparrow\rangle\langle\uparrow| + \hbar\omega_e|e\rangle\langle e| + \hbar\omega_M a^\dagger a \qquad H_I = -\sum_{i=b,r} e\vec{r}\cdot\vec{E}_i$$

$$\vec{E}_i = \hat{\epsilon}_i E_{i0}\cos(\vec{k}_i\cdot\vec{X}_M - \omega_i t + \phi_i), \qquad i \in \{b, r\}$$

$$\vec{X}_M = \hat{x}x_0(a + a^\dagger), \quad x_0 = \sqrt{\hbar/2m\omega_M}$$

zero-point wavefunction spread

$$\Psi = \sum_{n=0}^{\infty} \left[C_{\downarrow,n} e^{-in\omega_M t} |\downarrow, n\rangle + C_{\uparrow,n} e^{-i[\omega_0 + n\omega_M]t} |\uparrow, n\rangle + C_{e,n} e^{-i[\omega_e + n\omega_M]t} |e, n\rangle \right]$$

$\langle\downarrow, n|(i\hbar\partial\Psi/\partial t = H\Psi)$ (+ rotating wave approximation) $\Rightarrow$

$$\dot{C}_{\downarrow,n} = ig_b \sum_{m=0}^{\infty} e^{i(\omega_b - \omega_e + (n-m)\omega_M)t} \langle n|e^{-i\vec{k}_b \cdot \vec{X}}|m\rangle C_{e,m}$$

similarly:

$$\dot{C}_{\uparrow,n} = ig_r \sum_{m=0}^{\infty} e^{i(\omega_r - (\omega_e - \omega_0) + (n-m)\omega_M)t} \langle n|e^{-i\vec{k}_r \cdot \vec{X}}|m\rangle C_{e,m}$$

$$\dot{C}_{e,m} = ig_b^* \sum_{p=0}^{\infty} e^{-i(\omega_b - \omega_e - (m-p)\omega_M)t} \langle m|e^{i\vec{k}_b \cdot \vec{X}}|p\rangle C_{\downarrow,p}$$

$$+ ig_r^* \sum_{p=0}^{\infty} e^{-i(\omega_r - (\omega_e - \omega_0) - (m-p)\omega_M)t} \langle m|e^{i\vec{k}_r \cdot \vec{X}}|p\rangle C_{\uparrow,p}$$

$$g_b \equiv \langle\downarrow|\hat{\epsilon}_b \cdot \vec{r}\,|e\rangle \frac{eE_{b0}e^{-i\phi_b}}{2\hbar} \qquad g_r \equiv \langle\downarrow|\hat{\epsilon}_r \cdot \vec{r}\,|e\rangle \frac{eE_{r0}e^{-i\phi_r}}{2\hbar}$$

$$\dot{C}_{e,m} = ig_b^* \sum_{p=0}^{\infty} e^{-i(\omega_b - \omega_e - (m-p)\omega_M)t} \langle m | e^{i\vec{k}_b \cdot \vec{X}} | p \rangle C_{\downarrow,p}$$

$$+ig_r^* \sum_{p=0}^{\infty} e^{-i(\omega_r - (\omega_e - \omega_0) - (m-p)\omega_M)t} \langle m | e^{i\vec{k}_r \cdot \vec{X}} | p \rangle C_{\uparrow,p}$$

"Adiabatic elimination":

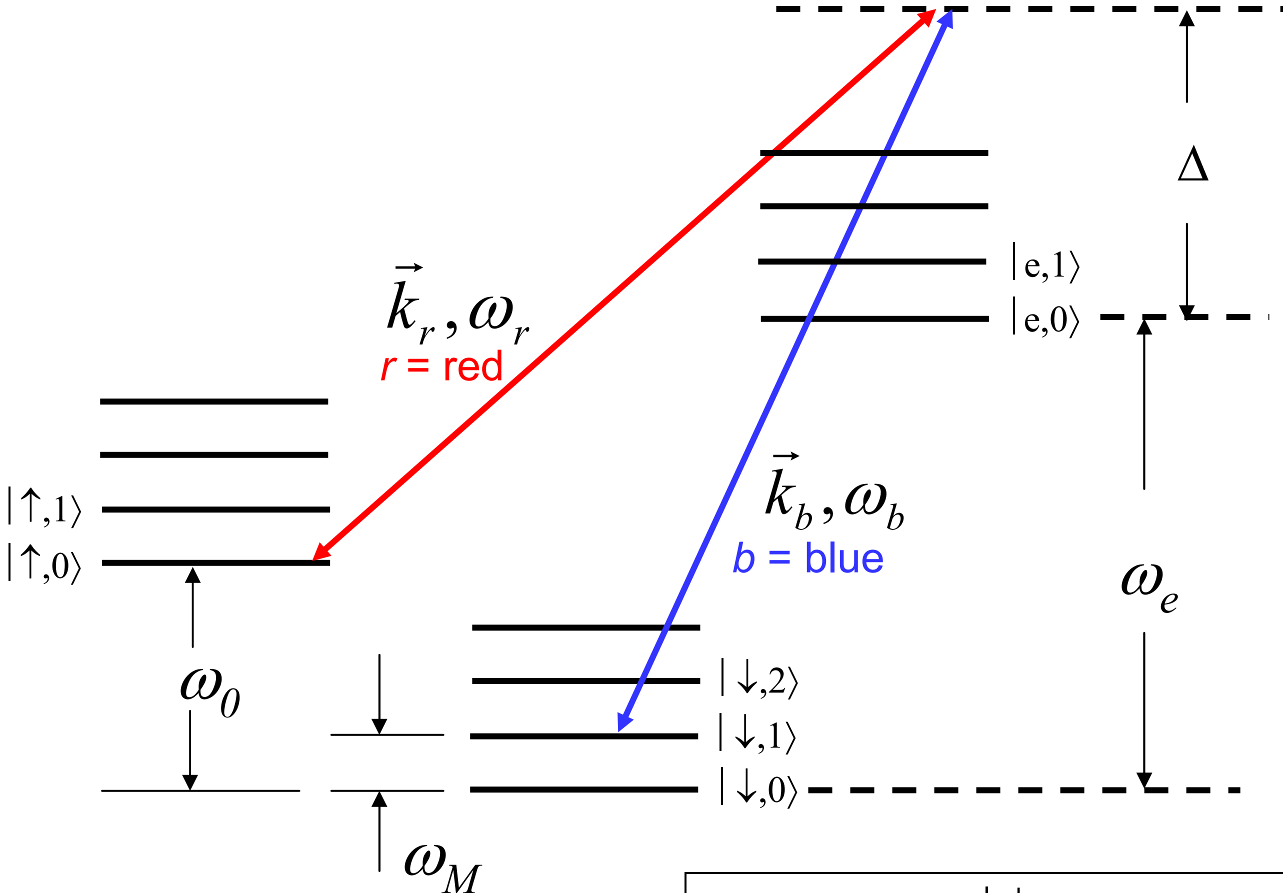
$$C_{e,m} \equiv e^{-i\Delta t} C'_{e,m}; \quad \dot{C}_{e,m} = e^{-i\Delta t}(\dot{C}'_{e,m} - i\Delta C'_{e,m})$$

make ansatz: $\Delta C'_{e,m} \gg \dot{C}'_{e,m}$

$$\Rightarrow C'_{e,m} = ie^{i\Delta t}\dot{C}_{e,m}/\Delta \qquad \Rightarrow C_{e,m} = i\dot{C}_{e,m}/\Delta$$

$$\dot{C}_{\downarrow,n} = ig_b \sum_{m=0}^{\infty} e^{i(\omega_b - \omega_e + (n-m)\omega_M)t} \langle n | e^{-i\vec{k}_b \cdot \vec{X}} | m \rangle C_{e,m}$$

$$\dot{C}_{\uparrow,n} = ig_r \sum_{m=0}^{\infty} e^{i(\omega_r - (\omega_e - \omega_0) + (n-m)\omega_M)t} \langle n | e^{-i\vec{k}_r \cdot \vec{X}} | m \rangle C_{e,m}$$



Stark shift of $|\downarrow\rangle$ from blue laser

$$\dot{C}_{\downarrow,n} = -i\frac{|g_b|^2}{\Delta}C_{\downarrow,n} - i\sum_{n'=0}^{\infty}\Omega_{n,n'}\ e^{i(\delta-(n'-n)\omega_M)t}C_{\uparrow,n'}$$

$$\delta \equiv \omega_b - \omega_r - \omega_0$$

$$\dot{C}_{\uparrow,n'} = -i\frac{|g_r|^2}{\Delta}C_{\uparrow,n'} - i\sum_{n=0}^{\infty}\Omega^*_{n',n}\ e^{-i(\delta+(n-n')\omega_M)t}C_{\downarrow,n}$$

Add in other Stark shifts

$$\dot{C}_{\downarrow,n} = -i\Delta_{S\downarrow} C_{\downarrow,n} - i\sum_{p=0}^{\infty} \Omega_{n,p}\ e^{i(\delta-(p-n)\omega_M)t} C_{\uparrow,p}$$

$$\dot{C}_{\uparrow,n} = -i\Delta_{S\uparrow} C_{\uparrow,n} - i\sum_{p=0}^{\infty} \Omega^*_{n,p}\ e^{-i(\delta+(p-n)\omega_M)t} C_{\downarrow,p}$$

$$\Delta_{S\downarrow} = |g_b|^2/\Delta + |g_{\downarrow,e,r}|^2/(\Delta-\omega_0), \quad \Delta_{S\uparrow} = |g_{\uparrow,e,b}|^2/(\Delta+\omega_0) + |g_r|^2/\Delta$$

absorb Stark shifts into wave function amplitudes

$$C_{\downarrow,n} = C'_{\downarrow,n} e^{-i\Delta\omega_{S\downarrow} t}, \quad C_{\uparrow,n} = C'_{\uparrow,n} e^{-i\Delta\omega_{S\uparrow} t}$$

near a resonance:

$$\delta_{n',n} \equiv \delta - (\Delta\omega_{S\uparrow} - \Delta\omega_{S\downarrow}) - (n'-n)\omega_M \ \simeq 0 \quad (\delta \equiv \omega_b - \omega_r - \omega_0)$$

$$\dot{C}'_{\downarrow,n} = -i\Omega_{n,n'} e^{i\delta_{n',n}t} C'_{\uparrow,n'} \quad \dot{C}'_{\uparrow,n'} = -i\Omega^*_{n',n} e^{-i\delta_{n',n}t} C'_{\downarrow,n}$$

$\Rightarrow$ Rabi flopping

$$\ddot{C}'_{\downarrow,n} + |\Omega_{n',n}|^2 C'_{\downarrow,n} = 0, \quad \ddot{C}'_{\uparrow,n'} + |\Omega_{n',n}|^2 C'_{\uparrow,n'} = 0$$

$$\Omega_{n,n'} \equiv \Omega\langle n|e^{-i(\vec{k}_b-\vec{k}_r)\cdot\vec{X}}|n'\rangle = \Omega\langle n|e^{-i\eta(a+a^\dagger)}|n'\rangle = \Omega_{n',n}$$

$$\eta \equiv (\vec{k}_b - \vec{k}_r)\cdot\hat{x}x_0$$ (Lamb-Dicke parameter)

$$\Omega \equiv g_b g_r^* / \Delta$$

Be$^+$: P = 1 mW, w_0 = 25 μm, $\Delta/2\pi$ = 100 GHz, $\Omega/2\pi \sim 0.5$ MHz

$$\langle n|e^{-i\eta(a+a^\dagger)}|n'\rangle = e^{-\eta^2/2}\sqrt{\frac{n_<!}{n_>!}}[-i\eta]^{|n'-n|}L_{n_<}^{|n'-n|}(\eta^2)$$

$$\simeq \langle n|1 - i\eta(a+a^\dagger) - \frac{\eta^2}{2}(1+2\tilde{n}+a^2+(a^\dagger)^2)|n'\rangle \quad (for\ \eta << 1)$$

Carrier transitions:

$$\Omega_{n,n} = \Omega(1-\eta^2(n+1/2)) \simeq \Omega\underbrace{e^{-\eta^2/2}}(1-n\eta^2)$$

Debye-Waller factor

Sideband transitions: n' = n ± 1

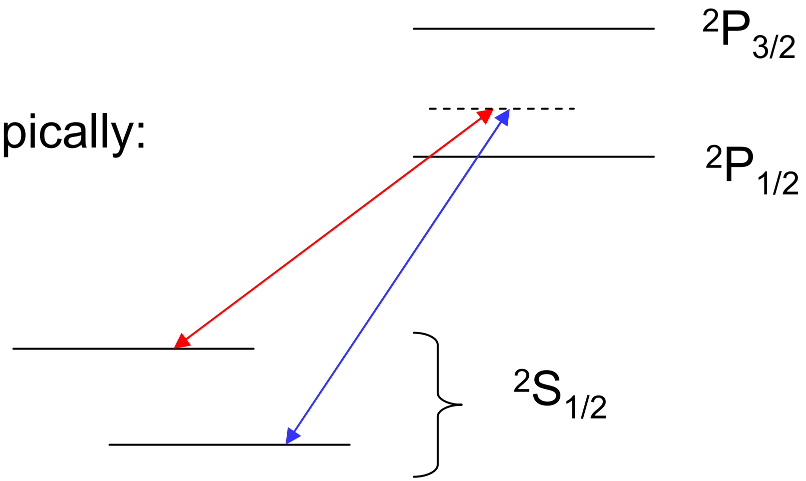
$$\Omega_{n',n} = -i\Omega\eta\sqrt{n_>}$$ ($n_>$ = larger of n and n')

red sideband (*n'* = *n*-1): can get from $H_I = \hbar\eta\Omega(|\downarrow\rangle\langle\uparrow|a^\dagger + h.c.$

Jaynes-Cummings Hamiltonian from cavity-QED
(see, e.g., Raimond, Brune, Haroche, Rev. Mod. Phys. **73**, 565 ('01))

Complete picture:

- Sum over excited states, typically:



  - can tune out differential Stark shifts
  - can tune out polarization sensitivity

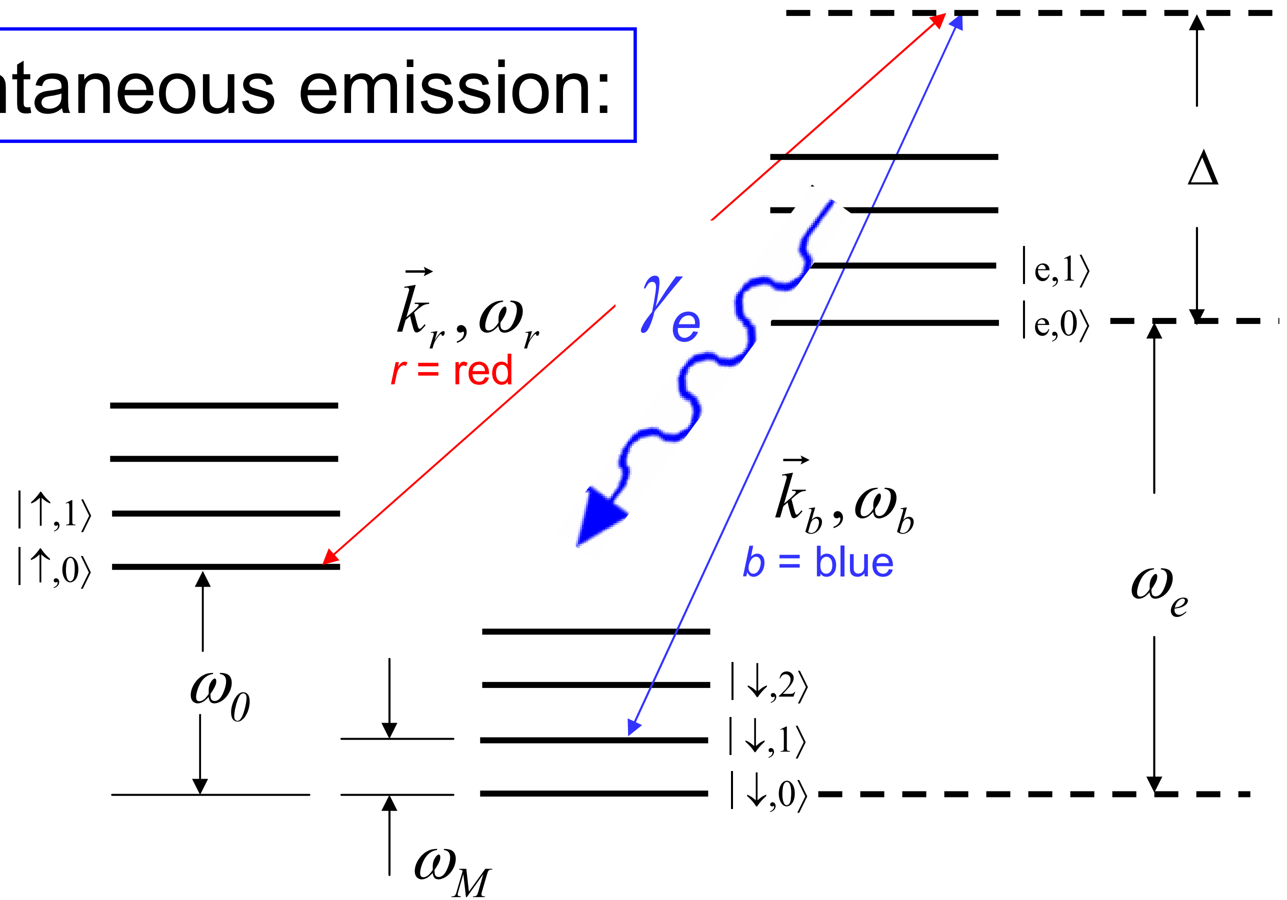
- For N ions, consider effects of 3N modes

  - Debye-Waller factors from “spectator” modes

$$\Omega_{n,n'} = \Omega\langle n|e^{-i\eta(a+a^\dagger)}|n'\rangle \rightarrow \Omega_{n'_k,n_k}\langle\{n_{p\neq k}\}|\prod_{p\neq k} e^{-i\eta_p(a_p+a_p^\dagger)}|\{n_{p\neq k}\}\rangle$$

  - sideband transitions: interference from two-mode transitions:
    e.g. $n\omega_p - m\omega_r = \omega_M$

Spontaneous emission:



$$R_{SE} = \gamma_e \sum_{m=0}^{\infty} |C_{e,m}|^2$$

$$\simeq \gamma_e \sum_{n=0}^{\infty} \left[|C_{\downarrow,n}|^2 \left(\frac{|g_b|^2}{\Delta^2} + \frac{|g_{\downarrow,e,r}|^2}{(\Delta - \omega_0)^2} \right) + |C_{\uparrow,n}|^2 \left(\frac{|g_r|^2}{\Delta^2} + \frac{|g_{\uparrow,e,b}|^2}{(\Delta - \omega_0)^2} \right) \right]$$

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