University of New Mexico
Department of Physics and Astronomy

Physics 511 - Graduate Electrodynamics
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Lecture I - Overview

I.A Introduction

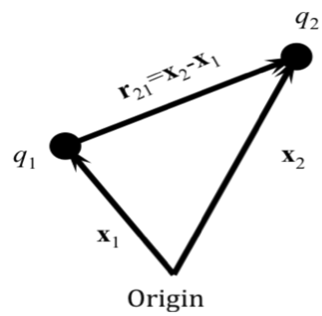
Electricity and magnetism is represented by one of the most beautiful and complete physical theories. During mostly the 18th and 19th centuries scientists amassed a huge collection of empirical data ranging from the effect of electricity on the reflex of a frog leg, to Ørsted and Ampère's discovery of the relation between electric current and magnetism, to Faraday's discovery of electromagnetic induction. This produced important knowledge of the phenomenology of electricity and magnetism and gave strong indication of the unity of these phenomena. This all culminated with Maxwell who reformulated the data in 1873 into a mathematical model that agreed with all measurements. But there was something asymmetric about the theory. A changing magnetic flux produced the electric field, but what about the reverse process? From purely theoretical grounds Maxwell added his now famous "displacement current" which in turn led him to PREDICT that light itself was a wave of the electromagnetic field, a prediction that was verified by Hertz in his 1885-1889 experiments, thereby unifying optics with E&M. The early 20th century gave us the null result of the Michaelson-Morley experiment in their search for the "luminiferous ether", followed by Einstein's revolutionary hypotheses at the foundation of relativity and the unification of space and time. Einstein's dream of the unification of gravity and electromagnetism was not achieved; he of course had deep philosophical problems with quantum field theory. Perhaps this unification will be one of the hallmarks of the 21st century.

The development of electromagnetic theory is a wonderful example of the scientific method at its best. Empirical studies are quantified and relations between observations are sought. Mathematical descriptions of these relations are formulated and NEW predictions are made. These predictions are put to the test in the laboratory. Either they hold up, or the theory must be modified accordingly. The ability to predict previously unobserved phenomena and put them into a consistent framework is the hallmark of a good theory. Often, the nonscientific community confuses the notion of a *theory* with a *hypothesis* or *hunch,* as is typified by the current debate over biological evolution.

I.B The Empirical Basis of Electromagnetism

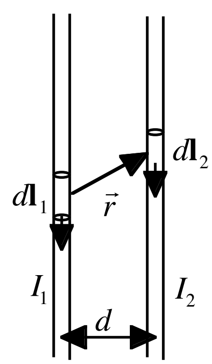
All electromagnetic phenomena are ultimately observed as forces on charges. The two basic forces are electrostatic and magnetostatic.

- Coulomb's Law (1.1):



$$\mathbf{F}_{2\,due\,to\,1} = k_1 \frac{q_1 q_2}{r_{21}^2}\hat{r}_{21} = k_1 \frac{q_1 q_2}{\left|\mathbf{x}_2-\mathbf{x}_1\right|^3}\left(\mathbf{x}_2-\mathbf{x}_1\right)$$

- Ampère (Biot-Savart) (1.2):



$$\mathbf{F}_{2\,due\,to\,1} = k_2 \frac{\left(I_2 d\mathbf{l}_2\right)\times\left(I_1 d\mathbf{l}_1 \times \hat{r}\right)}{r^2}$$

$$\frac{dF}{dl} = k_2 \frac{2 I_1 I_2}{d}$$

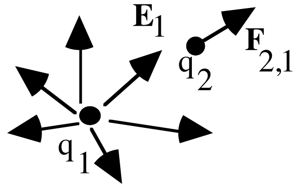
(1.2)

The constants k_1 and k_2 determine the systems of units. That is, they relate the units of *charge* and *current* to the mechanical unit of force (see below).

I.C Abstraction : Electric and Magnetic Fields

Electric and magnetic forces act at a distance between charges and currents. This model is perfectly consistent. We can consider instead, however, the charges and currents as sources of electric and magnetic fields, produced locally, and then acting locally on distant charges and currents. In the static case, this notion is a matter of convenience, but in the dynamic case we will see the fields taking on a *reality* of their own - they can detach from the source and radiate to infinity carrying energy, momentum, and angular momentum with them! The notion of conservation laws in electromagnetism will play a central role in this course.

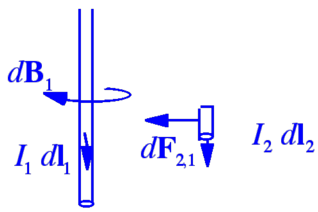
• Electric field (static definition): Force on a "test charge" q_2 per unit charge.



$$\mathbf{F}_{2,1} = q_2 \mathbf{E}_1(\mathbf{x}_2)$$

$$\Rightarrow \mathbf{E}_1(\mathbf{x}_2) = k_1 \frac{q_1}{|\mathbf{x}_2 - \mathbf{x}_1|^3}(\mathbf{x}_2 - \mathbf{x}_1) \quad (1.3)$$

• Magnetic field (static definition):



$$d\mathbf{F}_{2,1} \equiv \frac{1}{\alpha}(I_2 d\mathbf{l}_2) \times d\mathbf{B}_1$$
$$d\mathbf{B}_1(\mathbf{x}_2) = \alpha k_2 \frac{(I_1 d\mathbf{l}_1) \times \hat{r}}{r^2} \quad (1.4)$$

The constant α fixes the unit of magnetic field relative to other units (see below).

I.D Units

Nothing plagues a discussion of electromagnetic theory like the choice of units. Of course we will use the "metric system". Here there are two usual choices - CGS (centimeters-gram-seconds) and MKS (meter-kilogram-seconds). The former is the system used by us "old timers" with the unit of force being the "dyne" and energy the "erg", whereas the MKS system has become the standard "SI" with the unit of force "Newtons" and energy "Joules". The issue is to relate the units of force from mechanics (mass times acceleration) to that of electric and magnetic forces. There are three parameters: k_1, k_2, and α. Note, however that *two are related by experiment*. The current I is the flow of charge/unit time. Thus, for any choice of unit for charge, by Eqs. (1.1) and (1.2), the constants k_1 and k_2 are related by,

$$\frac{k_1}{k_2} = c^2, \quad (1.5)$$

where c is a constant with unit of velocity. In fact $c \approx 3(\times 10^{10}\, cm/s$: CGS or $\times 10^{8}\, m/s$: MKS). Of course this is recognized as the speed of light. Amazingly, this comes out of SOLELY from static force measurements.

There are two "simple" but archaic choices of units:

• Electrostatic Units (ESU): $k_1 = 1 \Rightarrow k_2 = 1/c^2, \alpha = 1$

• Electromagnetic Units (EMU): $k_2 = 1 \Rightarrow k_1 = c$ $k_1 = 1 \Rightarrow k_2 = 1/c^2, \alpha = 1$

The standard for these older system of units is CGS. The unit of charge in ESU is called an "esu". It itself is related to the fundamental units as: $1\,\text{esu} = \sqrt{\text{dyne}\cdot\text{cm}^2} = \sqrt{\text{g}\cdot\text{cm}^3\cdot\text{s}^{-2}}$. In the EMU system one defines a new unit - the current is measured in abamps (which used to be called "absolute amperes", but that name is recalimed in the SI system). The definition is given as follows: given two parallel currents separated by one centimeter, the force per distance of wire is 2 dyne/cm. The abcoulomb is the charge flowing in 1 sec. given 1 abamp.

A third choice of units is the one we will use:

• Gaussian (mixed units): $k_1 = 1 \Rightarrow k_2 = 1/c^2, \alpha = c$: CGS units

This system, is very similar to ESU except that the magnetic for the choice of α. This choice is "natural" because then the **E** and **B** fields have the SAME UNITS. This makes good physical sense since we know from relativity that the **E** and **B** fields are in fact two different manifestations of the SAME THING. *Of course* they should have the same units.

A fourth choice is SI, or "Système International" (since the French make the rules):

• SI or Rationalized MKSA: $k_1 \equiv \dfrac{1}{4\pi\varepsilon_0}, k_2 \equiv \dfrac{\mu_0}{4\pi} \Rightarrow \dfrac{1}{\mu_0\varepsilon_0} = c^2, \alpha = 1$: MKS units

The "rationalized" has to do with the normalization by 1/4π, which eliminates this factor from Maxwell's Equations. The "A" in "MKSA" stands for Amp. Like the EMU system, a NEW unit of current is defined, A=ampère = 0.1 abamps. The unit of charge is then the coulomb: 1 C = 1 Amp·sec. The constants ε_0, μ_0 are known respectively as the "permitivity" and "permeability" of free space. These are really archaic terms harking back to the idea of the "ether" as a kind of substance with material properties (another reason I do not like SI units for E&M). In the SI system, the speed of light is *defined* as $c = 2.998792458 \times 10^8$ m/s. Length and time are then defined experimentally through the atomic clock cesium-standard. The permeability of free space is defined $\mu_0 / 4\pi = 10^{-7}$ N/A^2, and thus $\varepsilon_0 = 10^7/(4\pi c^2)$.

The SI system makes use of the familiar historical units: Amps, Volts, Coulomb, etc. However, in theoretical analysis it is cumbersome, especially when dealing with relativity. This is evident by the fact that although in the 3rd edition of Jackson, the first half of the book was converted to SI units, the second "relativistic" half was left in CGS-Gaussian units. This is a mess, but get used to it. One challenge will be for you to switch back and forth between the two systems when necessary. We will be using exclusively Gaussian units in class.

One quick conversion trick for going between Gaussian and SI is to substitute: $\varepsilon_0 \Leftrightarrow \frac{1}{4\pi}$, $\mu_0 \Leftrightarrow \frac{4\pi}{c}$. Note that these conversions take into account the different choice of α. This holds for "free space". All hell breaks loose for the macroscopic Maxwell equations in dielectrics and magnetic materials. For a complete set of "rules" for converting between units and a detailed background, see J.D. Jackson, "Appendix on Units and Dimensions".

I.D Maxwell's Equations:

The whole of electromagnetism is summarized in Maxwell's Equations, given here in both integral and differential form for the general set of units

Gauss' Law:	$\oint_S \mathbf{E}\cdot d\mathbf{a} = 4\pi k_1 Q_{enc} = 4\pi k_1 \int_V \rho\, d^3x$	$\nabla\cdot\mathbf{E} = 4\pi k_1 \rho(\mathbf{x})$
No magnetic monopoles:	$\oint_S B\cdot d\mathbf{a} = 0$	$\nabla\cdot\mathbf{B} = 0$
Faraday's Law:	$\oint_C E\cdot d\mathbf{l} = -\frac{1}{\alpha}\frac{d}{dt}\int \mathbf{B}\cdot d\mathbf{a}$	$\nabla\times\mathbf{E} = -\frac{1}{\alpha}\frac{\partial\mathbf{B}}{\partial t}$
Ampère-Maxwell Law:	$\oint_C B\cdot d\mathbf{l} = \int_S \left(4\pi k_2 \alpha \mathbf{J} + \frac{k_2\alpha}{k_1}\frac{\partial\mathbf{E}}{\partial t}\right)\cdot d\mathbf{a}$	$\nabla\times\mathbf{B} = 4\pi k_2\alpha\mathbf{J} + \frac{k_2\alpha}{k_1}\frac{\partial\mathbf{E}}{\partial t}$

The differential form relates the **E** and **B** fields to each other and to the local sources, the charge density ρ and the current density **J**. The integral forms follow from the theorems of vector calculus. These equations are then supplemented by the "Lorentz Force Law", which tells us how the charges are effected by the fields

$$\mathbf{F} = q\mathbf{E} + q\frac{\mathbf{v}}{\alpha}\times\mathbf{B}\,.$$

Gaussian Units:

$k_1 = 1, k_2 = 1/c^2, \alpha = c$

Gauss' Law:	$\oint_S \mathbf{E}\cdot d\mathbf{a} = 4\pi Q_{enc} = 4\pi \int_V \rho\, d^3x$	$\nabla\cdot\mathbf{E} = 4\pi\rho(\mathbf{x})$
No magnetic monopoles:	$\oint_S B\cdot d\mathbf{a} = 0$	$\nabla\cdot\mathbf{B} = 0$
Faraday's Law:	$\oint_C E\cdot d\mathbf{l} = -\frac{1}{c}\frac{d}{dt}\int \mathbf{B}\cdot d\mathbf{a}$	$\nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}$
Ampère-Maxwell Law:	$\oint_C B\cdot d\mathbf{l} = \int_S \left(\frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}\right)\cdot d\mathbf{a}$	$\nabla\times\mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}$

$$\mathbf{F} = q\mathbf{E} + q\frac{\mathbf{v}}{c}\times\mathbf{B}.$$

SI Units:

$k_1 = \frac{1}{4\pi\varepsilon_0}, k_2 = \frac{\mu_0}{4\pi} \Rightarrow \frac{1}{\mu_0\varepsilon_0} = c^2; \quad \alpha = 1$

Gauss' Law:	$\oint_S \mathbf{E}\cdot d\mathbf{a} = \frac{Q_{enc}}{\varepsilon_0} = \int_V \frac{\rho}{\varepsilon_0} d^3x$	$\nabla\cdot\mathbf{E} = \frac{\rho(\mathbf{x})}{\varepsilon_0}$
No magnetic monopoles:	$\oint_S B\cdot d\mathbf{a} = 0$	$\nabla\cdot\mathbf{B} = 0$
Faraday's Law:	$\oint_C E\cdot d\mathbf{l} = -\frac{d}{dt}\int \mathbf{B}\cdot d\mathbf{a}$	$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$
Ampère-Maxwell Law:	$\oint_C B\cdot d\mathbf{l} = \int_S \left(\mu_0\mathbf{J} + \mu_0\varepsilon_0\frac{\partial\mathbf{E}}{\partial t}\right)\cdot d\mathbf{a}$	$\nabla\times\mathbf{B} = \mu_0\mathbf{J} + \mu_0\varepsilon_0\frac{\partial\mathbf{E}}{\partial t}$

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v}\times\mathbf{B}.$$

Our goal for the semester is to understand the meaning of these equations, their implications, and to study the rich variety of phenomena that flow directly from them.