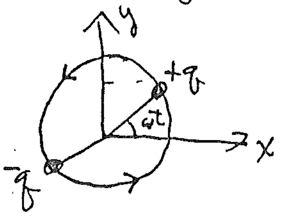
Physics 511. Problem Set #9 Solutions

(1) Radiation by a rotating electric dipole



$$\vec{p}(t) = p_0 \cos(\omega t)\,\hat{x} + p_0 \sin(\omega t)\,\hat{y}$$

Using the principle of superposition, we can add the dipole radiation associated with the dipole along x to that for the dipole along y.

For a monochromatic oscillating dipole, we have the complex amplitude for the electric field

$$\underset{\sim}{\vec{E}} = k^2 \frac{e^{ikr}}{r} \underset{\sim}{\vec{p}}_{0\perp} \qquad \text{where } \vec{p}(t) = \mathrm{Re}(\underset{\sim}{\vec{p}_0}\, e^{-i\omega t})$$

$\Leftarrow$ component $\perp$ to direction of observation

$$= k^2 \frac{e^{ikr}}{r} \left(\underset{\sim}{\vec{p}_0} - \hat{r}\,(\hat{r}\cdot \underset{\sim}{\vec{p}_0}) \right)$$

In this case: $\vec{p}_0(t) = \mathrm{Re}\left(p_0 e^{-i\omega t}\,\hat{x} + i p_0 e^{-i\omega t}\,\hat{y}\right)$

$\Uparrow$ 90° out of phase

$$\Rightarrow \underset{\sim}{\vec{E}} = k^2 \frac{e^{ikr}}{r} p_0 \left\{ \left(\hat{x} - \hat{r}(\hat{r}\cdot\hat{x})\right) + i\left(\hat{y} - y(\hat{r}\cdot\hat{y})\right) \right)$$

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Cartesian basis vectors in terms of those in spherical coordinates (which depends upon the point of observation)

$$\hat{x} = \sin\theta\cos\phi\,\hat{r} + \cos\theta\cos\phi\,\hat{\theta} - \sin\phi\,\hat{\phi}$$

$$\Rightarrow \hat{x} - \hat{r}(\hat{x}\cdot\hat{r}) = \cos\theta\cos\phi\,\hat{\theta} - \sin\phi\,\hat{\phi}$$

$$\hat{y} = \sin\theta\sin\phi\,\hat{r} + \cos\theta\sin\phi\,\hat{\theta} + \cos\phi\,\hat{\phi}$$

$$\Rightarrow \hat{y} - \hat{r}(\hat{y}\cdot\hat{r}) = \cos\theta\sin\phi\,\hat{\theta} + \cos\phi\,\hat{\phi}$$

Plug into the expression for the electric field complex amplitude

$$\Rightarrow \vec{\underset{\sim}{E}} = k^2 p_0 \frac{e^{ikr}}{r}\Big((\cos\theta\cos\phi + i\cos\theta\sin\phi)\,\hat{\theta} + (-\sin\phi + i\cos\phi)\,\hat{\phi} \Big)$$

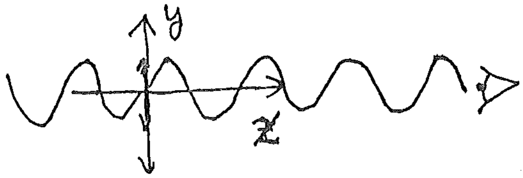
$$= k^2 p_0 \frac{e^{ikr}}{r}\Big(\cos\theta\,(\underbrace{\cos\phi + i\sin\phi}_{=e^{i\phi}})\,\hat{\theta} + i\,(\underbrace{\cos\phi + i\sin\phi}_{=e^{i\phi}})\,\hat{\phi} \Big)$$

$$\Rightarrow \boxed{\underset{\sim}{\vec{E}} = k^2 p_0 \left(\cos\theta\,\hat{\theta} + i\,\hat{\phi}\right) \frac{e^{i(kr+\phi)}}{r}}$$

The phase of the wave $kr + \phi = \omega\frac{r}{c} + \phi$, where (r, ϕ) are the coordinates of the observer (in the radiation zone). This makes sense: The ~~fist~~ first term is the retarded time effect. The phase of oscillation should depend on the phi coordinate because the dipole in rotating in the ϕ-direction. Thus observers at different ϕ will see the wave at a different phase of the oscillation.

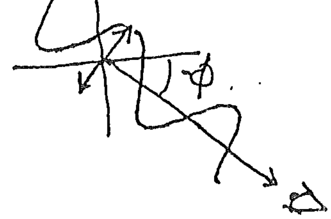
Comment on wave polarization

An observer on the x-axis sees only the projection of the rotating dipole along the y-axis, and vice-versa for observers on the y-axis



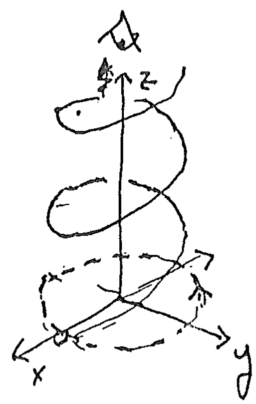
Generally, any where in the x-y plane

$$\vec{E}(\vec{r},t) = \frac{1}{4\pi\epsilon_0} k^2 p_0 \cos(\omega t - k\sqrt{x^2+y^2} + \phi)\,\hat{\phi}$$



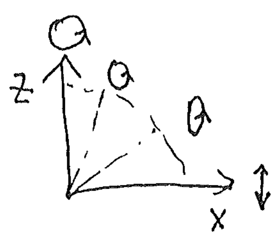
Linear polarization along $\hat{\phi}$, phase of oscillation depends on ϕ

On the z-axis

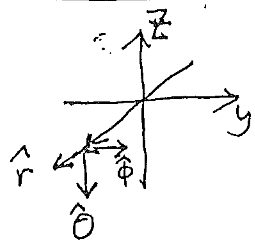


Traveling helix (right hand cork screw)
$\Rightarrow$ R.H. circular polarization

For an arbitrary point not along one of the axes the polarization is <u>elliptical</u> since the observer does not see an equal projection of oscillations along the two directions $\perp$ to the direction of observation



On the x-axis: $\theta = \frac{\pi}{2}, \ \phi = 0, \ y = z = 0 \Rightarrow r = x$



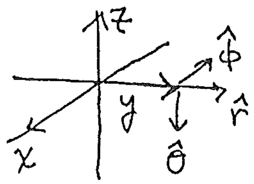
$$\hat{r} = \hat{x}, \quad \hat{\theta} = -\hat{z}, \quad \hat{\phi} = \hat{y}$$

$$\Rightarrow \vec{E}(\vec{r},t) = k^2 p_0 \, \mathrm{Re}\left(i \, \hat{y} \, \frac{e^{i(kx-\omega t)}}{x}\right)$$

$$\Rightarrow \boxed{\vec{E}(\vec{r},t) = k^2 p_0 \frac{\sin(\omega t - kx)}{x} \hat{y}}$$

Linear polarization along $\hat{y}$

On the y-axis: $\theta = \frac{\pi}{2}, \ \phi = \frac{\pi}{2}, \ x = z = 0 \Rightarrow r = y$



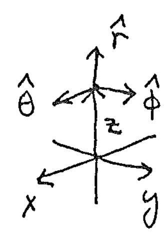
$$\hat{r} = \hat{y}, \quad \hat{\theta} = -\hat{z}, \quad \hat{\phi} = -\hat{x}$$

$$\Rightarrow \vec{E}(\vec{r},t) = k^2 p_0 \, \mathrm{Re}\left(-i \, \hat{x} \, \frac{e^{i(ky-\omega t)}}{y} \underbrace{e^{i\pi/2}}_{=i}\right)$$

$$\Rightarrow \boxed{\vec{E}(\vec{r},t) = k^2 p_0 \frac{\cos(\omega t - ky)}{y} \hat{x}}$$

Linear polarization along $\hat{x}$

On the z-axis $\theta = 0, \ \phi = 0, \ x = y = 0 \Rightarrow r = z$



$$\Rightarrow \hat{r} = \hat{z}, \quad \hat{\theta} = \hat{x}, \quad \hat{\phi} = \hat{y}$$

$$\Rightarrow \vec{E}(\vec{r},t) = k^2 p_0 \, \mathrm{Re}\left[(\hat{x} + i\hat{y}) \frac{e^{ikz}}{z} e^{-i\omega t}\right]$$

$$\Rightarrow \boxed{\vec{E}(\vec{r},t) = k^2 p_0 \left(\frac{\cos(\omega t - kz)\,\hat{x} + \sin(\omega t - kz)\,\hat{y}}{z}\right)}$$

Right hand circular polarization

(c) The intensity

$$\langle|\vec{S}|\rangle = \frac{c}{8\pi}\,\underset{\sim}{\vec{E}}^{*}\cdot\underset{\sim}{\vec{E}} = \frac{c}{8\pi}\left(k^4\, p_0^2\,(\cos^2\theta+1)\,\frac{1}{r^2}\right)$$

$\Rightarrow$ differential Power / solid angle :
$$\boxed{\frac{d\mathcal{P}}{d\Omega} = \langle|\vec{S}|\rangle r^2 = \frac{ck^4}{8\pi}\, p_0^2\,(1+\cos^2\theta)}$$

The momentum flux density radiated to infinity

$$\frac{d\vec{P}}{dt\,d\Omega} = -\left(\overleftrightarrow{T}\cdot\hat{r}\right)r^2, \qquad \overleftrightarrow{T} = \frac{1}{4\pi}\left(\vec{E}\vec{E}+\vec{B}\vec{B}-\frac{\overleftrightarrow{\mathbb{1}}}{2}(E^2+B^2)\right)$$

(Maxwell stress tensor)

$$\frac{d\vec{P}}{dt\,d\Omega} = \frac{\hat{r}}{8\pi}(E^2+B^2)$$

Time average: $\langle E^2\rangle = \langle B^2\rangle = \frac{1}{2}\,\underset{\sim}{\vec{E}}^{*}\cdot\underset{\sim}{\vec{E}}$

$$\Rightarrow \boxed{\frac{d\langle\vec{P}\rangle}{dt\,d\Omega} = \left(\frac{r^2}{8\pi}\,\underset{\sim}{\vec{E}}^{*}\cdot\underset{\sim}{\vec{E}}\right)\hat{r} = \frac{\langle\vec{S}\rangle r^2}{c}}$$

This make sense knowing what we do about quantum mechanics

Energy/photon $\hbar\omega$

momentum/photon $= \hbar\vec{k} = \frac{\hbar\omega}{c}\hat{k} = \frac{U}{c}\hat{k}$

Now what about the angular momentum?

This one is very tricky. First recall angular momentum flux density

$$\vec{\mathcal{L}} = \vec{r} \times \overleftrightarrow{T} \cdot \hat{r}$$

$\Rightarrow$ Angular momentum flowing out through patch of a sphere $d\vec{a} = r^2 d\Omega\, \hat{r}$

$$\frac{d\vec{L}}{dt} = -\vec{\mathcal{L}} \cdot d\vec{a} = -\left[\vec{r} \times (\overleftrightarrow{T} \cdot \hat{r})\right] \cdot \hat{r}\, r^2 d\Omega$$

$$= -\left[\hat{r} \times (\overleftrightarrow{T} \cdot \hat{r})\right] \cdot \hat{r}\, \underline{\underline{r^3}}\, d\Omega$$

The factor of r^3 here means that we must retain terms in the stress tensor up to $\frac{1}{r^3}$

$\Rightarrow$ Interference of induction and radiation fields contribute to the flow of angular momentum.

In fact, the radiation terms alone carry no angular momentum as we will see

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Thus, to $\frac{1}{r^3}$, the stress tensor is

$$\overleftrightarrow{T} = \frac{1}{4\pi}\Big[(\vec{E}_{rad}+\vec{E}_{ind})(\vec{E}_{rad}+\vec{E}_{ind}) - \frac{1}{2}\overleftrightarrow{\mathbb{1}}\left(|\vec{E}_{rad}+\vec{E}_{ind}|^2 + |\vec{B}_{rad}+\vec{B}_{ind}|^2\right) + (\vec{B}_{rad}+\vec{B}_{ind})(\vec{B}_{rad}+\vec{B}_{ind})\Big]$$

$$= \frac{1}{4\pi}\left[\vec{E}_{rad}\vec{E}_{rad} + \vec{B}_{rad}\vec{B}_{rad} - \frac{1}{2}\overleftrightarrow{\mathbb{1}}\left(|\vec{E}_{rad}|^2 + |\vec{B}_{rad}|^2\right)\right]$$

$$+ \frac{1}{4\pi}\left[\vec{E}_{rad}\vec{E}_{ind} + \vec{E}_{ind}\vec{E}_{rad} - \overleftrightarrow{\mathbb{1}}\left(\vec{E}_{rad}\cdot\vec{E}_{ind} + \vec{B}_{rad}\cdot\vec{B}_{ind}\right) + \vec{B}_{rad}\vec{B}_{ind} + \vec{B}_{ind}\vec{B}_{rad}\right]$$

where $\vec{E}_{rad}, \vec{B}_{rad}$ are the "radiation fields" $\sim \frac{1}{r}$

$\vec{E}_{ind}, \vec{B}_{ind}$ are the "induction fields" $\sim \frac{1}{r^2}$

Let us consider just the radiation term to start

$$\hat{r}\times\overleftrightarrow{T}_{rad\text{-}rad}\cdot\hat{r} = \frac{1}{4\pi}\Big[(\hat{r}\times\vec{E}_{rad})\cancelto{0}{(\hat{r}\cdot\vec{E}_{rad})} + (\hat{r}\times\vec{B}_{rad})\cancelto{0}{(\hat{r}\cdot\vec{B}_{rad})} - \frac{1}{2}\cancelto{0}{\hat{r}\times\hat{r}}\left(|\vec{E}_{rad}|^2 + |\vec{B}_{rad}|^2\right)\Big]$$

Thus, as promised the purely radiation piece carries no angular momentum

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Now for the interference term:

$$\hat{r} \times \overleftrightarrow{T}_{rad\text{-}ind} \cdot \hat{r} = \frac{1}{4\pi}\Big[(\hat{r} \times \vec{E}_{rad})(\hat{r} \cdot \vec{E}_{ind})$$
$$+ (\hat{r} \times \vec{B}_{rad})(\hat{r} \cdot \vec{B}_{ind})\Big]$$

Where I have used $\hat{r} \cdot \vec{E}_{rad} = \hat{r} \cdot \vec{B}_{rad} = 0$

Now, what about the induction field?
From class, and Jackson 9.18

$$\left\{\begin{aligned} \vec{\underset{\sim}{B}}_{ind} &= ik\,(\hat{r} \times \vec{\underset{\sim}{p}}_0)\,\frac{e^{ikr}}{r^2} \\ \vec{\underset{\sim}{E}}_{ind} &= \left[3\hat{r}(\hat{r} \cdot \vec{\underset{\sim}{p}}_0) - \vec{\underset{\sim}{p}}_0\right]\left[\frac{ike^{ikr}}{r^2}\right] \end{aligned}\right\}$$

where $\vec{B}_{ind} = Re(\vec{\underset{\sim}{B}}_{ind}\, e^{-i\omega t_0})$

$t_0 = t - \frac{r}{c}$

etc.

Thus, once we take time average

$$\hat{r} \times \langle \overleftrightarrow{T}_{rad\text{-}ind} \rangle \cdot \hat{r} = \frac{Re}{8\pi}\left[(\hat{r} \times \vec{\underset{\sim}{E}}^{*}_{rad})(\hat{r} \cdot \vec{\underset{\sim}{E}}_{ind})\right]$$

Having used our favorite rule $\langle f(t) g(t) \rangle = \frac{1}{2} Re\,(\tilde{f}^{*} \tilde{g})$
for sinusoidal oscillating functions

And also $\underset{\sim}{B}_{ind} \cdot \hat{r} = 0$

Plugging in: $$Re\left[(\hat{r} \times \vec{\underset{\sim}{E}}^{*}_{rad})(\hat{r} \cdot \vec{E}_{ind})\right] = \frac{2k^3}{r^3}\, Re\left[-i(\hat{r} \times \vec{\underset{\sim}{p}}_0^{*})(\hat{r} \cdot \vec{\underset{\sim}{p}}_0)\right]$$

$$= \frac{2k^3}{r^3}\, Im\left[(\hat{r} \times \vec{\underset{\sim}{p}}_0^{*})(\hat{r} \cdot \vec{\underset{\sim}{p}}_0)\right]$$

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Aside: Recall $\underset{\sim}{\vec{p}_0} = p_0(\hat{x}+i\hat{y}) = p_0 e^{i\phi}(\sin\theta\,\hat{r} + \cos\theta\,\hat{\theta} + i\hat{\phi})$

when $\hat{x}, \hat{y}$ are expressed in $\hat{r}, \hat{\theta}, \hat{\phi}$

$$\Rightarrow \operatorname{Im}\left[(\hat{r}\times\vec{p}_0^{\,*})(\hat{r}\cdot\vec{p}_0)\right] = p_0^2 \sin\theta\,\hat{\theta}$$

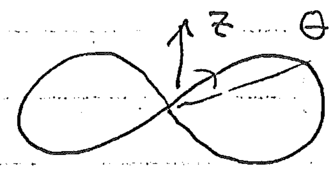
$$\Longrightarrow \quad \hat{r}\times\langle\overleftrightarrow{T}_{rad-ind}\rangle\cdot\hat{r} = \frac{2k^3 p_0^2}{8\pi r^3}\sin\theta\,\hat{\theta}$$

Rate of flow of angular momentum to ∞ / solid angle

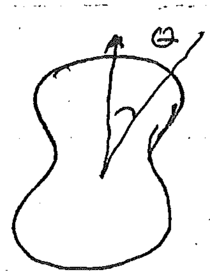
$$\Longrightarrow \boxed{\frac{d\langle\vec{L}\rangle}{dt\,d\Omega} = -\left(\hat{r}\times\langle\overleftrightarrow{T}\rangle_{rad-ind}\cdot\hat{r}\right) r^3 = -\frac{k^3 p_0^2 \sin\theta}{4\pi}\,\hat{\theta}}$$

Z-component $\quad \hat{\theta} = \cos\theta\,\hat{\rho} - \sin\theta\,\hat{z}$

$$\Rightarrow \frac{d\langle L_z\rangle}{dt\,d\Omega} = \frac{k^4 p_0^2 \sin^2\theta}{4\pi}$$

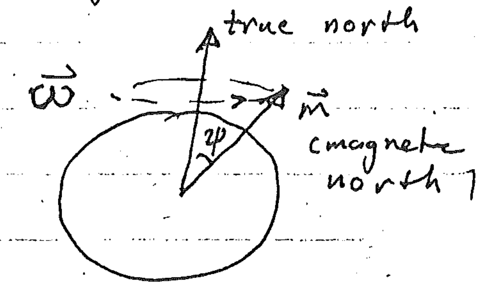


$$\frac{d\,\text{Energy}}{dt\,d\Omega} \sim (1+\cos^2\theta)$$



The energy radiated in the z-direction is <u>twice</u> that in the x-y plane because along z <u>both</u> $\hat{x}$ and $\hat{y}$ directions of oscillation contribute

Problem 2   Magnetic Dipole Radiation



Rotating neutron star ⇒ pulsar

$$\vec{m}(t) = m_0 \cos\psi\, \hat{z} + \underbrace{m_0 \sin\psi \left(\cos\omega t\, \hat{x} + \sin\omega t\, \hat{y}\right)}_{\text{rotating component}}$$

This rotating magnetic dipole will radiate energy at the rate

$$\mathcal{P}_{rad} = \frac{2\langle|\ddot{\vec{m}}|^2\rangle}{3c^3} = \frac{2m_0^2 \sin^2\psi}{3c^3}\omega^4$$

From Problem set # 4, Problem 3, we have the magnetic field at the surface of a sphere with mag. moment and radius R:

$$B_0 = \frac{2m_0}{R^3}$$

$$\Rightarrow \mathcal{P}_{rad} = \frac{1}{3}\sin^2\psi \; \frac{B_0^2 R^6 \omega^4}{c^3}$$

Units:
B^2 = Energy/Volume
$\Rightarrow (B^2R^3\omega)\left(\frac{\omega R}{c}\right)^3$ = Power

To estimate, we are given

$R \sim 10^4\,\text{m} = 10^6\,\text{cm}$, $\quad B_0 \sim 10^8\,\text{Tesla} = 10^{12}\,\text{Gauss}$

Rotation period: $T = 10^{-3}\,\text{sec} \Rightarrow \omega = \frac{2\pi}{T}$

Take $\psi \sim 10$ degrees ~ 0.1 rad

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Thus, the rate of energy loss

$$\mathcal{P}_{rad} \cong \frac{1}{3}(0.1)^2 \left[10^{24} \frac{erg}{cm^3}\right] (10^{36}\, cm^6) \frac{(2\pi \times 10^3)^4 s^{-4}}{(3\times 10^{10})^3 \frac{cm^3}{s^3}}$$

$\Rightarrow$ Order of magnitude $\boxed{\mathcal{P} \sim 10^{41} \frac{erg}{s} \sim \frac{10^{34}}{\pi} \frac{erg}{year}}$

(1 year ~ $\pi \times 10^7$ sec)

Spin down rate: $\frac{d\mathcal{E}}{dt} = -\frac{d}{dt}\left(\frac{1}{2} m\omega^2 R^2\right) = -m\omega R^2 \dot{\omega}$
$= \mathcal{P}_{rad}$

$$\Rightarrow \dot{\omega} = -\frac{\mathcal{P}_{rad}}{m\omega R^2}$$

Let us calculate the spin down rate at $t=0$

$$\frac{\dot{\omega}}{\omega}(0) = -\frac{\mathcal{P}_{rad}}{m\omega_0^2 R^2} \cong -\frac{10^{41}}{(1.5\times 10^{33} g)\left(\frac{2\pi}{10^{-3} s^2}\right)(10^6 cm)^2}$$

$$\Rightarrow \frac{\dot{\omega}}{\omega}(0) \sim 1.6 \times 10^{-12} s^{-1} \equiv \frac{1}{\tau_{life}}$$

$$\Rightarrow \boxed{\tau_{life} \sim 0.6 \times 10^{12} s \sim 1.9 \times 10^4 \text{ years}}$$

Problem 3 Radiation from hydrogen atom

A hydrogen atom in a superposition of 1s + 2p has a nonzero electric dipole momentum that will radiate. The purpose of this problem is to show that the rate at which energy is carried off by E-M radiation gives us a prediction of the spontaneous decay rate of the excited 2p state.

The effective charge density associated with the electron

$$\rho(r,\theta,t) = Re\left(-e\,\psi^*_{2p,0}(\vec{x},t)\,\psi_{1s}(\vec{x},t)\right) = Re\left(\frac{2e}{\sqrt{6}} a_0^{-4}\, r\, e^{-3r/2a_0}\; Y_{00}\, Y_{1,0}(\theta)\, e^{-i\omega_0 t}\right)$$

(Note typo in problem)

where $\omega_0 = \frac{E_{2p} - E_{1s}}{\hbar}$ = Bohr frequency

(a) The multipole moments (electric)

$$q_{\ell,m}(t) = \int \underbrace{d^3x}_{r^2 dr d\Omega}\; \rho(r,\theta,t)\, r^\ell\, Y^*_{\ell,m}(\theta,\phi)$$

Because $Y_{00} = \frac{1}{\sqrt{4\pi}}$, constant

and $\int d\Omega\, Y_{\ell,m}(\theta,\phi)\, Y^*_{\ell',m'}(\theta,\phi) = \delta_{\ell,\ell'}\,\delta_{m,m'}$

the only nonvanish moment is the electric dipole component $\ell=1$, $m=0$

$$\tilde{q}_{1,0}(t) = \frac{2e}{\sqrt{6}}\frac{a_0^{-4}}{\sqrt{4\pi}} \int_0^\infty r^4 e^{-3r/2a_0}\,dr\; e^{-i\omega_0 t}$$

Aside: $\int_0^\infty \frac{r^4}{a_0^4} e^{-3r/2a_0} dr = \frac{256}{81} a_0$

$$\Rightarrow \boxed{\tilde{q}_{10} = \frac{1}{\sqrt{6}\pi} \frac{256}{81} e a_0}$$

There are no other electric multiple moment

Note: There ~~are~~ is a non-zero magnetic dipole moment. However, because $ka_0 << 1$ this can be neglected

(b) In the electric dipole approximation we neglect the magnetic dipole contribution to radiation.

First note $\tilde{q}_{10} = \sqrt{\frac{3}{4\pi}}\, p_z \leftarrow$ z-component of electric dipole

According to the Larmor formula

$$P = \overline{\text{Power radiated}} = \frac{\omega_0^4}{3c^3} p_0^2$$

$$= \frac{\omega^4}{3c^3} \left(\frac{\sqrt{2}}{3} \frac{256}{81}\right)^2 (e a_0)^2$$

$$= \left(\frac{2^{17}}{3^{11}}\right)\left(\frac{\omega^4}{c^3}\right)(e a_0)^2$$

Now, for the hydrogen energy levels

$$E_n = -\frac{1}{2n^2}\frac{e^2}{a_0}, \qquad n = \text{principle quantum \#}$$

$$\Rightarrow \hbar\omega_0 = E_{2p} - E_{1s} = \frac{3}{8}\frac{e^2}{a_0} \Rightarrow \omega_0 = \frac{3}{8}\frac{e^2}{a_0}$$

Express in units of $\hbar\omega_0 \frac{\alpha^4 c}{a_0}$, $\alpha = \frac{e^2}{\hbar c} = \frac{1}{137}$

(Fine structure)

$$\Rightarrow \frac{P}{\hbar\omega_0\left(\frac{\alpha^4 c}{a_0}\right)} = \left(\frac{2^{17}}{3^{11}}\right)\frac{\omega_0^3}{\hbar c^4}\frac{e^2 a_0^3}{\alpha^4}$$

$$= \left(\frac{2^{17}}{3^{11}}\right)\left(\frac{3}{8}\right)^3 \frac{1}{\hbar}\left(\frac{e^2}{a_0\hbar}\right)^3\left(\frac{e^2 a_0^3}{c^4}\right)\left(\frac{\hbar c}{e^2}\right)^4 = \left(\frac{2}{3}\right)^8$$

$$\Rightarrow \boxed{\text{Power radiated} = \left(\frac{2}{3}\right)^8 \frac{\alpha^4 c}{a_0}\hbar\omega_0}$$

Numerically with $\alpha = \frac{1}{137}$ $a_0 = 0.5\,\text{Å}$

$$\boxed{P \cong 1 \text{ nWatt}}$$

(c) The spontaneous emission ~~radiate~~ rate Γ

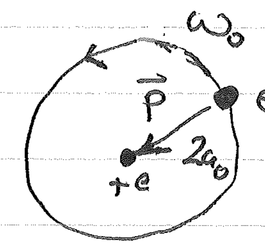
$$\text{Power radiated} = \Gamma \hbar \omega_0$$

$$\Rightarrow \Gamma = \frac{P}{\hbar\omega_0} = \left(\frac{2}{3}\right)^8 \frac{\alpha^4 c}{a_0} = 6.27 \times 10^8 s^{-1}$$

$$\Rightarrow \text{Lifetime } \boxed{\tau = \frac{1}{\Gamma_{\text{Quant}}} = 1.59 \text{ ns} = \text{Quantum value!}}$$

Note: Quantum mechanically this is exact because the magnetic dipole cannot contribute by the parity selection rule.

(d) We now look at the classical radiation of the point-particle electron orbiting the nucleus.



The total power radiated is the sum of the powers radiated into all modes.

From P.S #1 $\quad \frac{dP}{d\Omega} = \frac{ck^4}{8\pi} p_0^2 (1 + \cos^2\theta)$

$$\Rightarrow P_{\text{class}} = \int d\Omega \frac{dP}{d\Omega} = \frac{2\omega_0^4}{3c^3} p_0^2$$

$$= \left(\frac{2^3}{3}\right) \left(\omega_0^4 / c^3\right) (ea_0)^2$$