Physics 511
Electrodynamics

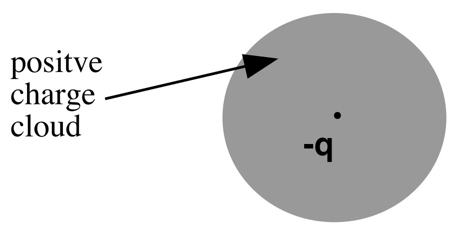
Problem Set #2: Due Thurs. Feb. 6

(1) The "Yukawa Potential" (20 Points)

The electric force dominates the interaction of particles because it is very long-range; i.e. the electrostatic potential falls off as an algebraic function of distance 1/r. In certain circumstances we would like to “cut-off” the Coulomb field after some distance. This can be achieved using the “Yukawa” potential.

$$\Phi(r) = G\,\frac{ae^{-r/a}}{r},$$

where G is some coupling constant, and a is a distance. This potential can be mimicked in electrostatics by a point charge q at the origin and "screened" by a smeared out negative charge distribution: $\rho(\mathbf{x}) = -q\delta^{(3)}(\mathbf{x}) + \frac{q}{4\pi a^2}\frac{e^{-r/a}}{r}$. (Here a is a constant with dimensions of length)



(a) Sketch Φ(r) showing the asymptotic form for r<<a and r>>a. Explain the physical significance of the constant a.

(b) Show that the overall charge distribution is neutral. Does this make sense given the long range form of the potential?

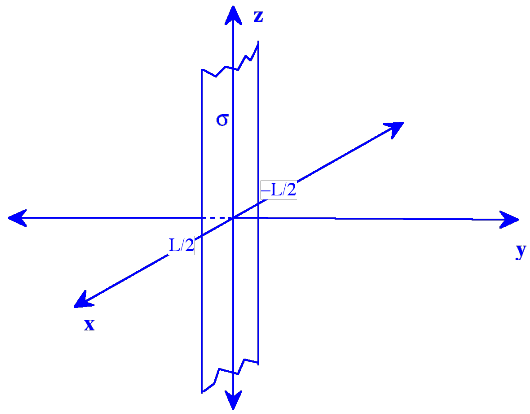
(c) Derive the Yukawa potential for this charge distribution.

(*Hint* Use Gauss' law to find the electric field first)

(d) What is the potential energy stored in this charge distribution? Explain your result.

(2) Potentials and contours (20 points)

An infinitely long strip of width L (shown below) carries a charge per unit area σ



(a) Show that the electrostatic potential and the electric field in the *x*-*y* plane is

$$\phi(x,y) = -\sigma \left[2y \left(\tan^{-1}\left(\frac{x+L/2}{y}\right) - \tan^{-1}\left(\frac{x-L/2}{y}\right)\right) + (x+\frac{L}{2})\log\left((x+\frac{L}{2})^2+y^2\right) - (x-\frac{L}{2})\log\left((x-\frac{L}{2})^2+y^2\right)\right] + C$$

C = constant.

Possibly useful integral:

$$\int du\ \log(u^2+v^2) = -2u + 2v\tan^{-1}\left(\frac{u}{v}\right) + u\log(u^2+v^2)$$

(b) Take the following limits of **E** and Φand explain why the result is what you expect.

(i) $\lim \frac{R}{L} \to 0$, (ii) $\lim \frac{R}{L} \to \infty$, where $R = \sqrt{x^2+y^2}$

Note: There are extra "infinities" due to the charge density at infinity. Where is ground?

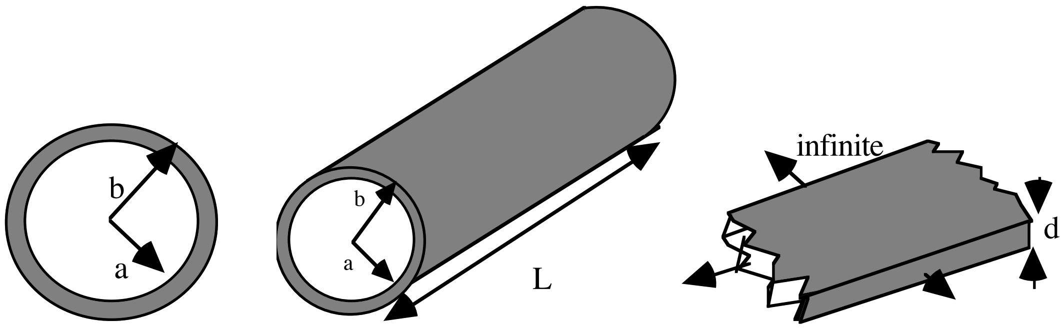
(c) Extra Credit 5 points: Plot the field and equipotential contours. Do this 3 times with different ranges for the plots: (i) x,y ~ L. (ii) x,y >>L. (iii) x,y<<L. Explain your Results

(d) Now suppose there are two strips, one with positive surface charge σ at y=0, and one with equal and opposite charge –σ, at y = –s Use the *principle of superposition* to find Φ, **E**

(e) Extra Credit 5 points: Again plot the contours and electric field. Do this for two cases (i) s<<L . (ii) s>>L Explain your results.

(3) Boundary condition at a surface charge (15 points)

(a) Each of the following geometries has a uniform charge density ρ. Find the electric field every in space. Sketch the plot of E as a function of the appropriate distance.



(i) Spherical shell, inner radius a, outer radius b,

(ii) Cylindrical shell, length $L>>a,b$

(iii) Infinite slab, *thickness* d

(b) Take the limit where thickness of each shell(slab) goes to zero, such that all of the charge is concentrated on a surface with density σ; i. e.,

(i) $b-a \to 0$
$\rho(b-a) \to \sigma$

(ii) $b-a \to 0$,
$\rho(b-a) \to \sigma$

(iii) $d \to 0$,
$\rho d \to \sigma$

Show that the electric field normal to the surface is discontinuous by $\Delta\mathbf{E}_{\perp} = 4\pi\sigma$