Physics 511
Electrodynamics
Problem Set #4: DUE Thursday Feb. 27

Problem 1: Motion of Charged Particle in External Fields (15 points)

(a) Show that in the presence of a time independent electric and magnetic field, the general equation of motion for the velocity of a charged particle q with mass m is:

$$\frac{d^2\mathbf{v}}{dt^2} = -\omega_c^2\mathbf{v} + \frac{q^2}{m^2 c}(\mathbf{E}\times\mathbf{B}) + \frac{q^2}{m^2 c^2}(\mathbf{B}\cdot\mathbf{v})\mathbf{B},$$

where $\omega_c = \dfrac{qB}{mc}$ is the cyclotron frequency.

(Note that the equation of motion only depends on the charge to mass *ratio*).

(b) Assume that the magnetic field is in the z-direction and the electric field is in the x-direction and the initial velocity is in the x-y plane. Show that the general solution is:

$$\mathbf{v}(t) = \mathrm{v}_0\cos(\omega_c t + \phi)\hat{\mathbf{x}} + \mathrm{v}_0\sin(\omega_c t + \phi)\hat{\mathbf{y}} + c\frac{\mathbf{E}\times\mathbf{B}}{B^2},$$

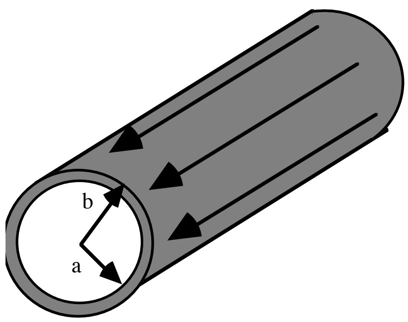
where v_0 and ϕ are constants that depend on the initial conditions outside the fields.

The first two terms are the "homogeneous solution" and represents the cyclotron motion The third term is the "particular solution", known as the "$\mathbf{E}\times\mathbf{B}$" drift.

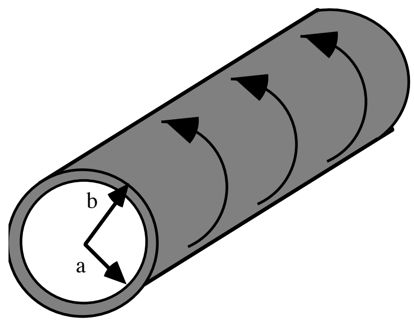
Sketch the trajectory of the particle.

Problem 2: Boundary Condition at a Surface Current (15 points)

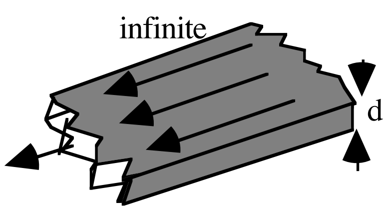
(a) Each of the following geometries has a uniform current density **J**. Find the magnetic field every in space. Sketch the plot of B as a function of the appropriate distance.



(ii) Infinite cylindrical shell,
current along axis



(ii)Infinite cylindrical shell
azimuthal current



(iii) Infinite slab,
thickness d

(b) Take the limit where thickness of each shell(slab) goes to zero, such that all of the charge is concentrated on a surface with current density K; i. e.,

(i) $b-a \to 0$
$J(b-a) \to K$

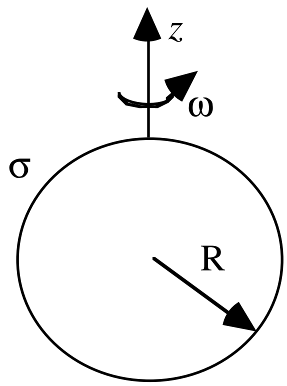
(ii) $b-a \to 0$,
$J(b-a) \to K$

(iii) $d \to 0$,
$Jd \to K$

Show that the magnetic field tangential to the surface is discontinuous by $\Delta \mathbf{B}_{\parallel} = \frac{4\pi}{c}\mathbf{K}$ where $\mathbf{B}_{\parallel} \equiv \hat{\mathbf{n}} \times \mathbf{B}$, with $\hat{\mathbf{n}}$ the normal to the surface.

Problem 3: Field of a Rotating Sphere (15 points)

A sphere of radius R carrying a uniform surface charge σ rotates with angular velocity ω about the z-axis.



(a) Use the Biot-Savart law to find the magnetic field at a point P, a distance z on the axis of rotation from the center of the sphere, both inside and outside the sphere.

(b) Find the vector potential everywhere in space and from it, find **B** everywhere.
Hint: Use the "Addition theorem" for spherical harmonics:

$$\frac{1}{|\mathbf{x}-\mathbf{x}'|}=\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\frac{4\pi}{2l+1}\left(\frac{r_<^l}{r_>^{l+1}}\right)Y_{l,m}^*(\theta',\phi')Y_{l,m}(\theta,\phi) \;.$$

Describe the nature of the field and sketch it.

Problem 4: Magnetic Dipole Moment (15 points)

(a) Explicitly calculate the magnetic dipole moment of the rotating sphere in Problem 3

(b) Show that in the limit that the radius of the sphere goes to zero while the charge density increases to kept the magnetic moment constant, the magnetic field from Problem 3 is:

$$\mathbf{B}(\mathbf{x}) = \frac{\mathbf{3\hat{r}(\hat{r}\cdot m)-m}}{r^3} + \frac{8\pi}{3}\mathbf{m}\delta(\mathbf{x})$$

The last term is know as the "contact" field, and plays an important role in the hyperfine spectrum of atoms.

(c) If the mass M of the sphere is concentrated at the radius (i. e. a massive spherical shell of radius R) and the total charge on the sphere is q, what is the "gyromagnetic ratio" : the ratio of the magnetic moment to the angular momentum.

(The permanent magnetic moment of the electron due to its "spin" is a factor of two different from this model: a classical spinning sphere of charge)